Unit 1

Numbers and Algebra

Ghiyath Al-Din Ibn Masoud Al-Kashi :

He was an Arab scientist who had many investigations in mathematics :

- He had invented the decimal fraction.
- He put a theory concerning the sum of the natural numbers that are raised to the fourth power.
- He reached a very accurate rate for the approximate ratio (π) that nearly equates the accuracy of the calculators.

Ghiyath Al-Din Ibn Masoud Al-Kashi (1380 A.D. - 1436 A.D.)

Lesson 1 Repeated multiplication

We had known before in the set of integers that : $\boxed{3^4 = 3 \times 3 \times 3 \times 3}$ where we found that the number 3 has repeated 4 times in the multiplication operation and we read it as «3 to the power 4»

Also, in the set of rational $\mathbb{Q}$, we find that :

- $\boxed{\left(\frac{2}{3}\right)^3} = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \boxed{\frac{2^3}{3^3}} = \frac{8}{27}$
- $\boxed{\left(\frac{2}{3}\right)^4} = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \boxed{\frac{2^4}{3^4}} = \frac{16}{81}$

From the previous, we deduce that :

If $\frac{a}{b}$ is a rational number and n is a positive integer , then : $\left(\frac{a}{b}\right)^n = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}$ ... to n times

It is read as « $\frac{a}{b}$ to the power n » or «the n^{th} power of the number $\frac{a}{b}$ »

i.e. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

For example: • $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$ • $(0.7)^2 = \left(\frac{7}{10}\right)^2 = \frac{7^2}{10^2} = \frac{49}{100}$

Remark

If : $\frac{a}{b}$ is a rational number , then : $\left(\frac{a}{b}\right)^0 = 1$ where $a \neq 0$

For example: • $\left(\frac{1}{5}\right)^0 = 1$ • $\left(-\frac{3}{7}\right)^0 = 1$

Remark

If $\frac{a}{b}$ is a rational number , and m is a positive integer , then :

1. $\left(-\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^m$ when m is an even number.

For example: $\left(-\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

2. $\left(-\frac{a}{b}\right)^m = -\left(\frac{a}{b}\right)^m$ when m is an odd number.

For example: $\left(-\frac{1}{2}\right)^3 = -\left(\frac{1}{2}\right)^3 = -\frac{1}{8}$

Unit 1

Try by yourself

Find each of the following in its simplest form :

1) $\left(\frac{1}{5}\right)^2 = \ldots\ldots = \ldots\ldots$

2) $\left(-\frac{2}{3}\right)^3 = \ldots\ldots = \ldots\ldots = \ldots\ldots$

3) $\left(-\frac{4}{5}\right)^4 = \ldots\ldots = \ldots\ldots = \ldots\ldots$

4) $\left(1\frac{1}{2}\right)^4 = \left(\frac{\ldots\ldots}{\ldots\ldots}\right)^4 = \ldots\ldots = \ldots\ldots$

Example 1 **Find each of the following in the simplest form :**

1) $\left(\frac{2}{3}\right)^2 \times \frac{9}{4}$

2) $\left(-\frac{5}{4}\right)^2 \times \left(\frac{2}{5}\right)^4$

3) $\left(3\frac{1}{2}\right)^2 \div \left(-10\frac{1}{2}\right)$

4) $\left(-\frac{2}{5}\right)^2 \times \left(-\frac{5}{2}\right)^3 \times \left(\frac{1}{5}\right)^0$

Solution

1) $\left(\frac{2}{3}\right)^2 \times \frac{9}{4} = \frac{2^2}{3^2} \times \frac{9}{4} = \frac{4}{9} \times \frac{9}{4} = 1$

2) $\left(-\frac{5}{4}\right)^2 \times \left(\frac{2}{5}\right)^4 = \frac{5^2}{4^2} \times \frac{2^4}{5^4} = \frac{25}{16} \times \frac{16}{625} = \frac{1}{25}$

3) $\left(3\frac{1}{2}\right)^2 \div \left(-10\frac{1}{2}\right) = \left(\frac{7}{2}\right)^2 \div \left(-\frac{21}{2}\right) = \frac{7^2}{2^2} \times \left(-\frac{2}{21}\right) = \frac{49}{4} \times \left(-\frac{2}{21}\right) = -\frac{7}{6}$

4) $\left(-\frac{2}{5}\right)^2 \times \left(-\frac{5}{2}\right)^3 \times \left(\frac{1}{5}\right)^0 = \frac{2^2}{5^2} \times \left(-\frac{5^3}{2^3}\right) \times 1 = \frac{4}{25} \times \left(-\frac{125}{8}\right) = -\frac{5}{2}$

Try by yourself

Find in the simplest form : $\left(-\frac{3}{9}\right)^2 \times \left(\frac{9}{4}\right)^2 \times \left(\frac{81}{16}\right)^0$

....

....

Example 2 If $x = -\frac{1}{2}$, $y = \frac{1}{4}$ and $z = 4$, find the value of : $(x + y)^3 \times z^3$

Solution $(x + y)^3 \times z^3 = \left(-\frac{1}{2} + \frac{1}{4}\right)^3 \times 4^3 = \left(-\frac{2}{4} + \frac{1}{4}\right)^3 \times 4^3$

$= \left(-\frac{1}{4}\right)^3 \times 4^3 = -\frac{1^3}{4^3} \times 4^3 = -1$

Try by yourself

If $x = -\frac{2}{3}$, $y = \frac{1}{2}$ and $z = -\frac{4}{3}$, find the value of : $x^2 - y^2 z$

....

....

Lesson 2 Non-negative integer powers

The first law

From the definition of repeated multiplication , we know that :

$$\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^4$$

$$\boxed{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}} \quad \boxed{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}} = \left(\frac{2}{3}\right)^7$$

Notice that :

$3 + 4 = 7$

i.e. When multiplying the like bases , we add their powers (indices).

i.e. **If $\frac{a}{b}$ is a rational number , n and m are non-negative integers , then**

$$\left(\frac{a}{b}\right)^n \times \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n+m}$$

For example:

- $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)^{3+2} = \left(\frac{2}{5}\right)^5$
- $\left(-\frac{1}{2}\right)^4 \times \left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)^{4+3} = \left(-\frac{1}{2}\right)^7$

You can check the truth of the previous law by using the calculator as follows :

- $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^2 = \frac{32}{3125}$

- $\left(\frac{2}{5}\right)^5 = \frac{32}{3125}$

Example 1

Calculate each of the following , then put the result in its simplest form :

1 $\frac{2}{3} \times \left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3$ **2** $\left(-\frac{1}{3}\right)^3 \times \left(\frac{1}{3}\right)^2$ **3** $\frac{3}{4} \times \left(-\frac{3}{4}\right)^2$

Solution

1 $\frac{2}{3} \times \left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)^{1+2+3} = \left(\frac{2}{3}\right)^6 = \frac{2^6}{3^6} = \frac{64}{729}$

2 $(-\frac{1}{3})^3 \times (\frac{1}{3})^2 = -(\frac{1}{3})^3 \times (\frac{1}{3})^2$

$= -(\frac{1}{3})^5 = -\frac{1^5}{3^5} = -\frac{1}{243}$

Notice that :
$(-\frac{1}{3})^3 = -(\frac{1}{3})^3$
because the index is an odd number.

3 $\frac{3}{4} \times (-\frac{3}{4})^2 = \frac{3}{4} \times (\frac{3}{4})^2$

$= (\frac{3}{4})^3 = \frac{3^3}{4^3} = \frac{27}{64}$

Notice that :
$(-\frac{3}{4})^2 = (\frac{3}{4})^2$
because the index is an even number.

The Second law

From the definition of repeated multiplication , we know that :

$$(\frac{1}{3})^7 \div (\frac{1}{3})^4 = \frac{\cancel{\frac{1}{3}} \times \cancel{\frac{1}{3}} \times \cancel{\frac{1}{3}} \times \cancel{\frac{1}{3}} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}}{\cancel{\frac{1}{3}} \times \cancel{\frac{1}{3}} \times \cancel{\frac{1}{3}} \times \cancel{\frac{1}{3}}}$$

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = (\frac{1}{3})^3$$

Notice that :
$7 - 4 = 3$
i.e. When dividing like bases , we subtract their powers (indices)

i.e. **If $\frac{a}{b}$ is a rational number , where $\frac{a}{b} \neq 0$, n and m are non-negative numbers , $n \geq m$, then $(\frac{a}{b})^n \div (\frac{a}{b})^m = (\frac{a}{b})^{n-m}$**

For example: • $(\frac{3}{8})^5 \div (\frac{3}{8})^2 = (\frac{3}{8})^{5-2} = (\frac{3}{8})^3$

• $(-\frac{2}{7})^4 \div (-\frac{2}{7})^2 = (-\frac{2}{7})^{4-2} = (-\frac{2}{7})^2$

You can check the truth of the previous law by using the calculator as follows :

• $(\frac{3}{8})^5 \div (\frac{3}{8})^2 = \frac{27}{512}$

• $(\frac{3}{8})^3 = \frac{27}{512}$

Example 2

Calculate each of the following , then put the result in the simplest form :

1 $(\frac{4}{5})^2 \times (\frac{4}{5})^5 \div (\frac{4}{5})^4$

2 $\frac{2^5 \times 2^4}{2^6}$

Solution

1 $\left[\left(\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5\right] \div \left(\frac{4}{5}\right)^4 = \left(\frac{4}{5}\right)^{2+5} \div \left(\frac{4}{5}\right)^4 = \left(\frac{4}{5}\right)^7 \div \left(\frac{4}{5}\right)^4 = \left(\frac{4}{5}\right)^{7-4} = \left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3} = \frac{64}{125}$

2 $\frac{2^5 \times 2^4}{2^6} = \frac{2^{5+4}}{2^6} = \frac{2^9}{2^6} = 2^{9-6} = 2^3 = 8$

Try by yourself

Find each of the following in the simplest form :

1 $\left(\frac{1}{5}\right)^2 \times \left(\frac{1}{5}\right)^2 =$ ....

2 $\left(\frac{3}{7}\right)^8 \div \left(\frac{3}{7}\right)^6 =$ ....

3 $\left(-\frac{2}{3}\right)^5 \times \left(-\frac{2}{3}\right)^2 \div \left(-\frac{2}{3}\right)^6 =$ ....

4 $\left(-\frac{1}{4}\right)^7 \div \left(\frac{1}{4}\right)^6 \times \frac{1}{4} =$ ....

Remarks

- **From the repeated multiplication , notice that :**

$$\left(\frac{3}{4} \times \frac{5}{7}\right)^3 = \left(\frac{3}{4} \times \frac{5}{7}\right) \times \left(\frac{3}{4} \times \frac{5}{7}\right) \times \left(\frac{3}{4} \times \frac{5}{7}\right)$$
$$= \left(\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right) \times \left(\frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}\right) = \left(\frac{3}{4}\right)^3 \times \left(\frac{5}{7}\right)^3$$

i.e. **If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , n is a non-negative integer , then :**

$$\boxed{\left(\frac{a}{b} \times \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \times \left(\frac{c}{d}\right)^n}$$

- **From the repeated multiplication, notice that :**

$$\left(\frac{2}{3} \div \frac{5}{11}\right)^4 = \left(\frac{\frac{2}{3}}{\frac{5}{11}}\right)^4 = \frac{\frac{2}{3}}{\frac{5}{11}} \times \frac{\frac{2}{3}}{\frac{5}{11}} \times \frac{\frac{2}{3}}{\frac{5}{11}} \times \frac{\frac{2}{3}}{\frac{5}{11}}$$
$$= \frac{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}}{\frac{5}{11} \times \frac{5}{11} \times \frac{5}{11} \times \frac{5}{11}} = \left(\frac{2}{3}\right)^4 \div \left(\frac{5}{11}\right)^4$$

i.e. **If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , $\frac{c}{d} \neq 0$, n is a non-negative integer , then :**

$$\boxed{\left(\frac{a}{b} \div \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \div \left(\frac{c}{d}\right)^n} \quad (\text{where } \tfrac{c}{d} \neq 0)$$

Example 3

Find the result of each of the following in its simplest form :

1 $\left(\frac{xy}{z}\right)^2$ **2** $\left(\frac{2x}{3y}\right)^4$

Solution

1 $\left(\frac{xy}{z}\right)^2 = \frac{(xy)^2}{z^2} = \frac{x^2y^2}{z^2}$ **2** $\left(\frac{2x}{3y}\right)^4 = \frac{2^4x^4}{3^4y^4} = \frac{16x^4}{81y^4}$

The Third law

From the definition of repeated multiplication , we know that :

$$\left[\left(\frac{1}{3}\right)^3\right]^2 = \left(\frac{1}{3}\right)^3 \times \left(\frac{1}{3}\right)^3 = \left(\frac{1}{3}\right)^{3+3} = \left(\frac{1}{3}\right)^6$$

i.e. **If $\frac{a}{b}$ is a rational number , n and m are non-negative integers , then** $\left[\left(\frac{a}{b}\right)^n\right]^m = \left(\frac{a}{b}\right)^{n \times m}$

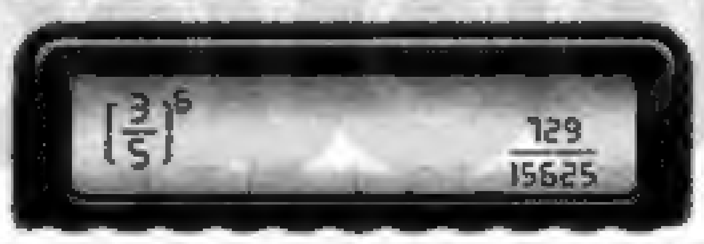
For example:

- $\left[\left(\frac{3}{5}\right)^3\right]^2 = \left(\frac{3}{5}\right)^{3\times2} = \left(\frac{3}{5}\right)^6$
- $\left[\left(-\frac{1}{2}\right)^4\right]^2 = \left(-\frac{1}{2}\right)^{4\times2} = \left(-\frac{1}{2}\right)^8$

You can check the truth of the previous law by using the calculator as follows :

- $\left[\left(\frac{3}{5}\right)^3\right]^2 = \frac{729}{15625}$

- $\left(\frac{3}{5}\right)^6 = \frac{729}{15625}$



Example 4

Calculate each of the following , then put the result in the simplest form :

1 $\left[\left(-2\frac{1}{2}\right)^2\right]^2$ **2** $\left(\frac{x^2}{y^3}\right)^3$ **3** $\frac{(-4x^3y^4)^2}{(-2xy^2)^4}$

Solution

1 $\left[\left(-2\frac{1}{2}\right)^2\right]^2 = \left(-2\frac{1}{2}\right)^{2\times2} = \left(-2\frac{1}{2}\right)^4 = \left(2\frac{1}{2}\right)^4 = \left(\frac{5}{2}\right)^4 = \frac{5^4}{2^4} = \frac{625}{16}$

2 $\left(\frac{x^2}{y^3}\right)^3 = \frac{(x^2)^3}{(y^3)^3} = \frac{x^{2\times3}}{y^{3\times3}} = \frac{x^6}{y^9}$

3 $\frac{(-4x^3y^4)^2}{(-2xy^2)^4} = \frac{(-4)^2 \times x^{3\times 2} \times y^{4\times 2}}{(-2)^4 \times x^4 \times y^{2\times 4}} = \frac{\cancel{16}x^6\cancel{y^8}}{\cancel{16}x^4\cancel{y^8}} = x^{6-4} = x^2$

Example 5

If $x = \frac{1}{2}$, $y = -\frac{3}{4}$ and $z = \frac{3}{2}$, find the numerical value of each of the following in the simplest form : **1** $(\frac{x^2}{z})^3$ **2** $(\frac{x^2 z}{y})^2$

Solution

1 $(\frac{x^2}{z})^3 = [(\frac{1}{2})^2 \div \frac{3}{2}]^3 = (\frac{1^2}{2^2} \times \frac{2}{3})^3$

$= (\frac{1}{4} \times \frac{2}{3})^3 = (\frac{1}{6})^3 = \frac{1^3}{6^3} = \frac{1}{216}$

Notice that : $\frac{x^2}{z} = x^2 \div z$

2 $(\frac{x^2 z}{y})^2 = \frac{x^{2\times 2}z^2}{y^2} = \frac{x^4 z^2}{y^2} = \frac{(\frac{1}{2})^4 \times (\frac{3}{2})^2}{(-\frac{3}{4})^2} = \frac{1^4}{2^4} \times \frac{\cancel{3^2}}{2^2} \times \frac{4^2}{\cancel{3^2}} = \frac{1}{\cancel{16}} \times \frac{\cancel{16}}{4} = \frac{1}{4}$

Try by yourself

Calculate each of the following , then put the result in the simplest form :

1 $[(\frac{1}{2})^2]^3$ **2** $(\frac{a^2 b^2}{c^3 d^4})^2$ **3** $(\frac{5^2 \times 5^4}{5^5})^2$

....
....
....
....

Unit 1

Lesson 3 Negative integer powers

Notic the table , then try to discover the pattern to complete it :

The power form	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
The value	32	16	8	4	2	1	....	....

÷2 ÷2 ÷2 ÷2 ÷2 ÷2 ÷2

From the previous pattern , we can deduce that :

$2^{-1} = 1 \div 2 = \frac{1}{2}$, $2^{-2} = \frac{1}{2} \div 2 = \frac{1}{4} = \frac{1}{2^2}$

Definition :

If a is a rational number , $a \neq 0$ and n is a positive integer , then $a^{-n} = \frac{1}{a^n}$ and $a^n = \frac{1}{a^{-n}}$

For example: • $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ • $\frac{2}{5^{-2}} = 2 \times 5^2 = 2 \times 25 = 50$

• $0.1 = \frac{1}{10} = 10^{-1}$, $0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$, ... and so on.

Remark

1 If a is a rational number , $a \neq 0$ and n is a positive integer , then :

$a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1$ (the multiplicative neutral)

i.e. each of a^n and a^{-n} is the multiplicative inverse of the other.

2 If $\frac{a}{b}$ is a rational number not equal to zero and n is a positive integer , then :

$\boxed{\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n}$ **For example:** $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Example 1 **Find the value of each of the following in the simplest form :**

1. 3×5^{-1}
2. $\frac{3}{7^{-1}}$
3. $\left(\frac{7^2}{7^{-2}}\right)^0$

Solution

1. $3 \times 5^{-1} = 3 \times \frac{1}{5} = \frac{3}{5}$
2. $\frac{3}{7^{-1}} = 3 \div 7^{-1} = 3 \div \frac{1}{7} = 3 \times 7 = 21$
3. $\left(\frac{7^2}{7^{-2}}\right)^0 = 1$

Example 2 Find each of the following in the simplest form :

1. $2^4 \times 2^{-2}$
2. $\frac{5^{-2}}{5^{-3}}$
3. $(3^2)^{-2}$
4. $\frac{6^{-3} \times 6^5}{6^2}$
5. $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2}$
6. $(7^3)^2 \times (7^{-2})^2$
7. $\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3}$

Solution

1. $2^4 \times 2^{-2} = 2^4 \times \frac{1}{2^2} = \frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4$
2. $\frac{5^{-2}}{5^{-3}} = \frac{5^3}{5^2} = 5^{3-2} = 5$
3. $(3^2)^{-2} = \frac{1}{(3^2)^2} = \frac{1}{3^4} = \frac{1}{81}$
4. $\frac{6^{-3} \times 6^5}{6^2} = \frac{6^5}{6^3 \times 6^2} = \frac{6^5}{6^5} = 1$
5. $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2} = \left(\frac{5^3 \times 5}{5^2 \times 5^4}\right)^{-2} = \left(\frac{5^4}{5^6}\right)^{-2} = \left(\frac{5^6}{5^4}\right)^2 = (5^{6-4})^2 = (5^2)^2 = 5^4 = 625$
6. $(7^3)^2 \times (7^{-2})^2 = (7^3)^2 \times \left(\frac{1}{7^2}\right)^2 = 7^6 \times \frac{1}{7^4} = 7^{6-4} = 7^2 = 49$
7. $\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{3}\right)^3 \div \left(\frac{5}{4}\right)^3 = \left(\frac{5}{3} \div \frac{5}{4}\right)^3 = \left(\frac{\cancel{5}}{3} \times \frac{4}{\cancel{5}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{4^3}{3^3} = \frac{64}{27}$

Remark

All laws of powers that we have studied in the previous lesson are correct in the case of the negative powers. So , the previous example can be solved by using laws of powers as follows :

1. $2^4 \times 2^{-2} = 2^{4+(-2)} = 2^2 = 4$
2. $\frac{5^{-2}}{5^{-3}} = 5^{-2-(-3)} = 5^{-2+3} = 5$
3. $(3^2)^{-2} = 3^{2\times(-2)} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$
4. $\frac{6^{-3} \times 6^5}{6^2} = 6^{-3+5-2} = 6^0 = 1$
5. $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2} = (5^{3+(-2)-(-1)-4})^{-2} = (5^{3-2+1-4})^{-2}$
   $= (5^{-2})^{-2} = 5^{(-2)\times(-2)} = 5^4 = 625$
6. $(7^3)^2 \times (7^{-2})^2 = (7^3 \times 7^{-2})^2 = (7^{3+(-2)})^2 = 7^2 = 49$
7. $\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3} = \left(\frac{3}{5} \div \frac{4}{5}\right)^{-3} = \left(\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{4}\right)^{-3} = \left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3 = \frac{4^3}{3^3} = \frac{64}{27}$

Unit 1

Try by yourself

Find the value of each of the following in the simplest form :

1. 5^{-3} = ...
2. $\left(\frac{3}{7}\right)^{-2}$ = ...
3. $(2^{-3})^2$ = ...
4. $\left(\frac{2^{-2} \times 2^6}{2^3}\right)^{-3}$ = ...

Example 3 Simplify each of the following to the simplest form where $x \neq 0$:

1. $x^5 \times x^{-2} \times x^{-3}$
2. $(x^2)^{-3} \div (x^{-1})^2$
3. $\left(\frac{x^4 \times x^{-3}}{x^{-4} \times x}\right)^{-2}$

Solution

1. $x^5 \times x^{-2} \times x^{-3} = x^{5+(-2)+(-3)} = x^{5-2-3} = x^0 = 1$
2. $(x^2)^{-3} \div (x^{-1})^2 = x^{-6} \div x^{-2} = x^{-6-(-2)} = x^{-6+2} = x^{-4} = \frac{1}{x^4}$
3. $\left(\frac{x^4 \times x^{-3}}{x^{-4} \times x}\right)^{-2} = (x^{4+(-3)-(-4)-1})^{-2} = (x^{4-3+4-1})^{-2}$
   $= (x^4)^{-2} = x^{-8} = \frac{1}{x^8}$

Try by yourself

Simplify each of the following to the simplest form putting the result in positive integer power where the denominator doesn't equal zero :

1. $(x^{-2})^{-5}$ = ...
2. $\left(\frac{a^4}{a^{-3}}\right)^{-2}$ = ...
3. $(y^5 \times y^{-2})^3$ = ...

Lesson 4 Scientific notation of the rational number

Do you know ?

Neptune planet is far from the sun by 2 800 000 000 miles.
(the mile = 1.6 km. approximately).

The diameter length of a virus ≈ 0.00000000025 cm.

- The scientific notation of the number is one easy method to deal with the very great numbers or the very small numbers like the numbers mentioned in the previous figures , given that most calculators can show these numbers in the scientific notation.
- **Before explaining how to write the numbers in their scientific notation , we should notice the following :**

1 $10 = 10^1$, $100 = 10 \times 10 = 10^2$, $1000 = 10 \times 10 \times 10 = 10^3$ and so on

Hence we find that : $2000 = 2 \times 1000 = 2 \times 10^3$, $50\,000 = 5 \times 10\,000 = 5 \times 10^4$

2 $0.1 = \frac{1}{10} = 10^{-1}$, $0.01 = \frac{1}{100} = \frac{1}{10 \times 10} = 10^{-2}$,

$0.001 = \frac{1}{1\,000} = \frac{1}{10 \times 10 \times 10} = 10^{-3}$ and so on

Hence we find that :

$0.03 = \frac{3}{100} = \frac{3}{10 \times 10} = 3 \times 10^{-2}$,

$0.0007 = \frac{7}{10\,000} = \frac{7}{10 \times 10 \times 10 \times 10} = 7 \times 10^{-4}$

The standard scientific notation of a number :

The number is written in the standard form as :

$$\boxed{a \times 10^n \text{ where } 1 \le |a| < 10 \text{ and } n \in \mathbb{Z}}$$

For example:

Each of the following two numbers is written in its standard form :

4.6×10^8 : 4.6 → a number included between 1 and 10 ; 10^8 → a number expresses a power of 10

5.236×10^{-6} : 5.236 → a number included between 1 and 10 ; 10^{-6} → a number expresses a power of 10

Unit 1

Example 1 **Put each of the following two numbers in the standard form :**

1. 8 200 000 000
2. 0.000000135

Solution

1. $8\,200\,000\,000.0 = 8.2 \times 10^9$

Moving the decimal point $\boxed{9}$ places towards $\boxed{\text{left}}$	Using the power $\boxed{9}$ of the number 10

2. $0.000\,000\,135 = 1.35 \times 10^{-7}$

Moving the decimal point $\boxed{7}$ places towards $\boxed{\text{right}}$	Using the power $\boxed{-7}$ of the number 10

Try by yourself

Write each of the following two numbers in the standard form :

1. 650 000 000 = ...
2. 0.00000102 = ...

Remarks

- Notice that the number 32.4×10^5 is not in the standard form because $32.4 > 10$ and to write it in the standard form , we move the decimal point one place towards left and multiply by 10
  i.e. $32.4 \times 10^5 = 3.24 \times 10^5 \times 10 = \boxed{3.24 \times 10^6}$ (the standard form)
- Notice that the number 0.032×10^{-4} is not in the standard form because $0.032 < 1$ and to write it in the standard form , we move the decimal point two places towards right and multiply by 10^{-2}
  i.e. $0.032 \times 10^{-4} = 3.2 \times 10^{-4} \times 10^{-2} = \boxed{3.2 \times 10^{-6}}$ (the standard form)
- Notice that the standard form of the number 1 is 1×10^0 , also the number 2 is 2×10^0 , and so on ...

Example 2 **Write each of the following numbers in the standard form :**

1. 45×10^8
2. 706.4×10^5

3 0.248×10^{-7}

4 -0.0015×10^{-9}

Solution

1 $45 \times 10^8 = 4.5 \times 10 \times 10^8 = 4.5 \times 10^9$

2 $706.4 \times 10^5 = 7.064 \times 10^2 \times 10^5 = 7.064 \times 10^7$

3 $0.248 \times 10^{-7} = 2.48 \times 10^{-1} \times 10^{-7} = 2.48 \times 10^{-8}$

4 $-0.0015 \times 10^{-9} = -1.5 \times 10^{-3} \times 10^{-9} = -1.5 \times 10^{-12}$

Try by yourself

In the following , determine the numbers that are not in the standard form , then write them in the standard form :

1 8.5×10^{-4} 2 17×10^8 3 0.5×10^{-7}

4 530.5×10^9 5 -0.999×10^{-5} 6 6×10^6

....

....

....

....

Example 3 (connecting with georaphy)

The following table shows the areas of the world Oceans. Arrange these areas descendingly.

The Ocean	Indian	Arctic	Pacific	Antarctic	Atlantic
The area (in k.m.^{2})	7.35×10^7	1.4×10^7	1.66×10^8	7.6×10^6	8.65×10^7

Solution

Pacific : 1.66×10^8 > Indian : 7.35×10^7, Arctic : 1.4×10^7, Atlantic : 8.65×10^7 > Antarctic : 7.6×10^6

$\because 8.65 > 7.35 > 1.4$

$\therefore 8.65 \times 10^7 > 7.35 \times 10^7 > 1.4 \times 10^7$

$\therefore$ Atlantic > Indian > Arctic

$\therefore$ The descendingly arrangement : pacific > Atlantic > Indian > Arctic > Antarctic

Example 4

Write the result of each of the following in the standard form :

1. $(1.2 \times 10^5) \times (4 \times 10^3)$
2. $(6.5 \times 10^4) \times (8 \times 10^2)$
3. $(2.4 \times 10^{11}) \div (1.2 \times 10^{-4})$
4. $(6.6 \times 10^7) \times (3 \times 10)^4$
5. $(2.3 \times 10^6) + (3.7 \times 10^5)$

Solution

1. $(1.2 \times 10^5) \times (4 \times 10^3) = (1.2 \times 4) \times (10^5 \times 10^3) = 4.8 \times 10^8$
2. $(6.5 \times 10^4) \times (8 \times 10^2) = (6.5 \times 8) \times (10^4 \times 10^2)$
   $= 52 \times 10^6$
   $= 5.2 \times 10^7$

   Notice that :
   52×10^6 is not in the standard form , then we should put it in the standard form.
3. $(2.4 \times 10^{11}) \div (1.2 \times 10^{-4}) = \frac{2.4}{1.2} \times \frac{10^{11}}{10^{-4}} = 2 \times 10^{15}$
4. $(6.6 \times 10^7) \times (3 \times 10)^4 = (6.6 \times 10^7) \times (3^4 \times 10^4) = (6.6 \times 3^4) \times (10^7 \times 10^4)$
   $= 534.6 \times 10^{11} = 5.346 \times 10^{13}$
5. $(2.3 \times 10^6) + (3.7 \times 10^5) = 10^5 (2.3 \times 10 + 3.7) = 10^5 (23 + 3.7)$
   $= 10^5 \times 26.7 = 2.67 \times 10^6$

Example 5

Write the result of each of the following in the standard form :

1. $30\,000 \times 400\,000$
2. $140\,000 \times 0.005$
3. $0.000015 \div 30$
4. $(50\,000)^3$
5. $(0.0003)^5$
6. $(-0.001)^6$

Solution

1. $30\,000 \times 400\,000 = (3 \times 10^4) \times (4 \times 10^5) = (3 \times 4) \times (10^4 \times 10^5)$
   $= 12 \times 10^9 = 1.2 \times 10^{10}$
2. $140\,000 \times 0.005 = (1.4 \times 10^5) \times (5 \times 10^{-3}) = (1.4 \times 5) \times (10^5 \times 10^{-3}) = 7 \times 10^2$
3. $0.000015 \div 30 = (1.5 \times 10^{-5}) \div 3 \times 10 = \frac{1.5}{3} \times \frac{10^{-5}}{10} = 0.5 \times 10^{-6} = 5 \times 10^{-7}$

4 $(50\ 000)^3 = (5 \times 10^4)^3 = 5^3 \times 10^{12} = 125 \times 10^{12} = 1.25 \times 10^{14}$

5 $(0.0003)^5 = (3 \times 10^{-4})^5 = 3^5 \times 10^{-20} = 243 \times 10^{-20} = 2.43 \times 10^{-18}$

6 $(-0.001)^6 = (0.001)^6 = (1 \times 10^{-3})^6 = 1^6 \times 10^{-18} = 10^{-18}$

Try by yourself

Write the result of each of the following in the standard form :

1 $(5.3 \times 10^7) \times (3 \times 10^5)$

2 $0.0006 \div 20$

3 $(400\ 000)^2$

4 $(3.2 \times 10^9) - (0.2 \times 10^8)$

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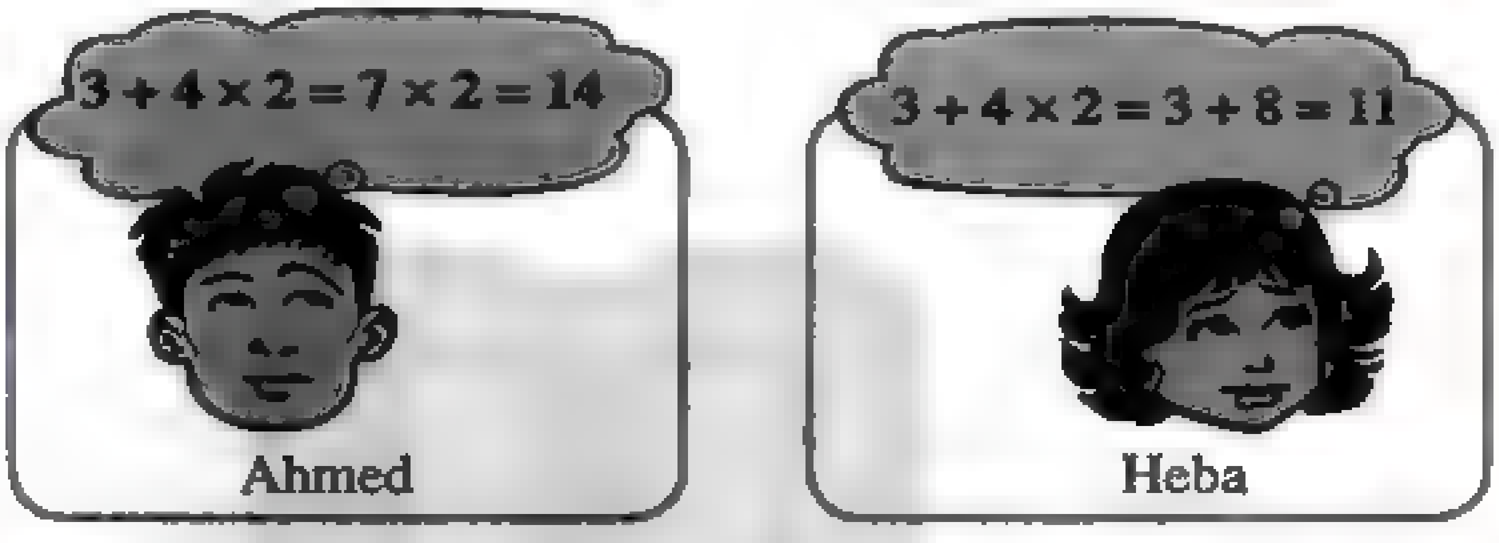
Lesson 5 Order of mathematical operations

- The following problem was given to each of Ahmed and Heba.

Calculate : $3 + 4 \times 2$

Their answers were as follows :



It is clear that each of them followed a different way to solve this problem and that led to the difference in the two answers :

- Ahmed performed the addition operation first , then he performed the multiplication operation.
- Heba multiplied first , then she added the two numbers.

In order to avoid this difference in these problems , it is agreed about some rules that determine the order of performing the mathematical operations for us , that are :

First : **Order of performing the mathematical operations in an expression has no parentheses**

1. Evaluate the powers.
2. Perform multiplications and divisions in order from left to right.
3. Perform additions and subtractions in order from left to right.

Second : **Order of performing the mathematical operations in an expression has parentheses**

1. Perform the operations within parentheses (interior parentheses then exterior ones).
2. Evaluate the powers.
3. Perform multiplications and divisions in order from left to right.
4. Perform additions and subtractions in order from left to right.

- According to these rules , we can determine that Heba held the correct answer because she performed the multiplication operation first , then the addition operation.

Remark

The scientific calculators and computers follow the same rules of ordering the operations.

For example , we can use the calculator in the previous problem by clicking the following buttons successively from the left and note the result.

[3] [+] [4] [×] [2] [=]

Example (1)

Calculate the value of each of the following :

1 $3 + 6 \times (5 + 4) \div 3 - 7$ **2** $9 - 5 \div (8 - 3) \times 2 + 6$

Solution

1 $3 + 6 \times (5 + 4) \div 3 - 7 = 3 + 6 \times 9 \div 3 - 7$ (parentheses)
$= 3 + 54 \div 3 - 7$ (multiplication)
$= 3 + 18 - 7$ (division)
$= 21 - 7$ (addition)
$= 14$ (subtraction)

2 $9 - 5 \div (8 - 3) \times 2 + 6 = 9 - 5 \div 5 \times 2 + 6$ (parentheses)
$= 9 - 1 \times 2 + 6$ (division)
$= 9 - 2 + 6$ (multiplication)
$= 7 + 6$ (subtraction)
$= 13$ (addition)

Example (2)

Calculate the value of each of the following :

1 $4 - 3\,[4 - 2\,(6 - 3)] \div 2$ **2** $16 \div [8 - 3\,(4 - 2)] + 1$

Solution

1 $4 - 3\,[4 - 2\,(6 - 3)] \div 2 = 4 - 3\,[4 - 2 \times 3] \div 2$ (the interior parentheses)
$= 4 - 3\,[4 - 6] \div 2$ (multiplication inside parentheses)
$= 4 - 3\,[-2] \div 2$ (subtraction inside parentheses)
$= 4 + 6 \div 2$ (multiplication by parentheses)
$= 4 + 3$ (division)
$= 7$ (addition)

2 $16 \div [8-3(4-2)]+1 = 16 \div [8-3\times 2]+1$ (the interior parentheses)

$= 16 \div [8-6]+1$ (multiplication inside parentheses)

$= 16 \div 2 + 1$ (subtraction inside the parentheses)

$= 8+1$ (division)

$= 9$ (addition)

Example 3

Calculate the value of each of the following :

1 $8\times 2^2 - 7\times(4+1)$

2 $2+3[5+(4-1)^2]$

3 $3[(3^2+1)-(2^3-2)]$

Solution

1 $8\times 2^2-7\times(4+1) = 8\times 2^2 - 7\times 5$ (addition inside parentheses)

$= 8\times 4 - 7\times 5$ (powers)

$= 32-35$ (multiplication)

$= -3$ (subtraction)

2 $2+3[5+(4-1)^2] = 2+3[5+3^2]$ (subtraction inside interior parentheses)

$= 2+3[5+9]$ (powers inside parentheses)

$= 2+3\times 14$ (addition inside parentheses)

$= 2+42$ (multiplication)

$= 44$ (addition)

3 $3[(3^2+1)-(2^3-2)] = 3[(9+1)-(8-2)]$ (powers)

$= 3[10-6]$ (the interior parentheses)

$= 3\times 4$ (subtraction inside parentheses)

$= 12$ (multiplication)

Remark

In the problems containing fractions , we should perform the operations in the numerator and denominator before division.

Example 4

Calculate the value of each of the following :

1. $\frac{36-6}{3+12}$
2. $\frac{11-(5-4)}{5^2-10\times 2}$
3. $7+8\div\frac{4+12-2}{3^2-2}-(2^3+2)$

Solution

1. $\frac{36-6}{3+12}=\frac{30}{15}=2$
2. $\frac{11-(5-4)}{5^2-10\times 2}=\frac{11-1}{25-20}=\frac{10}{5}=2$
3. $7+8\div\frac{4+12-2}{3^2-2}-(2^3+2)=7+8\div\frac{14}{7}-(2^3+2)$

$=7+8\div 2-10=7+4-10=1$

Try by yourself

Calculate the value of each of the following :

1. $20\div(12-2)\times 3^2-2$
2. $\frac{6\times 3+10\div 5}{2-(10-2^2)}$

....

....

....

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Unit 1

Lesson 6 The square root of a perfect square rational number

- **We know that :** $3^2 = 3 \times 3 = 9$ and it is read as the square of the number 3 is 9

And the square root is the inverse operation of finding the square of the number.

For example: To find the square root of the number 36 , we search for a number whose square is 36

- **We find that this number is :** 6 (because : $6^2 = 36$) or -6 [because : $(-6)^2 = 36$]

i.e. The number 36 has two square roots which are ± 6

- 6 is the positive square root of 36 and it is written as : $\sqrt{36} = 6$
- -6 is the negative square root of 36 and it is written as : $-\sqrt{36} = -6$

From the previous , we can deduce the following definition :

Definition :

The square root of the perfect square rational number «a» is the number whose square equals «a».

Generally :

Notice that : $\sqrt{\text{Zero}} = \text{Zero}$

- **The positive square root of the number a is symbolized by** $\sqrt{a}$

For example: The positive square root of 25 is $\sqrt{25} = 5$

- **The negative square root of the number a is symbolized by** $-\sqrt{a}$

For example: The negative square root of 16 is $-\sqrt{16} = -4$

- **The two square roots of the number a is symbolized by** $\pm\sqrt{a}$ **, and each of them is the additive inverse of the other.**

For example: The two square roots of 49 are $\pm\sqrt{49} = \pm 7$

Remarks

1. It is meaningless to find $\sqrt{a}$ if a is a negative rational number because there is no rational number if it is multiplied by itself , the result will be negative.
2. $\sqrt{\left(\frac{a}{b}\right)^2} = \left|\frac{a}{b}\right|$

   For example: • $\sqrt{(-3)^2} = |-3| = 3$ • $\sqrt{\left(-\frac{4}{5}\right)^2} = \left|-\frac{4}{5}\right| = \frac{4}{5}$
3. $\sqrt{a^2 b^2} = \sqrt{(ab)^2} = |ab|$ **For example:** $\sqrt{a^4 b^6} = \sqrt{(a^2 b^3)^2} = |a^2 b^3|$
4. If : $x^2 = a$ "where a is a perfect square rational number" , then : $x = \pm\sqrt{a}$

Example 1

Find each of the following in the simplest form :

1. $\sqrt{36}$
2. $-\sqrt{\frac{16}{25}}$
3. $\pm\sqrt{2\frac{1}{4}}$
4. $\sqrt{(-\frac{2}{7})^2}$
5. $-\sqrt{0.25}$
6. $\pm\sqrt{\frac{3.6}{10}}$
7. $\sqrt{16+9}$
8. $\sqrt{100-36}$
9. $\sqrt{\frac{36\,a^8}{49\,d^4}}$

Solution

1. $\sqrt{36}=6$ because $6^2=36$
2. $-\sqrt{\frac{16}{25}}=-\frac{4}{5}$ because $(\frac{4}{5})^2=\frac{16}{25}$
3. $\pm\sqrt{2\frac{1}{4}}=\pm\sqrt{\frac{9}{4}}=\pm\frac{3}{2}$
4. $\sqrt{(-\frac{2}{7})^2}=|-\frac{2}{7}|=\frac{2}{7}$
5. $-\sqrt{0.25}=-\sqrt{\frac{25}{100}}=-\frac{5}{10}=-\frac{1}{2}$
6. $\pm\sqrt{\frac{3.6}{10}}=\pm\sqrt{\frac{36}{100}}=\pm\frac{6}{10}=\pm\frac{3}{5}$
7. $\sqrt{16+9}=\sqrt{25}=5$
8. $\sqrt{100-36}=\sqrt{64}=8$
9. $\sqrt{\frac{36\,a^8}{49\,d^4}}=\frac{6\,a^4}{7\,d^2}$

Notice that :

When there is an addition or a subtraction operation under the square root , it must be performed first before finding the square root.

Try by yourself

Complete the following :

1. $\sqrt{64}=$ ....
2. $-\sqrt{900}=$ ....
3. $-\sqrt{\frac{36}{25}}=$ ....
4. $\pm\sqrt{6\frac{1}{4}}=$ ....
5. $\sqrt{0.64}=$ ....
6. $\sqrt{100-64}=$ ....

Remarks

If the operation of finding the square root of a number directly is difficult , then we factorize this number into its prime factors , then we take one factor from each two equal factors , then the product of these taken factors is the square root of this number.

Example 2 Find : $\sqrt{441}$

Solution

$\because 441 = \underbrace{3 \times 3}_{\downarrow} \times \underbrace{7 \times 7}_{\downarrow}$

$\therefore \sqrt{441} = 3 \times 7$

$= 21$

③	3	441
	3	147
⑦	7	49
	7	7
		1

Example 3

Simplify each of the following to the simplest form :

1. $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2$
2. $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0$
3. $\left(2\,\frac{7}{9}\right)^2 \div \sqrt{\frac{25}{9}}$

Solution

1. $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2 = -\frac{2}{7} \times \frac{7}{2} \times \frac{4}{49} = -\frac{4}{49}$
2. $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$
3. $\left(2\,\frac{7}{9}\right)^2 \div \sqrt{\frac{25}{9}} = \left(\frac{25}{9}\right)^2 \div \frac{5}{3} = \left(\left(\frac{5}{3}\right)^2\right)^2 \div \frac{5}{3} = \left(\frac{5}{3}\right)^4 \div \frac{5}{3} = \left(\frac{5}{3}\right)^{4-1} = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$

Try by yourself

Simplify each of the following to the simplest form :

1. $\left(\frac{2}{3}\right)^2 \times \sqrt{\frac{81}{16}} \times \left(\frac{7}{9}\right)^0 =$ ....
2. $\frac{5}{7} \times \sqrt{\frac{49}{36}} \div \left(-\frac{5}{3}\right)^2 =$ ....

Example 4

A square whose area equals the area of the triangle whose base length is 16 cm. and its corresponding height is 8 cm.

Find the side length of the square.

Solution

$\because$ The area of the triangle = $\frac{1}{2}$ of the base length × its corresponding height.

$\therefore$ The area of the triangle = $\frac{1}{2} \times 16 \times 8 = 64 \text{ cm}^2$.

$\therefore$ The area of the square = 64 cm^2.

$\therefore$ The side length of the square = $\sqrt{64} = 8$ cm.

Try by yourself

The area of a square is 1.44 cm^2.

Find its perimeter.

Unit 1

Lesson 7 Solving equations of the first degree in one unknown in $\mathbb{Q}$

Prelude

The equation is a mathematical statement which contains one variable as x (or more as x and y) and contains equality relation « = »

as : $2x = 6$, $x + 3 = 5$, $2x - y = 3$ and $x^2 = 25$

The degree of the equation is determined by the heighest degree of the terms forming the equation.

For example:

- $5x + 2 = 7$ is an equation of the first degree in one unknown x
- $x^2 + x - 3 = 0$ is an equation of the second degree in one unknown x
- $2x + 3y = 5$ is an equation of the first degree in two unknowns x and y

The substituting set : is the set that contains the probable values of the unknown.

The solution set (the S.S.) : is the set whose elements satisfy the equality of the equation and it is a subset of the substitution set.

For example:

- If $x + 3 = 5$ and the substitution set is $\{2 , 3\}$

- Putting $x = 2$, we get that the left side $= 2 + 3 = 5 =$ the right side.

  i.e. $x = 2$ is a solution to the equation.

- Putting $x = 3$, we get that the left side $= 3 + 3 = 6 \neq$ the right side.

  i.e. $x = 3$ is not a solution to the equation.

  $\therefore$ The S.S. $= \{2\}$ and it is a subset of the substitution set $\{2 , 3\}$

The previous method for solving the equation is called substitution method and we notice that it is a long way and it may be impossible if the number of elements of the substitution set is infinite as we see in the natural numbers set $\mathbb{N}$ and the integers set $\mathbb{Z}$ and the rational numbers set $\mathbb{Q}$

Therefore , we will use another easier method that will need studying the properties of the equality relation to enable us to make the unknown x in one side of the equation alone.

The properties of the equality relation

- We can add any rational number to both sides of the equation.

For example: If $x - 1 = 5$, then $x - 1 \underline{+1} = 5 \underline{+1}$

i.e. $x = 6$

- We can subtract any rational number from both sides of the equation.

For example: If $x + 3 = 2$, then $x + 3 \underline{-3} = 2 \underline{-3}$

i.e. $x = -1$

- We can multiply both sides of the equation by the same rational number.

For example: If $\frac{1}{5} x = 2$, then $\frac{1}{5} x \underline{\times 5} = 2 \underline{\times 5}$

i.e. $x = 10$

- We can divide both sides of the equation by the same rational number not equal to zero.

For example: If $7x = 14$, then $\frac{7x}{7} = \frac{14}{7}$

i.e. $x = 2$

Then by applying any of the previous properties in any equation , then we will get an equivalent equation to the origin equation that has the same solution.

Generally :

If a , b and c are three rational numbers , then these numbers have the following properties :

1. If $a = b$, then $a + c = b + c$
2. If $a + c = b + c$, then $a = b$
3. If $a = b$, then $a \times c = b \times c$
4. If $a \times c = b \times c$, $c \neq 0$, then $a = b$

The following examples show how to use the equality properties to solve an equation of the first degree in one unknown.

Example 1

Find the solution set of the equation $x + 5 = 4$ if the substitution set is :

1. $\mathbb{Z}$
2. $\mathbb{N}$

Solution

1 If the substitution set is $\mathbb{Z}$

$\because x+5=4$

Adding -5 to both sides

(-5 is the additive inverse of 5)

$\therefore x+5+(-5)=4+(-5)$

i.e. $x+0=-1$ *i.e.* $x=-1$

Another method :

You can imagine that 5 is moved from the left side to the right side and became -5

$x+5=4 \Rightarrow x=4-5$

You can check the truth of your solution by substituting by $x=-1$ in the origin equation , you will get the left side $=-1+5=4=$ the right side.

$\therefore$ The S.S. $=\{-1\}$

2 If the substitution set is $\mathbb{N}$

$\because x+5=4$

Subtracting 5 from both sides

$\therefore x+5-5=4-5$ $\therefore x=4-5$

$\because$ The subtraction $(4-5)$ is impossible in $\mathbb{N}$

$\therefore$ The S.S. in $\mathbb{N}$ is $\varnothing$

Example 2

Find the solution set of each of the following equations in $\mathbb{Q}$:

1 $2x-5=13$, **2** $2\frac{1}{2}-\frac{3}{2}x=5$

Solution

1 $\because 2x-5=13$

Adding 5 to both sides

(it is the additive inverse of (-5))

$\therefore 2x-5+5=13+5$

i.e. $2x=18$

Another method :

You can imagine that 2 is moved from the left side to the right side and it became divisor

$2x=18 \Rightarrow x=\frac{18}{2}$

Dividing both sides by 2 $\therefore \frac{2x}{2}=\frac{18}{2}$

$\therefore x=9$ $\therefore$ The S.S. $=\{9\}$

"Check the truth of the solution"

2 $\because 2\frac{1}{2} - \frac{3}{2}x = 5$ Subtracting $2\frac{1}{2}$ from both sides

$\therefore 2\frac{1}{2} - \frac{3}{2}x - 2\frac{1}{2} = 5 - 2\frac{1}{2}$ $\therefore -\frac{3}{2}x = 2\frac{1}{2}$

$\therefore -\frac{3}{2}x = \frac{5}{2}$

Multiplying both sides by $(-\frac{2}{3})$ (it is the multiplicative inverse of $-\frac{3}{2}$)

$\therefore -\frac{3}{2}x \times (-\frac{2}{3}) = \frac{5}{2} \times (-\frac{2}{3})$ $\therefore x = -\frac{5}{3}$

$\therefore$ The S.S. $= \{-\frac{5}{3}\}$ "Check the truth of the solution"

Example (3)

Find the S.S. of each of the following equations :

1 $2(x+3) = 4$, where $x \in \mathbb{Z}$ **2** $5(x+2) - 1 = 19$, where $x \in \mathbb{Q}$

Solution

1 $\because 2(x+3) = 4$ Dividing both sides by 2

$\therefore \frac{2(x+3)}{2} = \frac{4}{2}$ $\therefore x + 3 = 2$

Adding (-3) to both sides $\therefore x + 3 - 3 = 2 - 3$

$\therefore x = -1$ $\therefore$ The S.S. $= \{-1\}$

2 $\because 5(x+2) - 1 = 19$

Using the distribution property

$\therefore 5x + 10 - 1 = 19$

$\therefore 5x + 9 = 19$

Notice that :

$5(x+2) - 1 = 19$, $5x + 9 = 19$ and $5x = 10$ are equivalent equations.

Adding (-9) to both sides $\therefore 5x + 9 - 9 = 19 - 9$

$\therefore 5x = 10$ Dividing both sides by 5

$\therefore \frac{5x}{5} = \frac{10}{5}$ $\therefore x = 2$

$\therefore$ The S.S. $= \{2\}$

Unit 1

Example 4

Find in $\mathbb{Q}$ the solution set of each of the following equations :

1 $3x+4=2(x+1)$ **2** $2(x+3)-(x-2)=4(x-1)+3$

Solution

Notice that the variable (x) exists in the two sides , then we try to collect it in one side (say the left side)

1 $\because 3x+4=2(x+1)$

Using the distribution property

$\therefore 3x+4=2x+2$

Subtracting $2x$ from both sides

$\therefore 3x-2x+4=2x-2x+2$

$\therefore x+4=2$

Subtracting 4 from both sides

$\therefore x+4-4=2-4$ $\therefore x=-2$

$\therefore$ The S.S. $=\{-2\}$

Another method :

(-4)

$3x+4=2x+2$

$(-2x)$

$\therefore 3x-2x=2-4$

i.e. $x=-2$

2 $\because 2(x+3)-(x-2)=4(x-1)+3$

Using the distribution property

$\therefore 2x+6-x+2=4x-4+3$

$\therefore x+8=4x-1$

Subtracting x from both sides

$\therefore x-x+8=4x-x-1$ $\therefore 8=3x-1$

Adding 1 to both sides

$\therefore 8+1=3x-1+1$

$\therefore 9=3x$

Dividing both sides by 3

$\therefore \frac{9}{3}=\frac{3x}{3}$

$\therefore 3=x$

$\therefore$ The S.S. $=\{3\}$

Another method :

$(+1)$

$x+8=4x-1$

$(-x)$

$\therefore 8+1=4x-x$

i.e. $9=3x$

Try by yourself

Find the solution set of each of the following equations :

1 $x - 5 = 2$, where $x \in \mathbb{N}$

....

....

....

2 $2x + 11 = 3$, where $x \in \mathbb{Z}$

....

....

....

3 $2x - 3 = 5x + 6$, where $x \in \mathbb{Q}$

....

....

....

The solution of the equations can help us in solving some of our life problems and we will show some of these in the following examples :

Example 5

Two natural numbers , one of them is thrice of the other If the sum of them is 16, find the two numbers.

Solution

- We give one of the two number the symbol x
- Using the information given , we form a first degree equation in one unknown.

$\because$ The other number is thrice of the number x

$\therefore$ The other number $= 3x$

$\therefore$ The sum of the two numbers $= 16$

$\therefore$ The equation is $x + 3x = 16$

- We solve the equation we get to find the value of the unknown.

$\because x + 3x = 16$ $\qquad$ $\therefore 4x = 16$

Dividing by 4 $\qquad$ $\therefore x = 4$

i.e. one of the two numbers = 4 the other number = $3 \times 4 = 12$

- We make sure that the solution is right by using the problem itself , not by using the equation

$\because$ 12 is the thrice of 4 , $12 + 4 = 16$

$\therefore$ The solution is right.

Remarks for solving life problems :

- If a number = x , then its twice = $2x$ and its three times = $3x$, ...
- If a number = x and another number exceeds it by 5
  , then the other number = $x + 5$
- If a number = x and another number decreases than it by 5 , then the other number = $x - 5$
- If the age of a man now = x years , then :
  * His age after 3 years = $(x + 3)$ years.
  * His age 3 years ago = $(x - 3)$ years.
- Three consecutive integers are ; x , $x + 1$ and $x + 2$
- Three consecutive natural (even or odd) numbers are x , $x + 2$ and $x + 4$
- The perimeter of a rectangle = 2 (length + width)
- The perimeter of a square = side length × 4
- The perimeter of the triangle = the sum of its sides lengths
- The area of the triangle = $\frac{1}{2}$ the base length × the height.
- The sum of measures of the interior angles of the triangle = 180°

Example 6

Three natural consecutive odd numbers whose sum is 27 , find these numbers.

Solution

Let the smallest odd number = x

$\because$ Each odd number exceeds the odd number just before it by 2

$\therefore$ The next odd number = $x + 2$ and the third odd number = $x + 4$

$\because$ The sum of the numbers = 27

$\therefore x + (x + 2) + (x + 4) = 27$ $\qquad$ $\therefore 3x + 6 = 27$

$\therefore 3x = 27 - 6$ $\qquad$ $\therefore 3x = 21$

$\therefore x = \frac{21}{3}$ $\qquad$ $\therefore x = 7$

i.e. The numbers are 7 , 9 and 11

To check the solution : the numbers 7 , 9 and 11 are natural consecutive odd numbers , $7 + 9 + 11 = 27$ $\qquad$ $\therefore$ The solution is true.

Example 7

A rectangle with length equals twice its width and its perimeter = 18 cm.
Find the dimensions of the rectangle.

Solution

Let the width of the rectangle = x cm. $\therefore$ Its length = $2x$ cm.

$\because$ The perimeter of the rectangle = 2 (length + width)

$\therefore 18 = 2(2x + x)$ $\therefore 18 = 2 \times 3x$

$\therefore 18 = 6x$ $\therefore x = 3$

i.e. The width of the rectangle = 3 cm. and its length = 6 cm.

To check the solution :

$\because$ The length of the rectangle = 6 cm. equals twice its width 3 cm.
, The perimeter of the rectangle = 2 (6 + 3) = 2 × 9 = 18 cm.

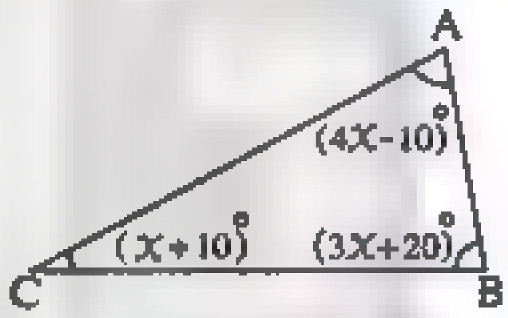
$\therefore$ The solution is true.

Example 8

In the opposite figure :

ABC is a triangle in which $m(\angle A) = (4x - 10)°$
, $m(\angle B) = (3x + 20)°$, $m(\angle C) = (x + 10)°$

Find the measures of the angles of the triangle.



Solution

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore (4x - 10) + (3x + 20) + (x + 10) = 180$

$\therefore 8x + 20 = 180$

Subtracting 20 from both sides

$\therefore 8x = 180 - 20$ $\therefore 8x = 160$

$\therefore x = \frac{160}{8}$ $\therefore x = 20$

$\therefore m(\angle A) = (4 \times 20) - 10 = 80 - 10 = 70°$

, $m(\angle B) = (3 \times 20) + 20 = 60 + 20 = 80°$

, $m(\angle C) = 20 + 10 = 30°$

To check the solution : $\because m(\angle A) + m(\angle B) + m(\angle C) = 70° + 80° + 30° = 180°$

$\therefore$ The solution is true.

Unit 1

Try by yourself

The difference between two integers is 4 and their sum is 14 Find the two numbers.

Solution

Let the small number = x

$\because$ The difference between the two numbers = 4 $\therefore$ The great number = x + ………

$\because$ Their sum is 14 $\therefore$ ……… + ……… = 14

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

Lesson 8 Solving inequalities in $\mathbb{Q}$

Prelude

- We had studied before some concepts as the substitution set and the solution set in equations which they are also the same concepts for inequalities.
- The solution set of the inequality is the set whose elements satisfy the inequality and it is a subset of the substitution set.

Illustrated example

Find the solution set of the inequality $x + 3 < 5$ if the substitution set is $\{-2 , -1 , 0 , 1 , 2 , 3\}$, then represent the S.S. on the number line.

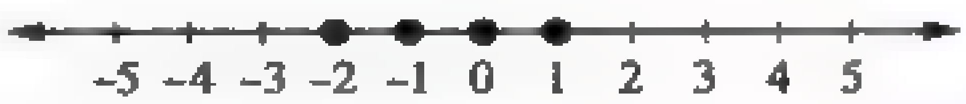
Solution

Substitute for the value of x by each element of the substitution set.

- At $x = -2$ $\quad\therefore$ The left side $= -2 + 3 = 1$
  $\because 1 < 5$ $\quad\therefore -2$ is a solution to the inequality
- At $x = -1$ $\quad\therefore$ The left side $= -1 + 3 = 2$
  $\because 2 < 5$ $\quad\therefore -1$ is a solution to the inequality.
- At $x = 0$ $\quad\therefore$ The left side $= 0 + 3 = 3$
  $\because 3 < 5$ $\quad\therefore 0$ is a solution to the inequality.
- At $x = 1$ $\quad\therefore$ The left side $= 1 + 3 = 4$
  $\because 4 < 5$ $\quad\therefore 1$ is a solution to the inequality.
- At $x = 2$ $\quad\therefore$ The left side $= 2 + 3 = 5$
  $\therefore 2$ is not a solution to the inequality.
- At $x = 3$ $\quad\therefore$ The left side $= 3 + 3 = 6$
  $\because 6 > 5$ $\quad\therefore 3$ is not a solution to the inequality.

From the previous :

$\therefore$ The solution set $= \{-2 , -1 , 0 , 1\}$



Remark

The substitution method which is followed in the previous example is a long method and difficult and it may be impossible if the substitution set is infinite. Therefore we use another easier method for the solution and that requires studying the properties of inequalities.

Unit 1

Properties of inequalities :

We know that $6 > -9$ is a true inequality.

But do the following operations lead to true inequalities ?

1 Add 2 to the two sides of the inequality

$\therefore 6 + 2 > -9 + 2 \longrightarrow 8 > -7$ (true inequality)

i.e. We can add a constant number to both sides of the inequality without change in the inequality relation.

2 Subtract 7 from the two sides of the inequality

$\therefore 6 - 7 > -9 - 7 \longrightarrow -1 > -16$ (true inequality)

i.e. We can subtract a constant number from the two sides of the inequality without change in the inequality relation.

3 Multiply the two sides of the inequality by 5 (positive number)

$\therefore 6 \times 5 > -9 \times 5 \longrightarrow 30 > -45$ (true inequality)

i.e. Multiplying the two sides of the inequality by a positive number does not change the inequality relation.

4 Divide the two sides of the inequality by 3 (positive number)

$\therefore \frac{6}{3} > -\frac{9}{3} \longrightarrow 2 > -3$ (true inequality)

i.e. Dividing the two sides of the inequality by a positive number does not change the inequality relation.

5 Multiply the two sides of the inequality by -1 (negative number)

$\therefore 6 \times (-1) > -9 \times (-1) \longrightarrow -6 > 9$ (false inequality) because $-6 < 9$

i.e. If we multiply the two sides of the inequality by a negative number , then we change the sign of the inequality to the opposite sign.

6 Divide the two sides of the inequality by -3 (negative number)

$\therefore \frac{6}{-3} > \frac{-9}{-3} \longrightarrow -2 > 3$ (false inequality) because $-2 < 3$

i.e. If we divide the two sides of the inequality by a negative number , then we change the inequality sign to the opposite sign.

We can summarize the properties of inequality that noticed before as follows:

Assuming that a , b , c are three rational numbers , then :

1. If $a < b$, then $a + c < b + c$
2. If $a < b$, then $a - c < b - c$
3. If $a < b$, c is a positive number , then $ac < bc$
4. If $a < b$, c is a positive number , then $\frac{a}{c} < \frac{b}{c}$
5. If $a < b$, c is a negative number , then $ac > bc$
6. If $a < b$, c is a negative number , then $\frac{a}{c} > \frac{b}{c}$

Remark

If a and b are two rational numbers not equal to zero and $a > b$, then : $\frac{1}{a} < \frac{1}{b}$

Example 1

Find the solution set of the inequality :

$x + 2 < 5$, where : **1** $x \in \mathbb{Z}$ **2** $x \in \mathbb{N}$

then represent the solution set on the number line in each case.

Solution

$\because x + 2 < 5$ Subtracting 2 from the two sides

$\therefore x + 2 - 2 < 5 - 2$ *i.e.* $x < 3$

1 When $x \in \mathbb{Z}$

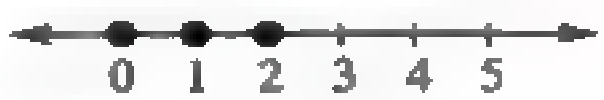
The solution set is all the integers which are less than 3

i.e. The S.S. = {2 , 1 , 0 , – 1 , ...}

2 When $x \in \mathbb{N}$

The solution set is all the natural numbers which are less than 3

i.e. The S.S. = {2 , 1 , 0}



We notice from the previous example that :

The solution set of the inequality depends on the substitution set, we find that :

The solution set in $\mathbb{N}$ differs from the solution set in $\mathbb{Z}$

Example 2

Find the solution set of the inequality $2x-5>5$, where :

1 $x \in \mathbb{Q}$

2 $x \in \mathbb{Z}$

Solution

$\because 2x-5>5$ — Adding 5 to both sides

$\therefore 2x-5+5>5+5$

$\therefore 2x>10$

Multiplying both sides by $\frac{1}{2}$

$\therefore \frac{1}{2} \times 2x > \frac{1}{2} \times 10$

i.e. $x>5$

1 When $x \in \mathbb{Q}$

The S.S. is all the rational numbers which are greater than 5 , then we write it by characterized property method because it is difficult to list all its members.

i.e. The S.S. $=\{x : x \in \mathbb{Q} , x>5\}$

2 When $x \in \mathbb{Z}$

The solution set is all the integers which are greater than 5

i.e. The S.S. $=\{6 , 7 , 8 , \cdots\}$

Example 3

Find in $\mathbb{Q}$ the solution set of each of the two following inequalities :

1 $4-2x \leq 2$

2 $7(x-1)>9x-6$

Solution

1 $\because 4-2x \leq 2$ — Adding -4 to both sides

$\therefore -4+4-2x \leq -4+2$

$\therefore -2x \leq -2$

Dividing both sides by (-2)

$\therefore \frac{-2x}{-2} \geq \frac{-2}{-2}$

Notice that: The change of inequality sign.

$\therefore x \geq 1$

i.e. The S.S. $=\{x : x \in \mathbb{Q} , x \geq 1\}$

2 $\because\ 7(x-1) > 9x-6$ $\qquad \therefore 7x-7 > 9x-6$

Subtracting $(9x)$ from both sides

$\therefore 7x-9x-7 > 9x-9x-6$ $\qquad \therefore -2x-7 > -6$

Adding 7 to both sides

$\therefore -2x-7+7 > -6+7$ $\qquad \therefore -2x > 1$

Dividing both sides by (-2)

$\therefore \frac{-2x}{-2} < \frac{1}{-2}$

Notice that: The change of inequality sign.

$\therefore x < -\frac{1}{2}$

i.e. The S.S. $= \{x : x \in \mathbb{Q} , x < -\frac{1}{2}\}$

Example 4

Find in $\mathbb{Z}$ the solution set of the inequality $-11 \le 3x-5 < 4$, then represent it on the number line.

Solution

$\because -11 \le 3x-5 < 4$

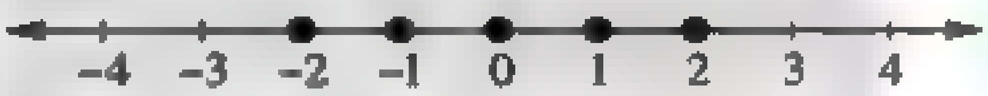
Adding 5 to the three sides

$\therefore -11+5 \le 3x-5+5 < 4+5$ $\qquad \therefore -6 \le 3x < 9$

Dividing all sides by 3

$\therefore \frac{-6}{3} \le \frac{3x}{3} < \frac{9}{3}$ $\qquad \therefore -2 \le x < 3$

i.e. The S.S. $= \{-2 , -1 , 0 , 1 , 2\}$



Try by yourself

Find the solution set of each of the following inequalities :

1 $2x-3 \ge 5$, where $x \in \mathbb{Q}$

....

....

....

....

2 $5x-10 < 2x-1$, where $x \in \mathbb{N}$

....

....

....

....

Unit 2

Statistics and Probability

Lesson One : Samples :

- Systematic sample.
- Random sample.

Lesson Two : Probability :

- Experimental probability.
- Theoretical probability.

Pierre Simon Marquis de Laplace (1749 - 1827)

He was a French mathematician and astronomer. His first work was published in 1771 starting with differential equations however he had already started to think about the mathematical and philosophical concepts of probability and statistics.

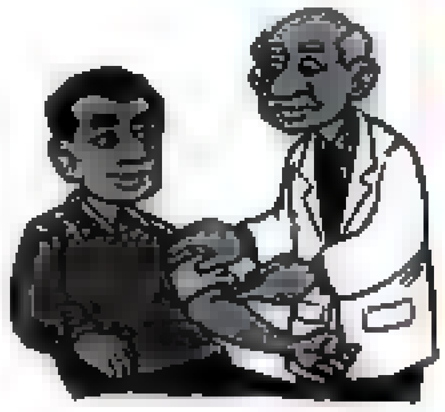
Pierre Simon Marquis de Laplace (1749 - 1827)

Lesson 1 Samples

Samples are greatly important for scientific and social studies and researches as when we study researches or different phenomena, these studies are not carried out on all the society but it is enough to study the results on samples representing the society under study.

For example :

- **In medical field :**



When we make blood analysis to a person, we take a sample of his blood to make the analysis for it , this sample is enough to make a true decision for the all of his blood.

- **In industrial field :**

When we check the production of a factory to know if its products are manufactured according to the certain specifications, we don't carry out this check to all the production of the factory but it is enough to check a sample of it on condition that this sample must represent the production of this factory completely, then we can generalize the results for all the production.

- **In the media field :**

When we do a survey to know which TV program is the most effective on the public opinion , we don't do the survey on all inhabitants but we choose a sample representing all inhabitants with different classes, then we generalize the results for all society.

Why are samples used ?

Samples are used for many reasons as :

1. Samples save time because if we try to check all the production of a factory, it will take a very long time.
2. Samples save money because the chosen produced units which are under check may be damaged and that leads to a great economic loss besides checking all the production needs a large number of researchers and a large number of laboratories.
3. Samples save efforts because checking all the production of a factory needs great efforts of a large number of researchers.

Unit 2

In this lesson , we will know the concept of the sample and its types and how it is chosen.

The concept of the sample :

A sample is a small part of a society. "society" means a collection , a set or a group of objects being studied and is selected randomly.

- From the previous , the selected sample should wholly represent the society (the object of study) and it shouldn't be based on a certain group and neglect the other , so that the results of the study can be near reality and we can make decisions according to these results , so we can generalise these results on the society as a whole.
- And the sample representing the society should be random.

- This sample can be classified according to the method of selecting its elements into :

1 Systematic sample :

Systematic sample is the sample whose elements are selected from the elements of a society distributed randomly by following a certain system or method in selection.

For example :

To select a systematic sample representing 10% of the marks of students in a preparatory school in the mid-year exam in maths , to study the standards of students , we will do as follows :

1. Students must be distributed randomly in a numbered list , *i.e.* selection shouldn't be from certain classes as excellent students' classes or selecting certain classes and neglecting others.
2. We select in a regular way the tenth student in each 10 student from the students list *i.e.* we select the mark of the tenth, twentieth, thirtieth, ... students is the list.

Remark

If the society (the object of study) is allready divided into classes or groups as the school is divided into classes for boys and others for girls , then we select a part of each group to represent the sample so that it can represent the society as a whole.

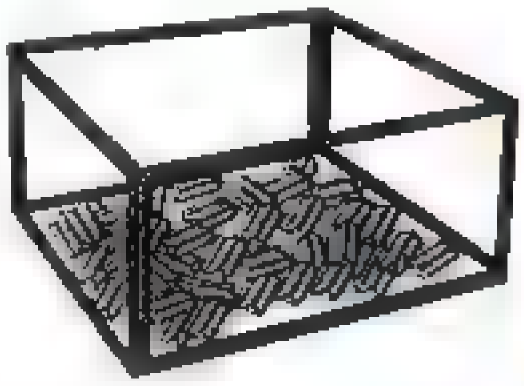
2 Random sample :

Random sample is the sample whose elements are selected from the elements of a society distributed randomly by following a random and irregular method or system of selecting.

- In this sample , each individual must get the same chance of selecting.

So , we can select its elements by two methods :

The first method (manual method) : It is done as follows :



1. Every member of the society is given a number , then this number is written on a piece of paper such that all pieces of paper are of the same colour and size.
2. Each piece of paper is folded perfectly such that the written number does not appear and is put in a bag or a box and mixed together very well.
3. The selection is carried out by drawing a piece after piece without looking inside the paper till the operation of drawing is finished when the required number of selected members is done.

The second method (using the scientific calculator) :

This method depends on using the random number function on the scientific calculator shown in the opposite figure by pressing the following keys successively from the left :

Then a random number in the range 0.000 to 0.999 will appear every time. Take the apparent numbers without the decimal point. The numbers which are greater than the whole number of the society under study should be ignored and , also if a number is repeated it must be ignored and taken once only.

In any survey , a 10% sample is considered adequate to provide reliable information about the whole society.

Example

A factory has 300 workers. The people in charge of the monthly magazine of this factory want to develop this magazine by doing a survey of a sample representing 10% of the total number of the workers in this factory. Show how the selection of this sample can be carried out using the calculator.

Unit 2

Solution

$\because$ The number of workers in the factory = 300 workers.

$\therefore$ The number of the random sample $= \frac{10}{100} \times 300 = 30$ workers.

Then we want to select 30 workers to do this survey. The selection operation can be carried out as follows :

1. Each worker in the factory is given a number from 1 to 300
2. Use the calculator to select 30 numbers randomly by the method mentioned before such that these numbers are included between 0 and 301 and the number that is more than 300 should be ignored.

For example :

By pressing the keys SHIFT → • → = successively.

- If we get the decimal 0.56 , then the number of the selected person is 56
- If we get the decimal 0.049 , then the number of the selected person is 49
- If we get the decimal 0.132 , then the number of the selected person is 132
- If we get the decimal 0.453 , it must be ignored because 453 is more than 300 and so on till we get 30 numbers.

Assuming that the calculator gave us the shown numbers in the opposite table , then the workers who carry these numbers are the selected sample to carry out this survey.

56	49	132	141	249	272
254	256	4	213	74	198
131	2	156	47	172	13
8	3	85	82	9	38
41	14	34	279	118	103

Remark

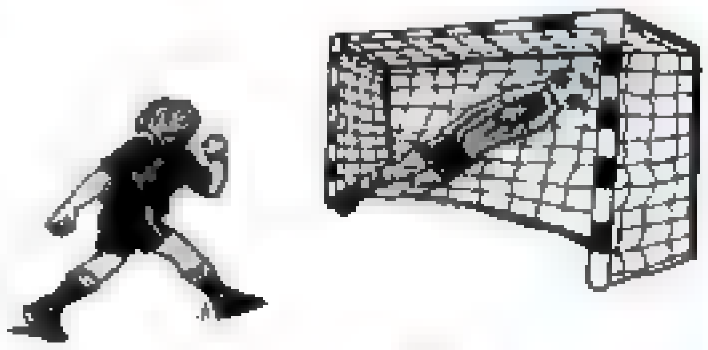
The random numbers can be generated by using «Excel» program that will be explained in details at the end of this book.

Lesson — Probability

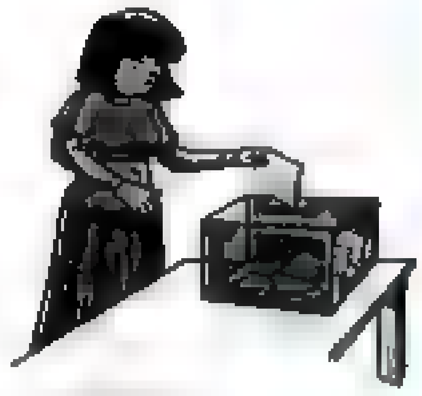
Prelude

In our daily life , many times we ask ourselves about some affairs that may happen in the future and that we cannot give a certain result for them.

For example :



- If the Egyptian football team get to the finals of African nations championship , what is its chance for winning the cup ?



- If an Egyptian citizen puts himself up for parliamentary elections in one of elections zones , what is his chance for winning in the elections ?

All these questions' answers are expectations to what may happen (occur) in the future referring to previous experience , studies or observations.

When we answer these questions , we use some words as «may be , chance or probable».

In mathematics , we call this probability. In this lesson , we will study : experimental probability , theoretical probability.

1 Experimental probability :

- If one of the Olympic swimmers wants to achieve a new record in the next Olympic Games , what is the probability that this swimmer achieves this record ?

The answer to this question cannot be got by expecting , hoping or by doing a survey of the opinions of the trainers or by asking the swimmer himself , but by trying.

i.e. The swimmer covers the needed distance in the race several times , then we record the number of times in which he could achieve the requested number and divide it by the total number of times , so the quotient is the probability of achieving the new record in the next Olympics.

i.e. The experimental probability depends on performing an experiment , then we record the results and use them to calculate the value of probability of an event occurrence using the rule :

$$\text{Experimental probability} = \frac{\text{Number of trials in which the outcome occurs}}{\text{Total number of trials}}$$

It is noticed that the more we carry out the experiment , the more we obtain an accurate value for the probability.

Example 1

If we tossed a piece of coin with double face 200 times and the results of appearance of a head or a tail in each toss were recorded in a table as shown :

H T

	Heads	Tails	Total
Statistics Tallies	𝍸 /	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ////	
Frequency	106	94	200

Calculate :

1. The probability of appearance of a head.
2. The probability of appearance of a tail.

Solution

1. The probability of appearance of a head $= \dfrac{\text{Number of getting heads}}{\text{Total number of tosses}} = \dfrac{106}{200} = 0.53$
2. The probability of appearance of a tail $= \dfrac{\text{Number of getting tails}}{\text{Total number of tosses}} = \dfrac{94}{200} = 0.47$

Try by yourself

A fair die is rolled 25 times. The results of appearance of a number on the upper face were recorded in the following table :

	1	2	3	4	5	6	Total
Statistics tallies							
Frequency							25

Calculate :

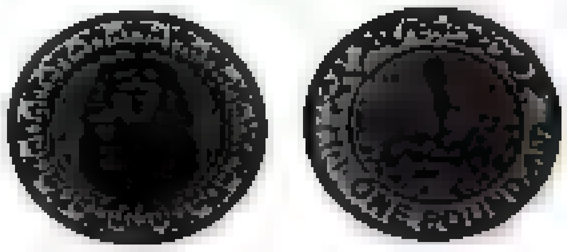
1. The probability of appearance of the number 4
2. The probability of appearance of the number 3

....

....

2 Theoretical probability :

In the previous , we carried out the experiment of tossing a piece of coin and we found that :



- The probability of appearance of a head = 0.53
- The probability of appearance of a tail = 0.47

But when we study this experiment theoretically , we find that :

If we tossed the coin piece once , then we obtain either a head or a tail.

i.e. the number of possible outcomes = 2 and there is one chance to obtain a head and also one chance to obtain a tail.

(*i.e.* all outcomes of the experiment have the same chance to happen).

i.e. the probability of appearance of a head $= \frac{1}{2} = 0.50$

and the probability of appearance of a tail $= \frac{1}{2} = 0.50$

Notice that :

We can express the probability by percentage.

i.e. we write the probability of appearance of a head = 50%

Remark

Notice the difference between the experimental probability of appearance of a head (0.53) and the theoretical probability of appearance of a head (0.50)

We can interpret this as :

The more the number of times of carrying out the experiment increases , the more the value of experimental probability approaches the value of theoretical probability.

Random experiment :

It is an experiment in which we can specify all its possible outcomes before carrying it out but we cannot determine certainly which of them will occur.

Sample space :

It is the set of all possible outcomes of a random experiment and it is denoted by S

For example:

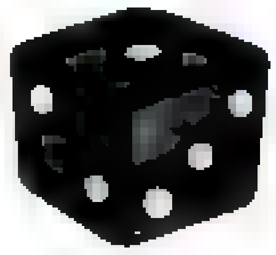
- When we toss a piece of coin once , then the sample space is $S = \{H , T\}$

- When we roll a fair die once observing the apparent number on the upper face , then the sample space is $S = \{1 , 2 , 3 , 4 , 5 , 6\}$

Event :

It is a subset of the sample space.

For example:



If A is the event of appearance of an odd number when rolling a fair die once and observing the apparent number on the upper face , then $A = \{1 , 3 , 5\}$, $A \subset S$

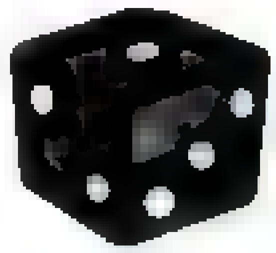
Generally

The probability of any event occurrence $A \subset S$ is denoted by P (A) and it is given by using the relation :

$$P(A) = \frac{\text{The number of elements of the event « A »}}{\text{The number of elements of sample space « S »}} = \frac{n(A)}{n(S)}$$

Example 2

If a fair die is rolled once and we observe the apparent number on the upper face , find the probability of each of the following events :



1. A is the event of appearance of a number more than 4 (Approximating the result to the nearest hundredth)
2. B is the event of appearance of an even number.
3. C is the event of appearance of a number equal to 5 (Approximating the result to the nearest tenth)
4. D is the event of appearance of a number equal to 7
5. E is the event of appearance of a number less than 7

Solution

$S = \{1 , 2 , 3 , 4 , 5 , 6\}$, $n(S) = 6$

1. $A = \{5 , 6\}$, $n(A) = 2$

   $\therefore P(A) = \frac{2}{6} = \frac{1}{3} \simeq 0.33$ (to the nearest hundredth)

2. $B = \{2 , 4 , 6\}$, $n(B) = 3$ $\therefore P(B) = \frac{3}{6} = 0.5$

3. $C = \{5\}$, $n(C) = 1$

   $\therefore P(C) = \frac{1}{6} \simeq 0.2$ (to the nearest tenth)

4. $D = \{\ \}$ or $\varnothing$, $n(D) =$ Zero

   $\therefore P(D) = \frac{0}{6} =$ Zero (the impossible event)

5. $E = \{1 , 2 , 3 , 4 , 5 , 6\}$, $n(E) = 6$

   $\therefore P(E) = \frac{6}{6} = 1$ (the certain event)

Remarks

1. The impossible event : is the event that has no chance for occurring.
   i.e. the probability of the impossible event = Zero
2. The certain event : is the event that has all the possible outcomes.
   i.e. the probability of the certain event = 1
3. The value of probability of any event is not less than zero and not more than one
   i.e. $0 \leq$ The probability of an event occurrence ≤ 1

Example 3

From the set of digits $\{3, 4, 5\}$, form a two-digit number, then find the probability of each of the following events :

1. A « the event that the unit digit is odd »
2. B « the event that the tens digit is even »
3. C « the event that the two digits are odd »
4. D « the event that the sum of the two digits = 8 »
5. E « the event that the product of the two digits = 20 »

Solution

$S = \{33, 43, 53, 34, 44, 54, 35, 45, 55\}$, $n(S) = 9$

1. $A = \{33, 43, 53, 35, 45, 55\}$, $n(A) = 6$ $\therefore P(A) = \frac{6}{9} = \frac{2}{3}$
2. $B = \{43, 44, 45\}$, $n(B) = 3$ $\therefore P(B) = \frac{3}{9} = \frac{1}{3}$
3. $C = \{33, 53, 35, 55\}$, $n(C) = 4$ $\therefore P(C) = \frac{4}{9}$
4. $D = \{53, 44, 35\}$, $n(D) = 3$ $\therefore P(D) = \frac{3}{9} = \frac{1}{3}$
5. $E = \{54, 45\}$, $n(E) = 2$ $\therefore P(E) = \frac{2}{9}$

Example 4

A bag has an amount of marbles of the same size and touch. If 2 marbles are red, 3 are blue and 5 are white and a marble is drawn randomly, calculate :

1. The probability of that (the drawn marble is red)
2. The probability of that (the drawn marble is blue)
3. The probability of that (the drawn marble is white)
4. The probability of that (the drawn marble is not blue)

Solution

The probability of occurrence of a certain outcome

$= \frac{\text{The number of possible chances to get this outcome}}{\text{The total number of chances}}$

The total number of marbles = 2 + 3 + 5 = 10

1 The probability of that (the drawn marble is red)

$= \frac{\text{The number of red marbles}}{\text{The total number of marbles}} = \frac{2}{10} = \frac{1}{5}$

2 The probability of that (the drawn marble is blue)

$= \frac{\text{The number of blue marbles}}{\text{The total number of marbles}} = \frac{3}{10}$

3 The probability of that (the drawn marble is white)

$= \frac{\text{The number of white marbles}}{\text{The total number of marbles}} = \frac{5}{10} = \frac{1}{2}$

4 The probability of that (the drawn marble is not blue)

$= \frac{\text{The number of marbles which aren't blue}}{\text{The total number of marbles}} = \frac{10-3}{10} = \frac{7}{10}$

Remark

In the previous example , notice that :

P (red marble) $= \frac{2}{10}$, P (blue marble) $= \frac{3}{10}$,

P (white marble) $= \frac{5}{10}$, $\because \frac{2}{10} + \frac{3}{10} + \frac{5}{10} = 1$

$\therefore$ the sum of probabilities of all outcomes of a random experiment = 1

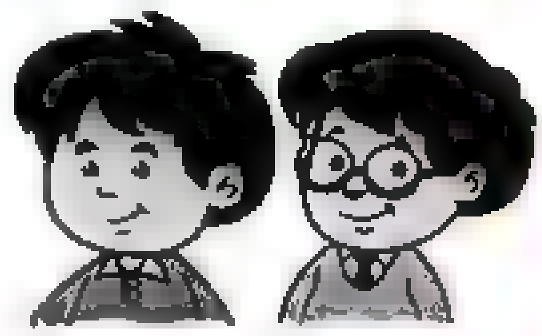
So , if the probability of occurrence of an event is a , then the probability that it doesn't occur = 1 – a

So , we can find the probability that the drawn marble is not blue as follows :

The probability that the drawn marble is not blue

= 1 – *the probability that it is blue* $= 1 - \frac{3}{10} = \frac{7}{10}$

Example 5



A Class has some students who wear glasses and other students who don't wear glasses.

If a student is chosen randomly from this class and the probability that this student wears glasses is 0.1

1 Find the probability that this student does not wear glasses.

2 If the number of students in this class is 30 students , find the expected number of students who wear glasses.

Solution

1 The probability that this student does not wear glasses = 1 − the probability that the student wears glasses = 1 − 0.1 = 0.9

2 The expected number of students who wear glasses = 0.1 × 30 = 3 students.

Notice that :

The expected number of outcomes of an event
= The probability of occurrence of this event × the total number of all possible outcomes.

Example 6

A spinner game was divided into some equal sectors. 2 of them are green , 4 are blue and the rest are red. If the probability that the pointer stops pointing at a green sector is $\frac{1}{6}$, then find the number of red sectors.

Solution

∵ The probability that the pointer stops pointing at a green sector

$= \frac{\text{The number of green sectors}}{\text{The number of all sectors}}$ $\qquad \therefore \frac{1}{6} = \frac{2}{\text{The number of all sectors}}$

∴ The number of all sectors = 2 × 6 = 12 sectors.

∴ The number of red sectors = 12 − (2 + 4) = 6 sectors.

Try by yourself

1 A box contains cards numbered from 1 to 15. If a card is drawn randomly , what is the probability that the written number on the card is divisible by 5 ?

Answer :

2 An experiment has 3 outcomes. If the probability of occurrence of the first outcome is 0.3 and the probability of the second is 0.45 , calculate the probability of the third outcome.

Answer :

3 A farm has 2000 cows. If the probability of cow madness infection in this farm is 0.17 , what is the expected number of infected cows ?

Answer :

Unit 1

Numbers and Algebra

From the school book

Exercise 1 On repeated multiplication

1 **Choose the correct answer from those given :**

(1) The multiplicative inverse of the number $\left(\frac{2}{5}\right)^0 =$ ....

(a) $\frac{5}{2}$ (b) $-\frac{2}{5}$ (c) 1 (d) 0

(2) The additive inverse of the number $(-3)^0$ is ....

(a) 1 (b) -3 (c) 3 (d) $-(3)^0$

(3) The multiplicative inverse of the number $(-1)^3$ is ....

(a) $(-1)^3$ (b) $(-1)^2$ (c) 1^3 (d) 1^2

(4) The additive inverse of the number $\left(-\frac{2}{5}\right)^2$ is ....

(a) $\frac{4}{25}$ (b) $-\frac{4}{25}$ (c) $\frac{25}{4}$ (d) $-\frac{25}{4}$

(5) $\left(\frac{1}{4}\right)^0 + \frac{1}{4} =$ ....

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{2}{4}$

(6) $\left(\frac{5}{3}\right)^2 \times \left(\frac{3}{5}\right)^0 =$ ....

(a) $\frac{5}{3}$ (b) $\frac{25}{9}$ (c) 0 (d) 1

(7) If $x = y$, then $\left(\frac{3}{5}\right)^{x-y} =$ ....

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) 1 (d) 0

(8) $\left(\frac{a}{b}\right)^2 \times \frac{b^2}{a^2} =$ .... (where $ab \neq 0$)

(a) ab (b) $\left(\frac{a}{b}\right)^4$ (c) $(ab)^0$ (d) $\frac{a}{b}$

(9) If $x = -\frac{1}{2}$ and $y = 3$, then $x^y =$ ....

(a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$

(10) If : $y^{26} + y^{27} = 0$, then $y =$ ....

(a) 1 (b) -1 (c) 2 (d) -2

2 **Calculate each of the following , then put the result in the simplest form :**

(1) $\left(\frac{1}{2}\right)^3$ (2) 📖 $\left(\frac{1}{3}\right)^4$ (3) $\left(\frac{3}{5}\right)^2$ (4) 📖 $\left(-\frac{1}{7}\right)^3$

(5) 📖 $\left(-\frac{3}{4}\right)^4$ (6) $\left(\frac{5}{9}\right)^0$ (7) $\left(1\frac{1}{5}\right)^2$ (8) $\left(-2\frac{1}{2}\right)^3$

(9) $(0.04)^2$ (10) $(1.5)^3$ (11) $(-3.2)^2$ (12) $\left(1-1\frac{2}{3}\right)^2$

3 **Calculate each of the following , then put the result in the simplest form :**

(1) $8\times\left(\frac{1}{2}\right)^3$ (2) 📖 $\left(-\frac{3}{4}\right)^2\times\frac{8}{27}$

(3) 📖 $\left(-\frac{3}{5}\right)^3\times\left(-\frac{25}{27}\right)$ (4) $\left(\frac{3}{5}\right)^2\div\left(-\frac{9}{125}\right)$

(5) $\left(\frac{4}{3}\right)^2\times\left(\frac{3}{2}\right)^3$ (6) 📖 $\left(-\frac{5}{6}\right)^2\div3\frac{3}{4}$

(7) $\left(2\frac{1}{2}\right)^2\times\frac{4}{25}$ (8) 📖 $2\frac{7}{9}\div\left(-1\frac{2}{3}\right)^2$

4 **Calculate each of the following , then put the result in the simplest form :**

(1) $\left(\frac{4}{5}\right)^2\times\frac{5}{16}\times\left(\frac{2}{3}\right)^0$ (2) $\frac{3}{4}\times\left(-\frac{2}{3}\right)^3\times\left(\frac{3}{2}\right)^2$

(3) $\left(-\frac{5}{3}\right)^4\times\left(-\frac{3}{5}\right)^3\times(-1)^7$ (4) 📖 $\left(-\frac{2}{3}\right)^3\times\left(\frac{1}{3}\right)^3\div\left(-\frac{2}{9}\right)^2$

(5) $\left[\left(\frac{5}{2}\right)^3\div\left(\frac{3}{2}\right)^4\right]\times\left(\frac{3}{5}\right)^3$ (6) 📖 $\left(-\frac{1}{2}\right)^3\div\left[8\times\left(-\frac{1}{2}\right)\times\frac{3}{4}\right]$

5 If $x=-\frac{2}{3}$ and $y=-\frac{1}{3}$, find the value of : x^2+y^3 « $\frac{11}{27}$ »

6 If $a=\frac{2}{3}$ and $b=-\frac{4}{3}$, find the value of : $|a^3+b^3|$ « $\frac{1}{8}$ »

7 If $x=0.5$, $y=-\frac{2}{3}$ and $z=-3$, find the value of : $9xy^2-z^3$ « 29 »

8 📖 If $a=-\frac{1}{2}$, $b=2$ and $c=\frac{3}{4}$, find the numerical value of : $a^3b^2+b^2c-8abc$ « $8\frac{1}{2}$ »

9 📖 If $x=-\frac{3}{2}$, $y=\frac{1}{2}$ and $z=-\frac{4}{3}$, find the numerical value of each of the following in its simplest form :

(1) $x^2y^2z^2$ « 1 » (2) $x^2\div z^2$ « $\frac{81}{64}$ »

(3) x^2-yz^2 « $\frac{49}{36}$ » (4) $\dfrac{x^2y^2z^2}{x+y}$ « -1 »

10 **Complete the following :**

(1) $\frac{8}{27} = (\frac{2}{3})^{\cdots\cdots}$

(2) $\frac{9}{16} = (\frac{3}{4})^{\cdots\cdots}$

(3) $-\frac{64}{125} = (-\frac{4}{5})^{\cdots\cdots}$

(4) $2\frac{1}{4} = \frac{\cdots\cdots}{\cdots\cdots} = (\frac{3}{2})^{\cdots\cdots}$

(5) $0.027 = \frac{\cdots\cdots}{\cdots\cdots} = (\frac{3}{10})^{\cdots\cdots}$

(6) $64\% = \frac{\cdots\cdots}{\cdots\cdots} = (\frac{4}{5})^{\cdots\cdots}$

(7) If $\frac{x}{y} = -\frac{2}{5}$, then $(\frac{x}{y})^3 = \cdots\cdots$

(8) If $c = -3$ and $d = -5$, then $(\frac{c}{d})^2 = \cdots\cdots$

(9) If : $\frac{a}{b} = 0.2$, then : $(\frac{a}{b})^3 = \cdots\cdots$

(10) If $x = \frac{1}{2}$ and $y = \frac{2}{3}$, then $x^2 y^2 = \cdots\cdots$

(11) 📖 $(-\frac{1}{2})^3 - (-\frac{1}{2})^2 = \cdots\cdots$

(12) $2^2 + 2^2 = 2^{\cdots\cdots}$

(13) $\frac{3}{4}$, $\frac{9}{16}$, $\frac{27}{64}$, ……… , ……… (in the same pattern)

(14) The greater number of the two numbers $(\frac{1}{4})^2$ and $(-\frac{8}{3})^5$ is ………

Geometric Application

11 If the volume of the cube is found from the relation $V = \ell^3$ where V is the cube volume and ℓ is its edge length , then calculate the volume of the cube whose edge length is $1\frac{1}{2}$ cm.

« $\frac{27}{8}$ cm^3 »

For excellent pupils

12 **Choose the correct answer from those given :**

(1) If $x \in \{0 , 1 , 2 , 3\}$, then the greatest value of the number $(\frac{1}{2})^x$ is when $x = \cdots\cdots$

(a) 0 (b) 1 (c) 2 (d) 3

(2) If $x \in \{0 , 1 , 3 , 4\}$, then the smallest value of the number $(-\frac{2}{5})^x$ is when $x = \cdots\cdots$

(a) 0 (b) 1 (c) 3 (d) 4

13 **Arrange the following numbers ascendingly without expanding :**

$(\frac{2}{3})^2$, $(-\frac{2}{3})^3$, $(-\frac{1}{3})^2$, $(-\frac{1}{3})^3$

From the school book

Exercise 2 — On non-negative integer powers

1 Calculate each of the following , then put the result in the simplest form :

(1) $\left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^2$

(2) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$

(3) $\left(-\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$

(4) $\frac{1}{5} \times \left(-\frac{1}{5}\right)^4$

(5) $\left(\frac{1}{6}\right)^9 \div \left(\frac{1}{6}\right)^8$

(6) $\left(\frac{2}{7}\right)^5 \div \left(\frac{2}{7}\right)^3$

(7) $\left(-\frac{3}{5}\right)^7 \div \left(\frac{3}{5}\right)^5$

(8) $\left(-\frac{5}{2}\right)^2 \div 2\frac{1}{2}$

(9) $\left(\frac{1}{2}\right)^2 \times \frac{1}{2} \times \left(\frac{1}{2}\right)^3$

(10) $\left(\frac{4}{5}\right)^8 \div \left(\frac{4}{5}\right)^6 \times \frac{4}{5}$

2 Calculate each of the following , then put the result in the simplest form :

(1) $\frac{3^7 \times 3^3}{3^6}$

(2) $\frac{2^6 \times 2}{2^3 \times 2^4}$

(3) $\frac{(-5)^4 \times 5^2}{5^3}$

(4) $\frac{(-2)^5 \times 2^4}{(-2)^3 \times 2^2}$

(5) $\frac{x^2 \times x^3 \times x^4}{x^7 \times x}$

(6) $\frac{(-3)^5 \times (-2)^7}{(-3)^3 \times (-2)^5}$

(7) $\frac{x^4 \times y^3 \times x^5}{x^6 \times y^2}$

(8) $\frac{x y^2 \times x^2 y}{x^2 y^2}$

(9) $\frac{\left(-1\frac{1}{2}\right)^5 \times \left(-1\frac{1}{4}\right)^8}{\left(-\frac{3}{2}\right)^4 \times \left(-\frac{5}{4}\right)^6}$

3 Calculate each of the following , then put the result in the simplest form :

(1) $\left(\frac{a\,b}{c}\right)^5$

(2) $\left(\frac{5\,x}{3\,y}\right)^2$

(3) $\left(-\frac{2\,a\,b}{3\,c}\right)^4$

(4) $\left(\frac{x^2}{y^3}\right)^2$

(5) $\left(\frac{a^3 b^2}{c^5}\right)^3$

(6) $\left(-\frac{c^2}{d}\right)^3$

(7) $\left(-\frac{x^3}{y^2}\right)^2$

(8) $\frac{(4\,x^3 y^2)^7}{(2\,x^2 y)^7}$

(9) $\frac{(2\,a)^3 \times (2\,a)^4}{(-2\,a)^6 \times a}$

4 Calculate each of the following , then put the result in the simplest form :

(1) $\left[\left(\frac{1}{2}\right)^2\right]^2$

(2) $\left[\left(-\frac{3}{2}\right)^2\right]^5$

(3) $\left[\left(2\frac{1}{2}\right)^3\right]^2$

(4) $\left(\left(-1\frac{1}{3}\right)^2\right)^3$

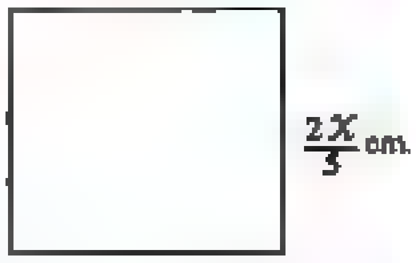
(5) $\left(\frac{3}{5}\right)^{10} \times \left(\frac{5}{3}\right)^{10}$

(6) $\left(\left(\frac{2}{7}\right)^2\right)^3 \times \left(\frac{7}{2}\right)^6$

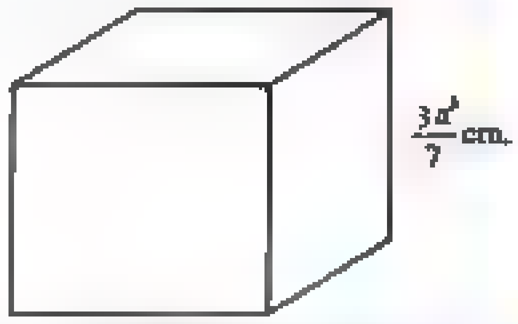
(7) $\left(2\frac{1}{2}\right)^2 \times \left(-\frac{2}{5}\right)^2$

(8) $\left(\frac{x^3}{y^2}\right)^{16} \div \left(\frac{x^2}{y^2}\right)^{16}$

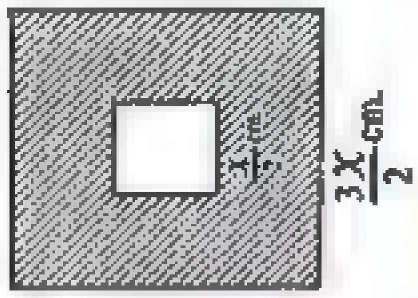
5 Find the area of the square whose side length is $\frac{2x}{5}$ cm.



6 Find the volume of the cube whose edge length is $\frac{3a^3}{7}$ cm.



7 **In the opposite figure :**
A square is drawn inside another square.
Find the area of the shaded part.



8 **Match each expression in column (A) with an equivalent expression in column (B) :**

Column (A)	Column (B)
(1) $(x^2)^n$	(a) x^{n^2}
(2) $(x^n)^n$	(b) $\frac{3m^c}{2n^c}$
(3) $(x y^a)^b$	(c) $27x^{3a}$
(4) $\left(\frac{x}{y^a}\right)^b$	(d) $\frac{3^c m^c}{2^c n^c}$
(5) $(-3x^a)^3$	(e) x^{2n}
(6) $(3x^a)^3$	(f) $-27x^{3a}$
(7) $\frac{3}{2}\left(\frac{m}{n}\right)^c$	(g) $\frac{n^b}{y^{ab}}$
(8) $\left(\frac{3m}{2n}\right)^c$	(h) $x^b y^{ab}$
	(i) $\frac{x^b}{y^{ab}}$
	(j) $x y^{ab}$

9 Choose the correct answer from those given :

(1) $3^2 \times 3^5 = \dots\dots$
(a) 3^7 (b) 3^3 (c) 3^{10} (d) 3^{25}

(2) $5^2 + 5^2 = \dots\dots$
(a) 10^2 (b) 10^4 (c) 5^4 (d) 50

(3) $3^5 \times 2^5 = \dots\dots$
(a) 5^{10} (b) 6^{10} (c) 6^5 (d) 6^{25}

(4) $(5a)^0 = \dots\dots$, $a \neq 0$
(a) 5 (b) a (c) 5 a (d) 1

(5) $3^{(2^3)} = \dots\dots$
(a) 3^6 (b) 3^5 (c) 3^8 (d) 3^{23}

(6) $(5^2)^3 = \dots\dots$
(a) 5^6 (b) 5^5 (c) 5^{23} (d) 5

(7) $3^{10} + 3^{10} + 3^{10} = \dots\dots$
(a) 3^{10} (b) 3^{30} (c) 9^{10} (d) 3^{11}

(8) $4^x + 4^x + 4^x + 4^x = \dots\dots$
(a) 4^{x+4} (b) 4^{4x} (c) 4^{x+1} (d) $4x^4$

(9) $\frac{(3^2)^5}{(3^5)^2} = \dots\dots$
(a) 3^{10} (b) 3^{52} (c) 3^{25} (d) 1

(10) $\frac{(x^2)^3}{x^3} = \dots\dots$, $x \neq 0$
(a) x^6 (b) x^2 (c) x^3 (d) x

(11) $(2y)^3 = \dots\dots$
(a) $2y^3$ (b) $8y$ (c) $8y^3$ (d) $23y$

(12) $(b^3)^4 = \dots\dots$
(a) b^{34} (b) b^7 (c) $b^3 \times b^3 \times b^3$ (d) $b^4 \times b^4 \times b^4$

(13) The quarter of the number $4^{20} = \dots\dots$
(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

Unit 1

10 Simplify to the simplest form :

$\frac{(2\,y)^4 \times (3\,y)^2}{12\,y^5}$, then find the value of the result at $y = -\frac{1}{6}$ « − 2 »

11 If $a = \frac{1}{2}$, $b = \frac{3}{4}$ and $c = -\frac{2}{3}$, find the numerical value of each of :

(1) $(c^2\,b)^3$ (2) $(4\,a^3\,c)^2$ (3) $(a^2\,b\,c^2)^2$ « $\frac{1}{27}$, $\frac{1}{9}$, $\frac{1}{144}$ »

12 If $a = \frac{5}{3}$, $b = -\frac{3}{2}$ and $c = \frac{2}{5}$, find the numerical value of each of :

(1) $\frac{(a^2\,c^2)^2}{b}$ (2) $\left(\frac{2\,a\,b}{5\,c}\right)^3$ « $-\frac{32}{243}$, $-\frac{125}{8}$ »

13 If $x = -\frac{1}{2}$, $y = \frac{3}{4}$ and $z = -\frac{3}{2}$,

find the numerical value of each of the following in the simplest form :

(1) $x^3\,y^2$ (2) $y^3\,x^2$ (3) $\frac{x^3}{y^2\,z^2}$ « $-\frac{9}{128}$, $\frac{27}{256}$, $-\frac{8}{81}$ »

14 Complete the following :

(1) $\left(\left(\frac{7}{9}\right)^3\right)^4 = \frac{7^{12}}{3^{\cdots\cdots}}$

(2) If : $\left(\frac{3}{4}\right)^5 \times x = \left(\frac{3}{4}\right)^7$, then $x =$ ....

(3) The greater number in the two numbers $((-3)^5)^3$ and $((-3)^2)^4$ is ....

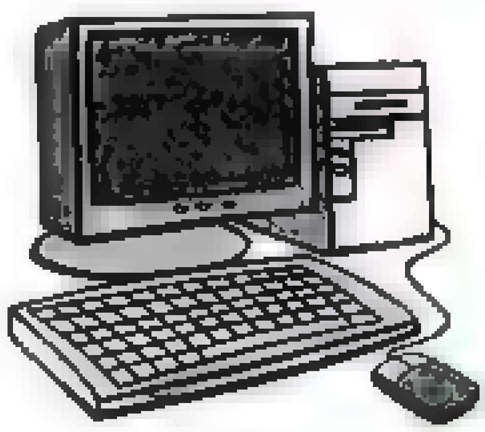
(4) $((-1)^5)^2 - ((-1)^3)^2 =$ ....

(5) $\frac{4^4}{4^3} + \frac{4^3}{4^2} + \frac{4^2}{4} + 4 = 2^{\cdots\cdots\cdots}$

(6) $2^{2x} \times 4^x = 4^{\cdots\cdots\cdots}$

Life Application

15 From computer technology , we know that :



1 kilobyte = 2^{10} bytes ,

1 megabyte = 2^{10} kilobytes ,

1 gigabyte = 2^{10} megabytes ,

1 terabyte = 2^{10} gigabyte ,

How many bytes are there in one terabyte ?

For excellent pupils

16 If four times a number is 4^3 , find $\frac{3}{4}$ this number. «12»

17 If $x = \left(\left(\frac{2}{3}\right)^5\right)^2$ and $y = 3\left(\frac{3}{2}\right)^9 - \left(\frac{3}{2}\right)^{10}$

, prove that the number x is the multiplicative inverse of the number y

18 If $x = \frac{1}{5}$ and y = 5 , find the value of : $x^{15}\, y^{14}$ «$\frac{1}{5}$»

19 Prove that : (1) $5^{x+2} - 5^{x+1} = 20 \times 5^x$

(2) $3^{15} + 3^{14}$ is divisible by 4

From the school book

Exercise 3 On negative integer powers

1 Evaluate each of the following :

(1) 4^{-1} (2) 5^{-2} (3) $\left(\frac{1}{2}\right)^{-1}$

(4) $\left(-\frac{2}{3}\right)^{-2}$ (5) $(0.2)^{-2}$ (6) $(1.2)^{-1}$

2 Evaluate each of the following :

(1) $3^{7} \times 3^{-3}$ (2) $2^{-2} \times 2^{-3}$ (3) $\frac{3}{3^{-2}}$ (4) $\frac{6^{-2}}{6^{-3}}$

3 Evaluate each of the following :

(1) $(5^{-1})^{-3}$ (2) $(3^{-2})^{2}$ (3) $(0.25)^{-2}$

(4) $(2^{-1} \times 2^{-2})^{3}$ (5) $\left(\frac{3^{-1}}{3}\right)^{2}$ (6) $\left(\frac{8^{4}}{8^{-4}}\right)^{0}$

4 Evaluate each of the following :

(1) $\frac{8 \times 8^{-2}}{8^{-3}}$ (2) $\frac{7^{-2} \times 7^{5}}{7^{3}}$ (3) $\frac{2^{5} \times 2^{-2}}{2^{-4} \times 2^{3}}$

(4) $\frac{2^{3} \times 2^{-3}}{(2^{2})^{2}}$ (5) $\frac{(3^{-2})^{3}}{3^{-2} \times 3^{-6}}$ (6) $\left(\frac{9^{3} \times 9}{9^{5}}\right)^{-3}$

(7) $\left(\frac{4^{-2} \times 3}{4^{-3}}\right)^{-3}$ (8) $\left(\frac{2^{5} \times 3^{2}}{3^{4} \times 2^{3}}\right)^{-1}$ (9) $(3^{0} \times 2^{-2})^{-2}$

(10) $(3^{0} - 2^{-2})^{-2}$ (11) $\frac{(10)^{2} \times (0.01)^{3}}{(10)^{-3}}$

5 Simplify each of the following and write the result in terms of positive exponents , where the denominator does not equal zero :

(1) $7x^{-1}$ (2) $x^{-1} y^{2}$ (3) $a^{-2} b^{-3}$ (4) $x^{3} \times x^{-5}$

(5) $x^{3} \times x^{-2} \times x^{-1}$ (6) $\frac{c^{-5}}{c^{2}}$ (7) $x^{7} \div x^{-5}$ (8) $(a^{-2})^{3}$

(9) $(b^{-1})^{-3}$ (10) $(a^{2} \times a^{-5})^{2}$ (11) $(x^{2})^{-3} \times (x^{-3})^{-2}$ (12) $\left(\frac{y^{5}}{y^{-2}}\right)^{-3}$

(13) $\frac{x^{2} \times x^{-3}}{x^{-4} \times x}$ (14) $\frac{(x^{2})^{-3} \times (x^{-1})^{2}}{x^{-3} \times x^{-4}}$ (15) $\left(\frac{x^{-2} \times y^{-1}}{y^{-3} \times x}\right)^{-1}$ (16) $\frac{a^{-1}}{b^{2}} \left(\frac{a^{-1}}{2b^{2}}\right)^{-2}$

(17) $(x + x^{-1})^{2}$

6 **Complete the following :**

(1) $2^{-3} \times C^0 = \cdots\cdots$

(2) 📖 $(b^{-1})^{-3} = b^{\cdots}$

(3) 📖 $2x^{-3} = \frac{2}{\cdots}$

(4) 📖 $(3x^{-1})^2 = 9x^{\cdots} = \frac{9}{\cdots}$

(5) $(3y^{-2})^{-2} = \cdots\cdots$

(6) 📖 $(3a^2)^{-1} = \frac{1}{\cdots}$

(7) 📖 $2x^{-2}y^{-3} = \frac{2}{\cdots}$

(8) $\frac{x^{-5}}{y^{-5}} = (\cdots\cdots)^5$

(9) $\left(\frac{1}{2}\right)^2 + 2^0 - (2)^{-2} = \cdots\cdots$

(10) $(x^2)^{\cdots} = \frac{1}{x^4}$

(11) $2^{10} \times 2^{-10} = 3^{\cdots}$

(12) $a^{-5} + 1 = a^{-5}(\cdots\cdots + \cdots\cdots)$, where $a \neq 0$

(13) If : $x = \frac{1}{2}$, $y = \frac{1}{4}$, then $(x - y)^{-1} = \cdots\cdots$

7 **Choose the correct answer from those given :**

(1) If $a^{-1} = \frac{2}{3}$, then $a = \cdots\cdots$

(a) $-\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) 1

(2) If $a = 7^{x}$ and $b = 7^{-x}$, then $a \times b = \cdots\cdots$

(a) 7^{2x} (b) 49^{2x} (c) 1 (d) 0

(3) $\frac{5^{x}}{5^{-y}} = \cdots\cdots$

(a) 5^{x+y} (b) 5^{x-y} (c) 5^{x+y} (d) $-\frac{x}{y}$

(4) 📖 $\frac{6a^2x^4}{2a^3x^3} = \cdots\cdots$

(a) $3ax$ (b) $3a^5x^7$ (c) $\frac{3x}{a}$ (d) $\frac{3}{ax}$

(5) 📖 $\frac{(-2s^2t)^3}{(-4st^2)^2} = \cdots\cdots$

(a) $-\frac{s^3}{2t}$ (b) $-\frac{s^4}{2t}$ (c) $\frac{s^5}{2t^2}$ (d) $\frac{s^4}{t}$

(6) 📖 $\left(\frac{m^2}{n^{-3}}\right)^{-1}\left(\frac{3m^{-2}}{n^{-2}}\right)^{-2} = \cdots\cdots$

(a) $\frac{9m^2}{n^7}$ (b) $\frac{m^2}{9n^7}$ (c) $\frac{m^2}{9n}$ (d) $\frac{9m^6}{n}$

(7) 📖 $\frac{(2ab^{-2})^0}{3^0a^{-2}b} = \cdots\cdots$

(a) $\frac{a^3}{3b^3}$ (b) a^2 (c) 1 (d) $\frac{a^2}{b}$

(8) If $a^{x} = 2$ and $a^{-y} = 3$, then $a^{x-y} = \cdots\cdots$

(a) 1 (b) -1 (c) $\frac{2}{3}$ (d) 6

(9) If $x y^{-1} = \frac{1}{2}$, then $\frac{y}{x} =$ ....

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) 2

(10) $3^{-1} + 3^{-1} + 3^{-1} =$ ....

(a) 3^{-3} (b) 3^3 (c) 9^{-3} (d) 1

(11) The multiplicative inverse of 5^{-1} is ....

(a) $\frac{1}{5}$ (b) 5 (c) -5 (d) $-\frac{1}{5}$

(12) $\left(\frac{3}{5}\right)^2 \times \left(\frac{5}{3}\right)^{-2} =$ ....

(a) $\left(\frac{3}{5}\right)^4$ (b) 1 (c) $\left(\frac{3}{5}\right)^{-4}$ (d) 0

8 Complete each of the following by the suitable sign of (>) , (<) or (=) :

(1) 2^{10} .... 2^{-10} (2) 3^{-20} .... 3^2 (3) 5^{-15} .... 2^{-15}

(4) $(-7)^{-2}$ .... $(-7)^{19}$ (5) $(-1)^{-6}$ .... $(-1)^{-9}$ (6) $(-1)^{-20}$ .... $(1)^{-10}$

9 Why is not b^{-3} defined when $b = 0$?

10 Calculate the value of $\left(-\frac{3}{5}\right)^x \times \left(\frac{3}{5}\right)^y$ in each of the following cases :

(1) $x = -2$ and $y = 2$ « 1 » (2) $x = -1$ and $y = 2$ « $-\frac{3}{5}$ »

11 If : $x = -\frac{1}{3}$, $y = \frac{2}{3}$, then find in the simplest form the numerical value of the expression : $\left(\frac{y}{x^2}\right)^{-2}$ « $\frac{1}{36}$ »

Life Applications

12 The flea can jump at a height of 200 times of its length. If a flea of length 2^{-4} inches can jump at a height of 2^3 inches , what does this height represent according to the length of the flea ?

13 The population of a city has been growing exponentially. It is estimated that in (t) years the population (p) will be : $p = 2\,(1.03)^t$ million.

(1) What will the population be in 2 years ? (2) What is the population now ?

(3) What was the population last year ?

For excellent pupils

14 Simplify to the simplest form : $\frac{2^{10} \times 3^4}{(12)^5}$ « $\frac{1}{3}$ »

15 Simplify to the simplest form :

$\frac{6^{2n+1} \times 4^{-n}}{2^n \times 3^{2n+1}}$, then find the value of the result when $n = 3$ « $\frac{1}{4}$ »

16 If $2^n = 3$, find the value of :

(1) 2^{n+1} (2) 4^n (3) 4^{-n} (4) 2^{n-1} « 6 , 9 , $\frac{1}{9}$, $\frac{3}{2}$ »

17 If $a = 5$ and $b = 5^{-1}$, find the value of : $a^{51}\ b^{50}$ « 5 »

18 Without finding the result , arrange the following ascendingly by inspection :

$(-2)^{-15}$, $(-5)^{20}$, $(-2)^{15}$, 2^{-20} , $(-5)^{15}$, $(-2)^{20}$

From the school book

Exercise 4 On scientific notation of the rational number

1 In each of the following , tick (✓) in front of the numbers which are in the standard form:

(1) 5.3×10^7 ☐ (2) 0.2×10^{-4} ☐ (3) 0.025×10^8 ☐
(4) 20.5×10^5 ☐ (5) 7×10^{-4} ☐ (6) 10×10^{-10} ☐
(7) 4.25×10 ☐ (8) 33.9×10^6 ☐ (9) -5.782×10^2 ☐
(10) -0.0003×10^3 ☐

2 Write each of the following numbers in the standard form :

(1) 📖 600 000 (2) – 20 000 (3) 📖 7 millions
(4) 19 millions (5) 46 870 000 (6) 58

3 Write each of the following numbers in the standard form :

(1) 📖 0.0006 (2) 📖 0.000053 (3) 📖 0.000864
(4) 0.421 (5) 25.0003 (6) – 300.501

4 The number of inhabitants in Egypt is about 83 000 000 persons.
Write this number in the standard form.

5 The area of the surface of the Earth is about 510 000 000 km^2.
Express this number by the standard form.

6 The mass of the Hydrogen atom is about 0.00000000000000000000000167 gm.
Write this number in the standard form.

7 The light velocity is about 300 000 km/s. Express this velocity by m/s. in the standard form.

8 Dr. Ahmed Zewail discovered the femto second which is a millionth of a miliardth of a second. Express it by the standard form.

9 At writting the number 2.74×10^{20} as a whole number, find the number of zeroes that are on the right of the digit 4

10 Write the following numbers in the standard form :

(1) 📖 68×10^5 (2) 📖 68×10^{-5} (3) 📖 720×10^6
(4) 📖 750×10^{-9} (5) -32.4×10^4 (6) -702.5×10^{-8}
(7) 📖 0.4×10^{-10} (8) 0.0005×10^{15} (9) 0.0036×10^{-4}
(10) 0.0020205×10^{12}

11 Put the suitable sign (<) or (>) :

(1) 6.4×10^3 ☐ 4.6×10^3 (2) 6.2×10^4 ☐ 4.1×10^5

(3) 0.0041 ☐ 3.2×10^{-2} (4) 4370 ☐ 3.41×10^4

(5) 2.10×10^{-5} ☐ 1.82×10^{-5} (6) 9.1×10^{-4} ☐ 1.2×10^{-5}

(7) 6.920×10^5 ☐ 96230 (8) 3.69×10^{-4} ☐ 0.0000623

12 Arrange the following numbers in a descending order :

3.6×10^{-3} , 5.2×10^{-5} , 1×10^{-2} , 8.35×10^{-2} , 6.08×10^{-8}

13 Choose the correct answer from those given :

(1) $3.04 \times 10^7 =$ ....

(a) 340 000 (b) 304 000 (c) 3 400 000 (d) 30 400 000

(2) $2.37 \times 10^{-4} =$ ....

(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

(3) If $0.00079 = 7.9\ a$, then $a =$ ....

(a) 10^{-1} (b) 10^{-3} (c) 10^{-4} (d) 10^4

(4) If $0.0000503 = m \times 10^{-5}$, then $m =$ ....

(a) 503 (b) 5.03 (c) 50.3 (d) 0.503

(5) If the thickness of a sheet of paper is 0.012 cm. , then a ream of 400 sheets is of height ....

(a) 48×10^{-3} cm. (b) 48×10^{-2} cm. (c) 4.8×10^0 cm. (d) 48 cm.

(6) Which of the following equals $\frac{1}{2}$ milliard ?

(a) 50×10^8 (b) 5×10^8 (c) 0.5×10^8 (d) 500×10^7

(7) Which of the following is the greatest ?

(a) 6.3×10^5 (b) 9.8×10^4 (c) 5.2×10^5 (d) 7.3×10^4

(8) Which of the following is the smallest ?

(a) 0.6×10^5 (b) 0.25×10^5 (c) 7×10^4 (d) 17.5×10^4

(9) $6\,000 \times 50 =$ ....

(a) 300×10^2 (b) 30×10^5 (c) 3×10^5 (d) 30×10^4

(10) $45 \times 900 =$ ....

(a) 4.05×10^2 (b) 4.05×10^3 (c) 4.05×10^4 (d) 45×10^2

(11) $0.7 \times 0.005 =$ ....

(a) 3.5×10^3 (b) 3.5×10^{-2} (c) 3.5×10^2 (d) 3.5×10^{-3}

14 **Write the result of each of the following in the standard form :**

(1) $(6.4 \times 10^8) \times (1.5 \times 10^5)$
(2) $(8.2 \times 10^7) \times (2.1 \times 10^{-4})$
(3) $(5.02 \times 10^{-4}) \times (0.1 \times 10^{-3})$
(4) $(4.4 \times 10^3) \times (2 \times 10)^5$
(5) 📖 $(3.8 \times 10^8) \div (1.9 \times 10^6)$
(6) $(125.5 \times 10^{-3}) \div (5 \times 10^4)$
(7) $(8.8 \times 10^{25}) \div (8.8 \times 10^{22})$
(8) $(5 \times 10)^4 \div (2.5 \times 10^{-3})$

15 **Write the result of each of the following in the standard form :**

(1) 📖 $(3.8 \times 10^5) + (4.6 \times 10^4)$
(2) $(4.54 \times 10^4) + (3.76 \times 10^3)$
(3) 📖 $(5.3 \times 10^8) - (0.8 \times 10^7)$
(4) $(2.65 \times 10^{-2}) - (6.34 \times 10^{-3})$

16 **Write the result of each of the following in the standard form :**

(1) 5000×3000
(2) 400×0.00007
(3) $8000 \div 0.004$
(4) $0.000033 \div 500$
(5) $(20\,000)^3$
(6) $(0.002)^2$
(7) $(0.1)^{-8}$

17 **Find the value of n in each of the following :**

(1) $800\,000 = 8 \times 10^n$
(2) 📖 $0.00000006 = 6 \times 10^n$
(3) $0.00052 = 5.2 \times 10^n$
(4) 📖 $0.000357 = 3.57 \times 10^n$
(5) 📖 $(0.004)^2 = 1.6 \times 10^n$
(6) $76293 = n \times 10^4$

Life Applications

18 If the diameter of the Earth equals 1.27×10^4 km. long and the length of the diameter of Mars is 6.79×10^3 km. Which of the two planets is the greater and what is the difference between the two diameter lengths in the standard form ?

19 📖 If light travels at a speed of 3×10^8 m/s. :

(a) Calculate the distance from the Sun to the Earth if you know that the light of the Sun takes 8 minutes to reach the Earth.

(b) If the distance between planet Venus and the Sun is 108 million kilometres , calculate the elapsed time (in minutes) that light takes to reach Venus from the Sun.

For excellent pupils

20 **Find the result of the following in the standard form :** $\dfrac{9.02 \times 10^3 + 4.98 \times 10^4}{2.5 \times 10^{-5}}$

21 **Without using the calculator ,**
write each of the following numbers in the standard form :

(1) $10^{29} - 10^{28}$
(2) $2^{19} \times 5^{15}$

22 If $X = 5 + (3 \times 10) + (4 \times 10^2) + (6 \times 10^3) + (9 \times 10^4) + (4 \times 10^5) + (2 \times 10^6)$
Write X in the standard form without using the calculator.

From the school book

Exercise 5 On order of mathematical operations

1 Calculate the value of each of the following :

(1) $3+12\div 6$ (2) $-5+2\times 3$ (3) $2\times 6-4\div 2$ (4) $4\times 7-3^2$
(5) $4\times 2^3-20$ (6) $9+4\times 3^2$ (7) $144-8\div 2^3$

2 Calculate the value of each of the following :

(1) $196\div(7-5)^2$ (2) $18\div(9-6)\times(1+2)$ (3) $20\div 5+8-(4-1)$
(4) $10\times 4-(2\times 6-8)$ (5) $(7-4)\times 2\div(5-3)$ (6) $(30-6)\div 6\times 30\div 3$
(7) $7(6^2\div 2\times 3)$ (8) $12(2^2)\div 24+3^2$ (9) $9(4)^2\div 2^2\times 3$
(10) $9\times 10+20\div 2-3$
(11) $6+9\div 3+2\times 3^2+11-8$
(12) $6-5+72\div 9+24\div 2^3+4\times 1+10\times 1+5$

3 Calculate the value of each of the following :

(1) $2-[(7-3)-2]$ (2) $[4-(5-2)]-1$
(3) $3+[5+2(8\div 4)]$ (4) $2^3+[4+(2-1)]$
(5) $[(2+23-7)\times 2]\div 4$ (6) $10\times 3\div[4-(9-8)]$
(7) $(26+1)\div[3(4-3)]$ (8) $(15\times 2)\div[5-(9-7)]$
(9) $2+3[4+(6\times 3-8)]\times 2$ (10) $2[(5^2+1)-(4^2-1)]$
(11) $5[(2^2-1)-(2^2-2)]$ (12) $6\div 3+[7+20\div(6-2^2)]$

4 Calculate the value of each of the following :

(1) $[-6\div(-3)]\times[-30\div(-3)]$ (2) $(-10+3)\div(-8+7)$
(3) $7-[10-(-8)]-3$ (4) $[(11-(-10)]\times 2\div(-6)$
(5) $2\div(-1)-(-4)^2$ (6) $-6-[-2-5]^2$

5 Calculate the value of each of the following :

(1) $\dfrac{15+7}{15-4}$ (2) $\dfrac{8+20-4}{8-4}$ (3) $\dfrac{-4\times(-10)}{-9+7}$

(4) $\dfrac{1+15}{8-(2-2)}$ (5) $\dfrac{11-(5-4)}{1+4}$ (6) $(3-1)^3+\dfrac{7\times 3}{-1-6}-\dfrac{2\times 15}{6}$

(7) $\dfrac{5^2-5\times 2}{(15+3)\div 6}$ (8) $\dfrac{5+2\times 5}{2^2+1}+5^2-5$ (9) $\dfrac{3^2\times 6\div 3}{2\times 1+(3+1)^2}$

Unit 1

6 **Calculate the value of each of the following :**

(1) $\left(\frac{3}{2} \times 3\frac{1}{2}\right) \div \left(\frac{6}{5} - 1\right)$

(2) 📖 $15 \div \frac{1}{3} - \frac{3}{4} \times 10^3 + 27$

(3) $16 + 4 \div 2 - 3 \times 10^{-2}$

(4) $9 \div \frac{1}{2} \times 2 - 3 + \frac{1}{5}$

---

7 📖 If $x = 3$, what is the numerical value of the expression : $2\left(\frac{5x+3}{4x-3}\right)$ « 4 »

---

8 📖 **Evaluate the expressions when t = 2 and s = 5 :**

(1) $(t + s)^2$

(2) $(s - t)^3$

(3) $\left(\frac{s}{t}\right)^3$

(4) $\frac{6^2}{s-1}$

(5) $\frac{s-t}{s^3}$

(6) $\frac{12}{4s^2}$

---

9 📖 **Evaluate :** $16t \div (4s) + 3st$, for $t = 9$ and $s = 6$ « 168 »

---

10 If $x = 4(5 + 6) - 6$ and $y = 9(36 \div 12) \div 3$,

find the value of the expression : $2x + 4y$ « 112 »

---

11 If $x = 3(5 + 7) - 4$ and $y = 4(8 + 2) + 5$,

find the numerical value of the expression : $x - 4y$ « zero »

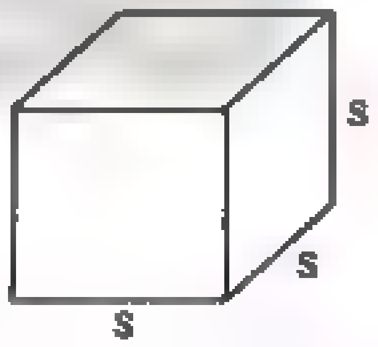
---

12 If $x = 18 - 4 \times 2 \div 2 + 1$ and $y = 8 + 9 \times 3 - 4^2 + 11$,

find the numerical value of the expression : $\left(\frac{y}{x}\right)^{-3}$ « $\frac{1}{8}$ »

Geometric Applications

13 📖 **In the opposite figure :**



The total area of a cube

is $T = 6s^2$, find T when :

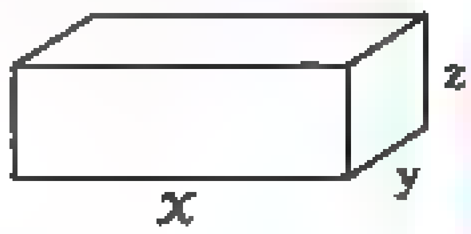
(1) s = 3 m.

(2) s = 0.8 cm.

« 54 m.^{2} , 3.84 cm.^{2} »

---

14 **In the opposite figure :**



The total area of a cuboid

is $T = 2(xy + yz + zx)$ find T when :

(1) $x = 2$ cm. , $y = 3$ cm. and $z = 5$ cm.

(2) $x = \frac{3}{5}$ m. , $y = 0.4$ m. and $z = \frac{1}{5}$ m.

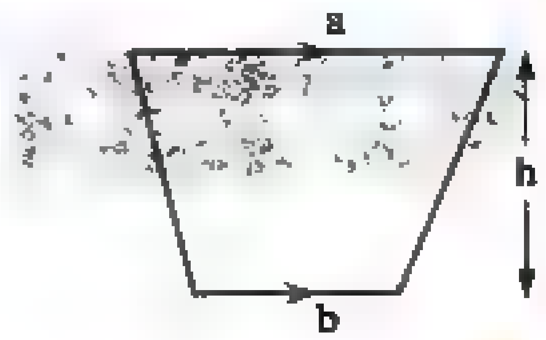
« 62 cm.^{2} , $\frac{22}{25}$ m.^{2} »

15 In the opposite figure :

The area of a trapezium is $A = \frac{1}{2} h (a + b)$, find A when :

(1) $h = 2$ metres , $a = \frac{3}{4}$ metre and $b = \frac{1}{4}$ metre.

(2) $h = 4$ metres , $a = \frac{1}{2}$ metre and $b = \frac{1}{2}$ metre.



« 1 m^2 , 2 m^2 »

For excellent pupils

16 Rewrite each of the following mathematical statements after putting the parentheses in the places which make them true :

(1) $3 + 96 \div 12 \times 4 = 5$

(2) $3 + 96 \div 12 \times 4 = 35$

(3) $3 + 96 \div 12 \times 4 = 33$

Unit 1

From the school book

Exercise 6 On the square root of a perfect square rational number

1 **Find each of the following :**

(1) $\sqrt{16}$ (2) $-\sqrt{25}$ (3) $\pm\sqrt{2500}$ (4) $\pm\sqrt{40000}$

(5) $\sqrt{\frac{9}{49}}$ (6) $-\sqrt{\frac{64}{25}}$ (7) $\sqrt{0.81}$ (8) $\pm\sqrt{1.44}$

(9) $\sqrt{6\frac{1}{4}}$ (10) $-\sqrt{1\frac{11}{25}}$ (11) $-\sqrt{4^2}$ (12) $\pm\sqrt{8^2}$

(13) $\sqrt{(\frac{81}{100})^2}$ (14) $\sqrt{(-\frac{3}{4})^2}$ (15) $\pm\sqrt{\frac{576}{1225}}$ (16) $-\sqrt{\frac{2.5}{40}}$

(17) $-\sqrt{\frac{49a^4}{25b^6}}$ (18) $\pm\sqrt{\frac{16b^8}{121h^2}}$ (19) $\sqrt{\frac{49a^4b^2}{9}}$ (20) $\sqrt{\frac{25x^2y^2}{36}}$

2 **Find the two square roots of each of the following numbers :**

(1) 64 (2) 144 (3) $\frac{9}{25}$

(4) $6\frac{1}{4}$ (5) 0.25 (6) 0.0049

3 **Find each of the following :**

(1) $\sqrt{9}+\sqrt{16}$ (2) $\sqrt{36+64}$ (3) $\sqrt{25-9}$

(4) $-\sqrt{225-81}$ (5) $\sqrt{3^2+4^2}$ (6) $-\sqrt{(10)^2-8^2}$

(7) $\sqrt{\frac{9}{16}+1}$ (8) $-\sqrt{\frac{1}{4}(1-\frac{3}{4})}$ (9) $\sqrt{\frac{5^4\times5^3}{5^5}}$

(10) $\sqrt{(\frac{1}{2})^2+(\frac{1}{5})^2}$ (11) $\sqrt{(\frac{1}{2})^4\times(\frac{1}{3})^4}$ (12) $\sqrt{(\frac{1}{4})^2\times(\frac{1}{4})^3}$

4 **Complete the following :**

(1) $\frac{3}{4}\times\sqrt{\frac{16}{9}}=$ ....

(2) $\sqrt{\frac{81}{49}}\times\frac{14}{27}=$ ....

(3) $\sqrt{\frac{9}{4}}-\frac{3}{2}+(\frac{3}{2})^{zero}=$ ....

(4) $\sqrt{36}+\sqrt{16}=\sqrt{\dots\dots}$

(5) $\sqrt{(81)^2-81\times2+1}=$ ....

(6) The multiplicative inverse of $\sqrt{\frac{4}{25}}$ in the simplest form equals ....

(7) The multiplicative inverse of $\sqrt{0.49}$ in the simplest form equals ....

(8) 📖 The multiplicative inverse of the rational number $\sqrt{\frac{10}{2.5}}$ equals ....

(9) The additive inverse of the number $-\sqrt{\frac{9}{16}}$ in the simplest form equals ....

(10) The rational number $6\frac{1}{4}$ in the form $\left(\frac{a}{b}\right)^2$ is ....

(11) $\sqrt{\frac{25}{64}} = \sqrt{\left(\frac{\cdots\cdots}{\cdots\cdots}\right)^2} = \cdots\cdots$

(12) $\sqrt{(-3)^2} = \cdots\cdots$

(13) $\sqrt{a^4b^8} = \cdots\cdots$

(14) If $a = \sqrt{\frac{1}{4}}$ and $b = 2$, then $ab = \cdots\cdots$

(15) If $a = -\frac{1}{2}$ and $b = -\frac{9}{8}$, then $\sqrt{ab} = \cdots\cdots$

(16) If $2x = \sqrt{36}$, then $x = \cdots\cdots$

(17) If $a = 0.000625$, then $\sqrt{a} = 2.5 \times 10^{\cdots\cdots}$

(18) $\sqrt{(2009)^2 + 2(2009) \times 213 + (213)^2} = \cdots\cdots$

5 Choose the correct answer from those given :

(1) $\sqrt{1\frac{9}{16}} = \cdots\cdots$

(a) $1\frac{3}{4}$ (b) $-1\frac{3}{4}$ (c) $1\frac{1}{4}$ (d) $-1\frac{1}{4}$

(2) 📖 $\sqrt{10^2 - 6^2} = \cdots\cdots$

(a) 4 (b) 8 (c) ± 4 (d) ± 8

(3) $\sqrt{18 \times 10 \times 10 \times 18} = \cdots\cdots$

(a) 18 (b) 180 (c) 10 (d) 100

(4) $\sqrt{\sqrt{81}} = \cdots\cdots$

(a) 81 (b) 27 (c) 9 (d) 3

(5) $\sqrt{2^2 + \sqrt{25}} = \cdots\cdots$

(a) 3 (b) -3 (c) 9 (d) -9

(6) If : $\frac{x}{2} = \frac{8}{x}$, then $x = \cdots\cdots$

(a) 4 (b) -4 (c) ± 4 (d) 16

(7) If : $x=\sqrt{\frac{1}{4}}$, then $x^3 =$ ....

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{64}$

(8) $\sqrt{(a+b)^3(a+b)} =$ ....

(a) $(a+b)^2$ (b) a^4+b^4 (c) $-(a+b)^2$ (d) $\pm(a+b)^2$

(9) $\sqrt{1}+\sqrt{4}+\sqrt{9}+\sqrt{16}+\sqrt{25}+\sqrt{36}+\sqrt{49}+\sqrt{64} =$ ....

(a) 6 (b) $\sqrt{204}$ (c) $\sqrt{81}$ (d) 6^2

(10) The side length of the square whose area is $16\,x^2$ cm.^{2} = ... cm.

(a) $8\,x$ (b) $4\,x$ (c) $2\,x$ (d) $8\,x^2$

6 **Simplify each of the following to the simplest form :**

(1) $(-\frac{1}{2})^3 \times \sqrt{\frac{64}{9}}$

(2) $\sqrt{\frac{49}{4}} \times (\frac{2}{7})^{zero} \times (-\frac{2}{7})^2$

(3) $\frac{2}{5} \times \sqrt{\frac{9}{16}} \div (-\frac{1}{2})^3$

(4) $(-\frac{1}{3})^2 + \sqrt{\frac{64}{81}} - (\frac{3}{4})^{zero}$

(5) $\frac{3}{4} \times (-\frac{2}{3})^3 \times \left(\frac{3}{\sqrt{4}}\right)^2$

(6) $\sqrt{(\frac{25}{4})^2 \times (\frac{2}{5})^2}$

7 **Simplify each of the following to the simplest form :**

(1) $\sqrt{16}+\sqrt{25}$

(2) $\sqrt{\sqrt{16}+\sqrt{25}}$

(3) $\sqrt{(\sqrt{16}+\sqrt{25})^2}$

8 **Find two rational numbers lying between :** $\sqrt{\frac{4}{9}}$ and $\frac{3}{4}$

9 **Which is greater :** $\frac{3}{5}$ or $\sqrt{\frac{4}{9}}$? Find the difference between them.

10 **Which is smaller :** $\sqrt{2\frac{1}{4}}$ or $(-\frac{2}{3})^2$? Find their difference.

11 **Find each of the following :**

(1) $\sqrt{5^2-2\times5+1}$

(2) $\sqrt{(\frac{1}{4})^2-2\times\frac{1}{4}+1}$

(3) $\sqrt{20\div5+8-(4-1)}$

(4) $\sqrt{8\times(5+11)\div(2+6)}$

(5) $\sqrt{2\times8+10-3+12+11\times6+88\div2^3+99\div11}$

(6) $\sqrt{6+3\sqrt{100}-\sqrt{121}}$

Geometric Applications

12 (1) $\overline{XY}$ is a line segment where $(XY)^2 = 25$ cm.^{2} , E is the midpoint of $\overline{XY}$

Find the length of : $\overline{XE}$ « 2.5 cm. »

(2) If : $(AB)^2 = 144$ cm.^{2} , $(BC)^2 = 625$ cm.^{2} and B $\in \overline{AC}$

Find the length of : $\overline{AC}$ « 37 cm. »

(3) The area of a square is 0.49 cm.^{2} Find its perimeter. « 2.8 cm. »

(4) The area of a square is equal to the area of a triangle with base = 9 cm. long and its height = 8 cm. find the side length of the square. « 6 cm. »

(5) The area of a circle 154 cm.^{2} Calculate its radius length $(\pi = \frac{22}{7})$ « 7 cm. »

(6) The area of a circle 78.5 cm.^{2} Calculate its diameter length $(\pi = 3.14)$ « 10 cm. »

(7) The area of a circle 616 cm.^{2} Calculate its circumference $(\pi = \frac{22}{7})$ « 88 cm. »

(8) If three quarters of the area of a square is $1\frac{11}{64}$ m.^{2}

Calculate the side length of the square. « $1\frac{1}{4}$ m. »

(9) The length of a rectangle is twice its width and its area is 24.5 cm.^{2}

Calculate each of its width and length. « 3.5 cm. , 7 cm. »

For excellent pupils

13 If $\frac{m}{n}$ is a rational number and $\frac{m^2}{n^2} = 0.16$, find the value of $(\frac{m}{n})^3$ « ± 0.064 »

Unit 1

From the school book

Exercise 7 On solving equations of the first degree in one unknown in $\mathbb{Q}$

1 Find the solution set of each of the following equations :

(1) $x - 7 = 3$ where $x \in \mathbb{N}$
(2) $x + 17 = 13$ where $x \in \mathbb{N}$
(3) $5x = 20$ where $x \in \mathbb{Q}$
(4) $\frac{2}{5} x = \frac{1}{5}$ where $x \in \mathbb{Q}$
(5) $-4 + y = 13$ where $y \in \mathbb{N}$
(6) $m - (-3) = 1$ where $m \in \mathbb{Z}$
(7) $x - 7 = 0$ where $x \in \mathbb{Z}$
(8) $y - (-5) = -3$ where $y \in \mathbb{Q}$
(9) $x - 6\frac{1}{4} = 12\frac{1}{2}$ where $x \in \mathbb{Q}$
(10) $8.91 + x = 11.09$ where $x \in \mathbb{Q}$

2 Solve each of the following equations :

(1) $2x - 1 = 5$ where $x \in \mathbb{Q}$
(2) $8x + 4 = 12$ where $x \in \mathbb{Q}$
(3) $3x - 13 = 26$ where $x \in \mathbb{N}$
(4) $2x + 14 = 14$ where $x \in \mathbb{N}$
(5) $8 + 2x = 14$ where $x \in \mathbb{Z}$
(6) $\frac{5}{6} x - 4 = 11$ where $x \in \mathbb{Q}$
(7) $8 - 2x = -2$ where $x \in \mathbb{Z}$
(8) $2 - 5x = 0$ where $x \in \mathbb{Q}$
(9) $2x + 3x + 25 = 5$ where $x \in \mathbb{Z}$
(10) $6x - 2x + 7 = 4$ where $x \in \mathbb{Z}$

3 Solve each of the following equations in $\mathbb{Q}$:

(1) $2(x - 3) = 4$
(2) $3x + 2(5x - 3) = 7$
(3) $7(x - 2) - 3(x + 1) = 3$
(4) $3(x + 2) + 7(x - 1) = 12$
(5) $4(x - 1) - (x + 3) = 0$
(6) $5(x - 2) + 2(x + 4) = -16$
(7) $2(x - 3) + 3(x - 2) - 4x = -3$
(8) $3y + 6(y + 3) - (8y - 16) = 60$

4 Find in $\mathbb{Q}$ the solution set of each of the following equations :

(1) $2x + 5 = x + 9$
(2) $5x - 4 = 2x + 11$
(3) $x + 3 = 18 - 3x$
(4) $3x + 6 = 30 - 5x$
(5) $4(x + 1) = 2(x - 1)$
(6) $3(x - 2) = 5x - 10$
(7) $a + 5a - 2 = 2(3 - a)$
(8) $3(2x - 8) - (2x + 2) = x - 3$
(9) $\frac{x + 1}{3} = \frac{x - 1}{4}$
(10) $\frac{5}{4 + 4x} = \frac{3}{1 - 2x}$

5 Complete the following :

(1) If $x + 5 = 7$, then $x =$ ....
(2) If $3t = 6$, then the value of $6t =$ ....
(3) If $2x = 5$, then the value of $4x =$ ....
(4) If $x + 9 = 11$, then the value of $7x =$ ....

(5) If $2t + 3 = 15$, then the value of $\frac{1}{3}$ t = ...

(6) If $Z - 1\frac{1}{4} = 5\frac{1}{2}$, then the value of $4Z - 18$ = ...

(7) If $\frac{P}{4} = \frac{2}{3}$, then the value of $\frac{P}{2}$ = ...

(8) If the age of a man now is X years , then his age 5 years ago is ....

(9) If the age of a man now is y years , then his age after 4 years is ....

(10) If the age of a man after 5 years is X years, then his age now is ....

(11) If the age of Youssef after 4 years is X years, then his age 2 years ago is ....

(12) A rectangle with length equals triple its width. if the length = X cm. , then its width = ... cm.

(13) The rectangle whose width = X cm. and its length is twice its width , then its perimeter = ... cm.

(14) Two integers, their sum is 5, if one of them is X, then the other one is ....

(15) Two integers, the difference between them is 2, if the small one is X, then the great one is ....

6 **Choose the correct answer from those given :**

(1) If $2X = 2$, then $3X - 1$ = ...
(a) 2 (b) 3 (c) 4 (d) 5

(2) If $2X = 0$, then X = ...
(a) 2 (b) 3 (c) 5 (d) zero

(3) If $2ab = 10$, then $3ab$ = ...
(a) 5 (b) 6 (c) 15 (d) 30

(4) If $0.2 + a = 5$, then $\frac{a}{4}$ = ...
(a) 4.8 (b) 1.3 (c) 1.2 (d) 19.2

(5) If $5X + 8X + 2X + 4X = 114$, then $5X + 3$ = ...
(a) 33 (b) 35 (c) 47 (d) $8X$

(6) The S.S. of the equation $\frac{2a}{3} = 8 + 4a$ in $\mathbb{Q}$ is ....
(a) $\{-2.4\}$ (b) $\{2.4\}$ (c) $\{-3\frac{1}{3}\}$ (d) $\{0\}$

(7) Which of the following equations is equivalent to the equation $X + 3 = 12$?
(a) $X - 3 = -12$ (b) $X + (-3) = 12$
(c) $X - (-3) = 12$ (d) $X - (-3) = -12$

(8) Which of the following equations is equivalent to the equation $X - 12 = 15$?
(a) $X + 12 = -15$ (b) $\frac{1}{3}X - 4 = 5$ (c) $X - 4 = -5$ (d) $X + 4 = 5$

Geometric Applications

7 Find the measure of each angle in each of the following triangles :

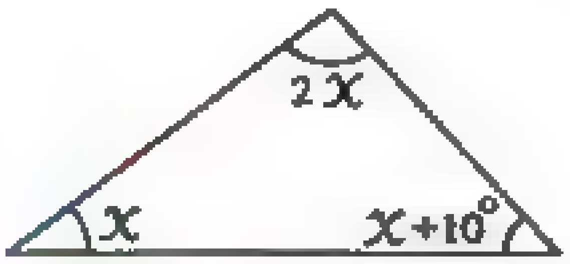


Fig. (1)

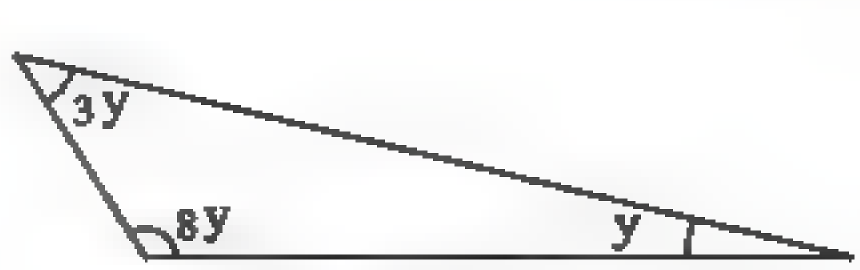


Fig. (2)

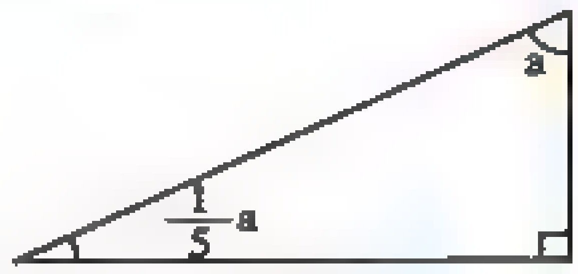
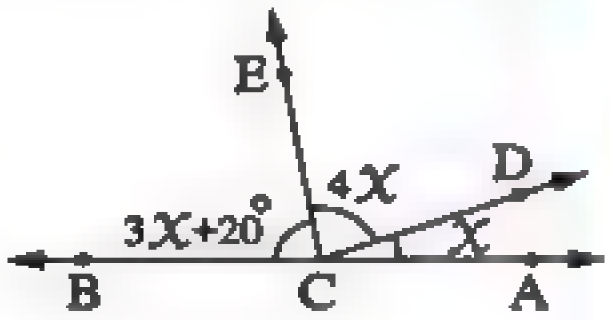


Fig. (3)

8 In the opposite figure :

If $C \in \overleftrightarrow{AB}$, find m ($\angle$ DCE)



« 80° »

9 The length of a rectangle exceeds its width by 4 metres and its perimeter is 68 metres. Find the dimensions of the rectangle. « 19 m. , 15 m.»

10 The length of a rectangle decreases than the twice of its width by 4 cm., if its perimeter equals the perimeter of a square of side length 7 cm.
Find the dimensions of the rectangle. « 6 cm. , 8 cm. »

11 The length of a rectangle is twice its width. If the length decreases 5 cm. and the width increases 6 cm. , then the rectangle becomes a square.
Find the area of the rectangle. « 242 cm^2 »

Life Applications

12 Two integers , the smaller number is $2x$ and the greater number is $7x$, if the difference between them is 25 , find the two integers. « 10 , 35 »

13 Two natural numbers, one of them is twice the other and their sum is 108
Find the two numbers. « 36 , 72 »

14 The difference between two natural numbers is 5 and their sum is 21
What are the two numbers ? « 13 , 8 »

15 Find the number that if it is added to its triple the result is 32 « 8 »

16 Find the number which if we subtract 9 from its triple , the result will be 6 « 5 »

17 Three consecutive natural numbers whose sum is 213
What are these numbers ? « 70 , 71 , 72 »

18 Find three consecutive odd numbers if their sum is 357 « 117 , 119 , 121 »

19 A man's age now is three times his son's age and after two years , the sum of their ages will be 52 years. What is the age of each now ? « 12 years , 36 years »

20 Three brothers , Amgad , Bassim and Ayman , the sum of their ages is 89 years.
If Amgad was born before Bassim by 2 years and Bassim was born before Ayman by 6 years , what is the age of each of them now ? « 25 years , 31 years , 33 years »

For excellent pupils

21 Find in $\mathbb{Q}$ the S.S. of each of the following equations :
(1) $5 - \frac{6}{x} = -1$ (2) $-\frac{3}{5} + \frac{x}{10} = -\frac{1}{5} - \frac{x}{5}$

22 Find in $\mathbb{Q}$ the S.S. of each of the following equations :
(1) $(x + 3)^2 - (x - 2)^2 = 15$ (2) $(2x + 3)(2x - 1) - (2x - 1)^2 = 14$

23 If the S.S. of the equation $12x + 3 = 39$ in $\mathbb{Q}$ equals the S.S. of the equation $ax - 12 = a$ in $\mathbb{Q}$, find the value of a « 6 »

24 If $a + 1$ is a solution of the equation $(x + a)(x - a) = x^2 - ax + 3$ in $\mathbb{Q}$, find the value of a « 3 »

25 Three brothers were born in 1980 , 1984 and 1986 , the required is finding the year in which the sum of their ages became 41 years. « 1997 »

From the school book

Exercise 8 On solving inequalities in $\mathbb{Q}$

1 Which number would you add to each side of the inequality to obtain x in one side of it ?

(1) $x+5>9$ (2) $x-4<6$ (3) $x-7<3$ (4) $x+9>12$

(5) $x-1.5\le 3.2$ (6) $4.8\le x+0.6$ (7) $1\frac{1}{2}>x-2\frac{1}{2}$ (8) $x+\frac{1}{3}>-\frac{1}{6}$

2 Find the solution set of the inequality $x+3\le 6$, where :

(1) $x\in\mathbb{Z}$ (2) $x\in\mathbb{N}$

then represent the solution set on the number line.

3 Find the solution set of the following inequalities in $\mathbb{Q}$:

(1) $x+2>5$ (2) $x+4>1$ (3) $y-5>7$

(4) $19<y+14$ (5) $-1\ge x-3$ (6) $-5\frac{1}{2}>a+1\frac{1}{4}$

(7) $-2x<12$ (8) $\frac{2}{3}x\ge 1$ (9) $-\frac{1}{4}x\le\frac{1}{4}$

4 Solve each of the following inequalities in $\mathbb{Q}$:

(1) $3x-2<1$ (2) $2x+3<9$ (3) $4x+2\ge -10$

(4) $3x-2\ge 5$ (5) $3x-9<0$ (6) $1+2x\le -3$

(7) $9-6x<15$ (8) $2-3x\le 4$ (9) $\frac{3x-2}{5}\ge\frac{1}{2}$

(10) $8x-3x+1\le 29$ (11) $4n-2(n-1)\ge 0$ (12) $-3m+6(m-4)>9$

5 Solve each of the following inequalities in $\mathbb{Q}$:

(1) $6d+1\le 5d-3$ (2) $6x+2\ge 14+5x$

(3) $3x-2<5x-8$ (4) $8-2x\le 5x$

(5) $5x+1\ge 2(x+2)$ (6) $3(x+2)<-x+4$

(7) $3(x+2)\ge -2(x+1)$ (8) $2-3(x-5)\ge x+7$

(9) $3(7y-\frac{1}{3})\le 20y-1$ (10) $\frac{x}{2}+3\le 2x+1$

(11) $4-5(x-2)\le -2(-9+2x)$ (12) $3(y+2)+8<10-(2-y)$

(13) $1-(4d-1)>2(d-3)$ (14) $x-3(2x+1)<5(1+x)+2$

6 Find the S.S. of each of the following inequalities :

(1) $9\le 4x+1\le 17$, $x\in\mathbb{Z}$ (2) $9\le 3x+2<12$, $x\in\mathbb{Q}$

(3) $9>x+6>2$, $x\in\mathbb{N}$

7 **Complete :**

(1) If $x > y$, then $x + z$ ...... $y + z$
(2) If $x < y$, then $x + z$ ...... $y + z$
(3) If $x < y$ and $y < z$, then $x <$ ......
(4) If $z > y$ and $y > x$, then $z >$ ......
(5) If $a - 3 < 0$, then ...... > ......
(6) If $a + 5 > 0$, then ...... > ......
(7) If $b < 0$, then $b + 3$ 3
(8) If $x > y$ and z is positive $(z > 0)$, then $x z$ ...... $y z$
(9) If $x < y$ and z is negative $(z < 0)$, then $x z$ ...... $y z$

8 **Choose the correct answer from those given :**

(1) If : $-x < 5$, then ....
(a) $x > 5$ (b) $x > -5$ (c) $x < 5$ (d) $x < -5$

(2) If $x \in \mathbb{N}$, then the S.S. of the inequality $-x > 3$ is ....
(a) $\{4 , 5 , ...\}$ (b) $\{-4 , -5 , ...\}$ (c) $\{-3\}$ (d) $\varnothing$

(3) $\frac{x}{3} < 4$ is equal to ....
(a) $x > \frac{4}{3}$ (b) $x < \frac{4}{3}$ (c) $x > 12$ (d) $x < 12$

(4) If $x \in \mathbb{Z}$, then the S.S. of the inequality $20 < 5x < 25$ is ....
(a) $\{4\}$ (b) $\{5\}$ (c) $\{4 , 5\}$ (d) $\varnothing$

(5) The S.S. of the inequality $-2x <$ zero in $\mathbb{Q}$ is ....
(a) $\varnothing$ (b) $\mathbb{Q}_+$ (c) $\mathbb{Q}_-$ (d) $\mathbb{Z}_+$

(6) The number of solutions of the inequality $\frac{1}{5} < x < \frac{2}{3}$, where $x \in \mathbb{Q}$ is ....
(a) zero (b) 1 (c) 2 (d) an infinite number.

(7) If $x > y$, then $\frac{1}{x}$ .... $\frac{1}{y}$, where $x \neq 0$, $y \neq 0$
(a) $>$ (b) $<$ (c) $=$ (d) $\geq$

(8) The number 2 belongs to the S.S. of the inequality .... where x is an integer.
(a) $x > 2$ (b) $x < 2$ (c) $-x > -3$ (d) $-x > 3$

(9) If $x > 5$, then $-x$ ....
(a) < -9 (b) ≥ -5 (c) < -5 (d) > -5

9 Show by using examples that if $a > b$ and $c > d$, then it is not always correct that $a - c > b - d$

10 **Put (✓) for the correct statement and (✗) for the incorrect statement, when a statement is false, give an example that shows why it is false (given that $x > y$) :**

(1) $y < x$ ( )
(2) $x > 0$ ( )
(3) $y^2 \geq 0$ ( )
(4) $y^2 > y$ ( )
(5) $x y > 0$ ( )
(6) $x + y > y$ ( )
(7) $y^2 > x$ ( )
(8) $y^2 < x y$ ( )
(9) $x y < x^2$ ( )
(10) $x^3 < y^2$ ( )

Life Application

11 Hany wants to buy a pair of shoes and some shirts , if Hany has L.E. 200 , the price of the pair of shoes is L.E. 70 and the price of one shirt is L.E. 40

What is the greatest number of shirts can Hany buy ?

« 3 »

For excellent pupils

12 If the S.S. of the inequality $a \leq 3x - 5 \leq b$ in $\mathbb{Q}$ is $\{x : x \in \mathbb{Q} , 2 \leq x \leq 5\}$, find the values of a and b « 1 , 10 »

---

13 If : $-4 \leq x \leq 5$ and $2 \leq y \leq 7$, where $x \in \mathbb{Q}$ and $y \in \mathbb{Q}$, find :

(1) The greatest possible value of the expression $x + y$ « 12 »

(2) The greatest possible value of the expression $y - x$ « 11 »

(3) The smallest possible value of the expression $x y$ « – 28 »

(4) The smallest possible value of the expression $x^2 + y^2$ « 0 »

General Exercises on Unit One "Numbers and Algebra"

First : Completion questions :

Complete the following :

(1) $-3\ a\ b^2 \times 2\ a^2\ b^3 =$ ....

(2) The additive inverse of $\left(\frac{-2}{3}\right)^3$ is ....

(3) If : $x = \frac{1}{4}$, $y = \frac{1}{8}$, then $(x - y)^{-1} =$ ....

(4) $3 \times 4 - 21 \div 7 =$ ....

(5) $\sqrt{9 + 16} =$ ....

(6) $\sqrt{100 - 64} =$ ....

(7) $\sqrt{10^2 - 6^2} =$ ....

(8) If we subtract twice the number x from 3 , then the result is ....

(9) If : $x + 9 = 11$, then $7\ x =$ ....

(10) The S.S. of the equation : $3\ x + 7 = 5$, $x \in \mathbb{Q}$ is ....

(11) If : $ac > bc$, then a .... b (where $c < 0$)

Second : Multiple choice questions :

Choose the correct answer from those given :

(1) $\frac{4\ a^2\ b^4}{2\ a^3\ b^3} =$ ....

(a) $2\ a\ b$ (b) $2\ a^5\ b^7$ (c) $\frac{2\ b}{a}$ (d) $\frac{2}{a\ b}$

(2) If : $a = b$, then $\left(\frac{3}{7}\right)^{b-a} =$ ....

(a) zero (b) 1 (c) $\frac{3}{7}$ (d) $\frac{7}{3}$

(3) $2^4 \times 3^4 =$ ....

(a) 5^4 (b) 6^4 (c) 6^8 (d) 6^{16}

(4) $2^7 \times 3^7 =$ ....

(a) 5^7 (b) 6^7 (c) 6^{14} (d) 6^{49}

(5) Quarter of $4^{20} =$ ....

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

(6) $3^{10} + 3^{10} + 3^{10} =$ ....

(a) 3^{10} (b) 3^{11} (c) 3^{20} (d) 3^{30}

(7) $7.35 \times 10^{-4} =$ ....

(a) 0.000735 (b) 0.00735 (c) 0.0735 (d) 7350

(8) Which of the following is the smallest number ?
(a) 314×10^3 (b) 3.14×10^4 (c) 31.4×10^5 (d) 0.314×10^6

(9) If : $x = 0.0009$, then $\sqrt{x} = \dots\dots$
(a) 0.0003 (b) 0.0081 (c) 0.003 (d) 0.03

(10) $\sqrt{\left(-\frac{2}{3}\right)^2} = \dots\dots$
(a) $\frac{-4}{9}$ (b) $\frac{-2}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

(11) If : $-x < 3$, then $\dots\dots$
(a) $x > 3$ (b) $x > -3$ (c) $x < 3$ (d) $x < -3$

(12) The age of Amer now is x years , then his age 5 years ago is $\dots\dots$
(a) $5x$ (b) $5 + x$ (c) $5 - x$ (d) $x - 5$

Third : Essay questions

1 Find the value of the following expression in the simplest form : $\frac{5^{-2} \times 5^5}{5^3}$

2 Put the following expression in the simplest form : $\frac{7^{-3} \times 7^5}{7^2}$

3 Put the expression : $\left(\frac{1}{2}\right)^2 \times \left(-\frac{1}{2}\right)^3$ in the simplest form.

4 If : $x = \frac{-3}{2}$, $y = \frac{-4}{3}$, find in the simplest form : $\left(\frac{x}{y}\right)^2$

5 If : $x = \frac{-1}{2}$, $y = \frac{3}{4}$, find the numerical value of the expression : $\left(\frac{y}{x^2}\right)^{-2}$ in the simplest form.

6 If : $300000 = 3 \times 10^x$ find the value of : x

7 Find the solution set of the following inequality in $\mathbb{Q}$: $4x + 7 \le 3$

8 Find in $\mathbb{Z}$ the S.S. of the inequality : $3 - 2x \ge 1$, then represent it on the number line.

9 Find the S.S. of the following inequality in $\mathbb{Q}$: $1 < x - 3 \le 6$

10 Three even consecutive numbers their sum is 204. Find these numbers.

11 A man's age now is three times his son's age , and after two years , the sum of their ages will be 52 years. What is the age of each now ?

12 The length of a rectangle is twice its width. If the length decreases by 5 cm. and the width increases by 6 cm. , the rectangle becomes a square. Find the area of the rectangle.

Unit 2

Statistics and Probability

Lesson One : Samples :

- Systematic sample.
- Random sample.

Lesson Two : Probability :

- Experimental probability.
- Theoretical probability.

Unit 2

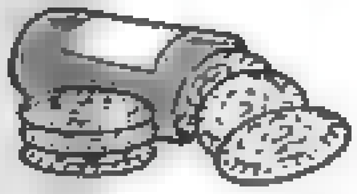
From the school book

Exercise 9 On samples

1 A factory's canteen service wanted to find the preferences of their 427 employees during their 15 – minute break. Each employee was given a number from 1 to 427

A 10% sample of the 427 were to be surveyed and asked to select a preference from :

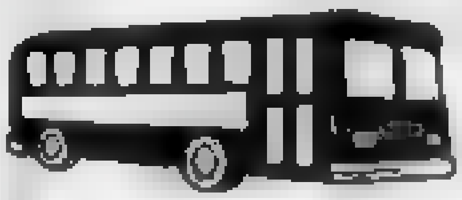
- Hot beverage.
- Hot soup with bread.
- Cold drink with biscuits.
- Fruit with fresh water.



The sample were determined by selecting 43 sample numbers in the range using calculator. Identify the sample numbers using a calculator.

2 A school makes a study about how the pupils come to school. If the number of pupils in the school is 320 , each pupil is given a number from 1 to 320 A sample of 10% from this number is selected as a sample to ask them how they come to school :

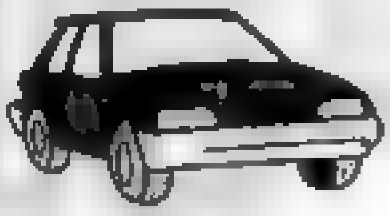
- On foot



- Public transport

- Taxi

- Bike

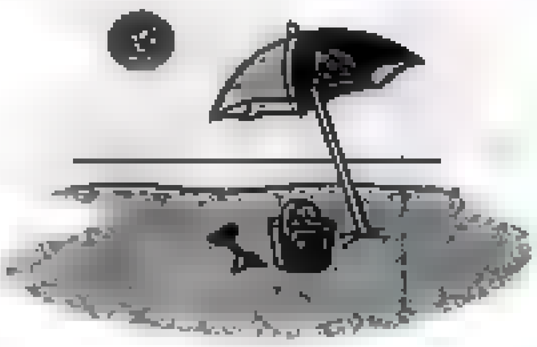


- Private car

determine the number of the sample using the calculator.

3 A company makes a study about the best places which the workers in the company prefer to spend their annual holiday among :

- Port Said
- Alexandria
- Matrouh
- The North Coast
- Ismailia

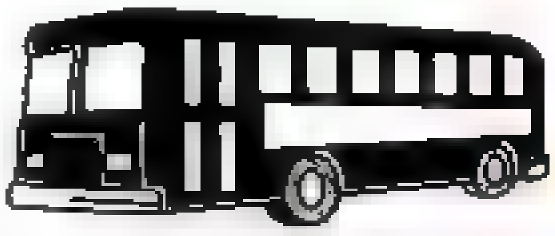


If the number of the workers in the company is 250 workers and a sample of 10% from the number of workers is selected to make a survey on it ,

determine the numbers of the sample using the calculator.

4 It is noticed that 230 persons use a public bus daily and the public transport authority wanted to collect some informations concerning with the using daily of this service. It is necessary to form a random sample representing 10% from the users of this bus to make a survey on them.

Determine the numbers of this sample using the calculator.



5 A sports survey was to be carried out among 318 students in a district to help decide on the type of entertaining services which would satisfy community youth needs.

Each student was given a number from 1 to 318

A sample of 10% of the 318 were to be surveyed and asked to select a preference from :

- Outdoor team games.
- Individual competitions.
- Indoor games.

Determine the sample by selecting 31 sample numbers using the Excel programme described in the activity at the end of the book.

6 A construction company wanted to ask their 362 workers about (on site) safety measures in terms of :

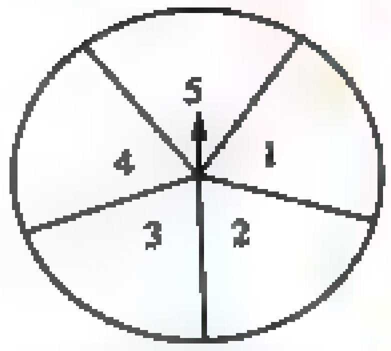
- Safety of emergency exit.
- Scaffold erection and maintenance.
- Rescue means positioning.

They asked 12% (to the nearest whole number) of their workforce with employment numbers from 20 to 382 to give their opinions. The employment numbers of the 12 % were identified by using a computer programme. Use a computer programme to identify the target employment numbers for the survey.

From the school book

Exercise 10 On probability

First : Problems on experimental probability

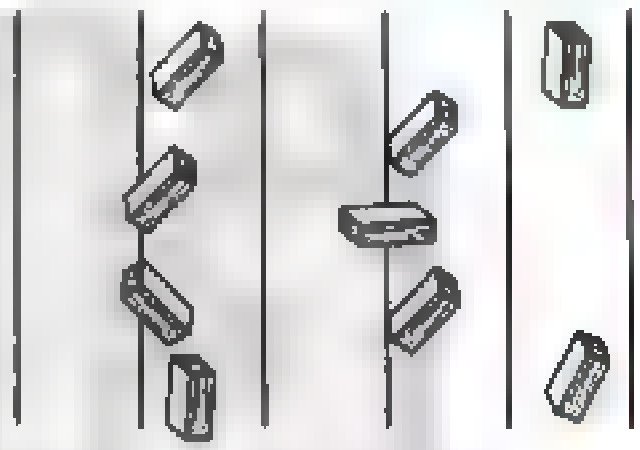


1 In the experiment of spinning game , roll the disc 50 times. In each time , record the number at which the pointer stops in the following table :

	1	2	3	4	5	Total
The tally sign						
Frequency						50

Calculate : (1) The probability that : The pointer stops at the number 2

(2) The probability that : The pointer stops at the number 5



2 (a) Draw six parallel lines with a distance of 2 cm. between each of them on a A_4 sheet of paper :

(b) Bring a piece of wood of length 2 cm.

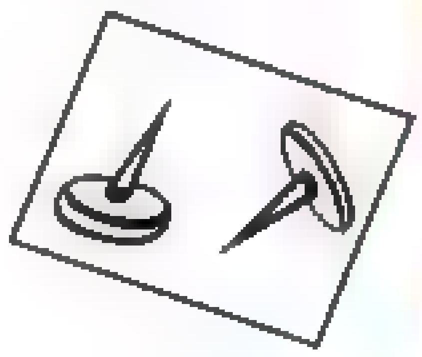
(c) Slightly toss the piece of wood in the air so that it falls from a suitable height on to the A_4 sheet.

(d) Repeat the trial 50 times.

(e) Record the number of times that the piece of wood falls across the line and also between the lines.

	Across	Between	Total
Tally			
Frequency			50

(f) Deduce the probability of the piece of wood falling between the lines.



3 (a) Drop a drawing pin 100 times from a suitable height.

(b) Record the number of times it lands with its point up and its point down :

	Up	Down	Total
Tally			
Frequency			100

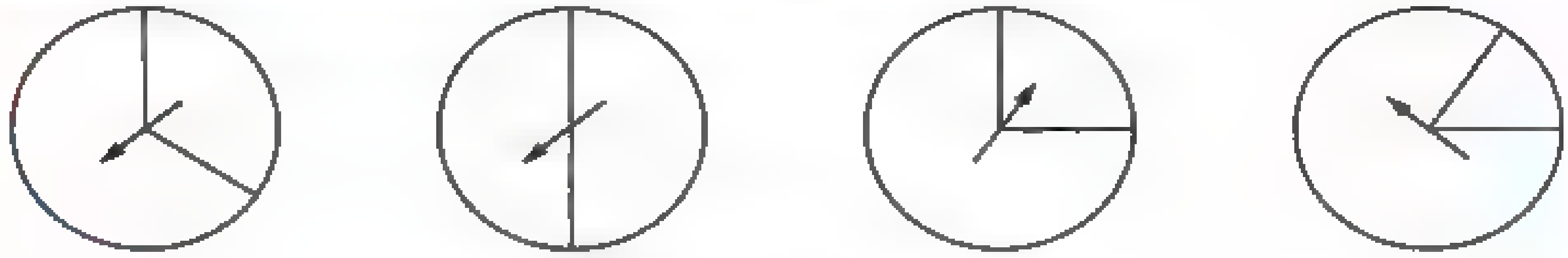
(c) Deduce the probability of the drawing pin landing point "UP" and point "Down"

4 **In the spinner game :**

The disc is divided into two unequal parts X and Y The Pointer is rolled 800 times.

It stands 197 times at the X-zone

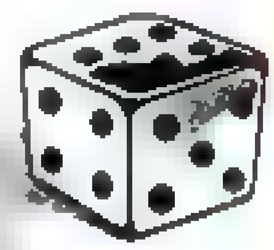
In which of the following figures the pointer points to X-zone ?



(a) (b) (c) (d)

Second | Problems on theoretical probability

1 **As throwing a fair die and observing the upper face , complete the following :**



(1) The probability of appearance a number greater than 2 = ...

(2) The probability of appearance a number less than 3 = ...

(3) The probability of appearance an even number = ...

(4) The probability of appearance the number 4 = ...

(5) The probability of appearance the number 7 = ...

(6) The probability of appearance a number less than or equal 6 = ...

(7) The probability of appearance a prime number = ...

(8) The probability of appearance a prime even number = ...

(9) The probability of appearance a number divisible by 5 = ...

(10) The probability of appearance the number 5 or the number 6 = ...

2 **Complete the following :**

(1) The probability of occuring the impossible event = ... and the probability of occuring the certain event = ...

(2) If a coin is flipped once , then the probability of appearance of a head = ...

(3) 10 cards numbered from 1 to 10. If a card is drawn randomly , then the probability that the card is numbered by an odd number = ...

(4) A box has 5 white balls , 7 red balls , 3 blue balls. If a ball is drawn randomly from the box , then the probability that the ball is blue = ...

(5) In the experiment of throwing a fair die once and observing the upper face , the probability that the apparent number is less than 1 = ...

(6) If one of the digits of the number 867742231 is selected randomly , then the probability that the selected number is even equals ....

(7) A box contains 48 oranges and 4 oranges of them are bad. If an orange is drawn randomly , then the probability that the drawn orange is bad = ... and the probability that the orange is good = ...

(8) If the probability of occuring an event is $\frac{5}{8}$, then the probability that the event doesn't occur = ...

(9) An activity room has 3 doors numbered from 1 to 3. If a student went out using one of them , then the probability that the student went out using the door number 2 is ....

(10) If the probability that a person get infected (in a city whose number of inhabitants 200000) with a disease is 0.003 , then the expected number of infected persons with the disease in this city is .... persons.

3 **Choose the correct answer from those given :**

(1) Which of the following is the probability of occurrence of an event ?

(a) 1.2 (b) −0.4 (c) 315% (d) 75%

(2) As throwing a fair die once, the probability of appearance of a number greater than 4 is ....

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

(3) A basket contains cards numbered from 1 to 20 If a card is drawn randomly , what is the probability that the number written on it is divisible by 6 ?

(a) $\frac{3}{20}$ (b) $\frac{4}{20}$ (c) $\frac{5}{20}$ (d) $\frac{6}{20}$

(4) A bag has 5 red balls and 3 white balls. If the balls are similar and a person draws a ball randomly , then the probability that the drawn ball is white = ...

(a) $\frac{3}{5}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{5}{3}$

(5) 📖 A letter is selected randomly from the name "ZAMALEK". The probability of selecting the letter A is ....

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

(6) 📖 Rashad is in grade 7 in a class of 36 students. 16 of them are girls. If a student is selected randomly from the class , what is the probability that the student is a boy ?

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) $\frac{5}{9}$ (d) $\frac{1}{36}$

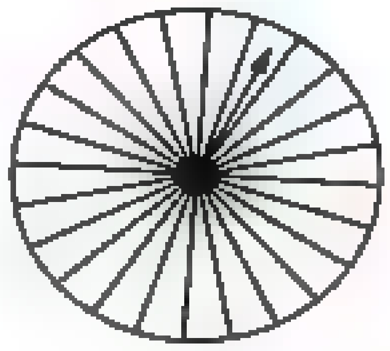
(7) A class has 25 boys and 20 girls. A pupil of them is selected randomly , then the probability that the pupil is a girl = ...

(a) $\frac{1}{20}$ (b) $\frac{4}{9}$ (c) $\frac{1}{25}$ (d) $\frac{5}{9}$

(8) If a die is tossed once , then the probability of getting a number satisfying the inequality $2 < x < 3$ equals ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) zero

(9) The opposite figure shows a spinner with 24 sectors.



When someone spins the arrow it is likely equal to stop on any sector. $\frac{1}{8}$ of sectors are blue , $\frac{1}{3}$ are red , $\frac{1}{2}$ are orange and $\frac{1}{24}$ are purple. If a person spins the arrow , at which colour of sector is the spinner least likely to stop ?

(a) Blue. (b) Purple. (c) Orange. (d) Red.

(10) If the probability of success of a student is 70% , then the probability of his failure = ...

(a) 0.7 (b) 0.07 (c) 0.3 (d) 0.03

4 A card is drawn from a bag of 25 cards numbered from 1 to 25
Calculate the probability that the drawn card carries :

(1) A number divisible by 5
(2) A number ≥ 20
(3) A perfect square number.

5 One card is selected randomly from 8 cards numbered from 1 to 8 Write down the sample space. Then find the probability of the following events :

(1) Getting an even number.
(2) Getting an odd number.
(3) Getting a number greater than or equal to 6
(4) Getting a number divisible by 3

6 A letter is selected randomly from the word "SAMEH" Calculate the probability of selecting the letter :

(1) S (2) E (3) R

7 A bag contains 5 red balls , 3 yellow balls and 2 black balls. If all balls are alike and a ball is drawn from the bag randomly , find :

(1) The probability that the drawn ball is yellow.
(2) The probability that the drawn ball is yellow or red.
(3) The probability that the drawn ball is not yellow.

8 A card is chosen randomly from ten cards numbered from 1 to 10. What is the probability that the selected card shows :

(1) An odd number. (2) A prime number.
(3) An even number. (4) An odd number greater than 3

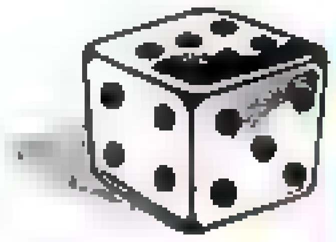
9 If a fair die is tossed once , what is the probability of each of the following events :

(1) Appearance of an even number less than or equal 4

(2) Appearance of a number between zero and 10

(3) Appearance of a number divisible by 7

(4) Appearance of a number that is not divisible by 2

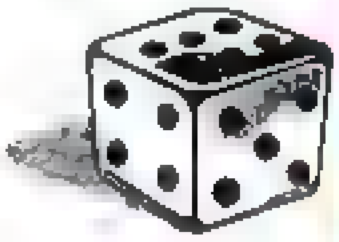


10 A fair die is rolled once and the number of dots on the upper face is observed. Write down the sample space , then find the probability of the following events :

(1) Getting a number greater than 6

(2) Getting a number satisfying the inequality : $1 \leq X \leq 6$

(3) Getting a number satisfying the inequality : $2 < X < 4$



11 8 cards , numbered by the opposite numbers , are put in a bag. Bassim drew a card from these cards randomly. Find :

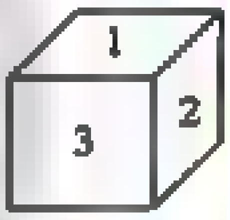
12	18	10	48
24	15	36	17

(1) The probability that the card carries a number whose tens digit is even.

(2) The probability that the card carries a number whose units digit is odd.

(3) The probability that the card carries a number multiple of 4

12 A cube is designed such that two opposite faces carry one of the digits 1 , 2 and 3 The cube is rolled and the apparent face is observed :



(1) Write down the sample space.

(2) What is the probability such that the number on the upper face is 2 ?

(3) What is the probability such that the number on the upper face is odd ?

13 A bag contains 30 similar marbles. Hani drew a marble randomly and he found it red. If the probability of drawing a red ball = $\frac{2}{5}$, find the number of red marbles in the bag.

14 A box contains 80 similar balls. Some of them are red and the rest is blue. If the probability of drawing a red ball is $\frac{1}{4}$, find the number of blue balls.

15 The set $\{2 , 3 , 5\}$ is used in writing a 2-digit number. Find the probability of the following events :

(1) The tens digit is odd.

(2) The units digit is odd.

(3) The sum of the two digits is 7

(4) The product of the two digits is 15

Lesson Two

16 Wael has a bag containing 22 marbles , 12 of them are black and the rest is red. If two marbles of them are drawn without returning them to the bag and they were red. Then he drew a third marble without looking at it. What is the probability that the last marble is black ?

17 A class has 50 students. The number of girls is less than the number of boys by 10
If one student is chosen randomly , find the probability that the student is a boy ?

18 **Choose the correct answer from those given :**

(1) A bowl contains 32 coloured beads. All beads are of the same size. Some of them are blue , some are green , some are red and the rest is yellow the probability of drawing a blue bead is $\frac{3}{8}$, how many blue beads are in the bowl ?

(a) 4 (b) 8 (c) 12 (d) 16

(2) A bag contains 3 white balls , 2 black and one red. If a ball is drawn randomly from the bag , then the probability that the drawn ball is not black equals ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

(3) A bag has a number of similar balls. Half of them is red , the third is black and the rest is white . A ball is drawn randomly , then the probability that the drawn ball is white = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) zero

(4) A box contains coloured balls (red , green , blue and yellow). If the box contains 20 yellow balls and the probability of drawing a yellow ball from the box randomly is $\frac{1}{4}$. What is the number of all balls in the box ?

(a) 5 (b) 25 (c) 60 (d) 80

(5) The number of pupils in a class (7 grade) is 36 pupils. The probability of choosing a pupil whose age is less than or equal to 13 is $\frac{1}{6}$. What is the number of pupils whose ages are more than 13 years old ?

(a) 23 (b) 24 (c) 30 (d) 32

(6) In a mixed school , if the ratio between the number of boys to the number of girls is 7 : 9. A student is selected randomly from the pupils of this school. The probability that the selected student is a boy equals ....

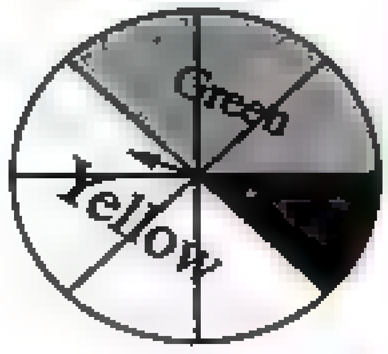
(a) zero (b) $\frac{7}{16}$ (c) $\frac{9}{16}$ (d) 7

(7) A small box contains 25 tickets numbered from 1 to 25 A large box contains 50 tickets numbered from 1 to 50 without looking at them , a ticket is picked from one of the two boxes. Which box would give the larger chance of picking a ticket with the number 17 ?

(a) The larger box. (b) The smaller box.

(c) Both would give the same chance. (d) The given information is not enough.

19 The opposite figure represents the spinning game. Find :



(1) The probability that the pointer stops at.

(a) Red colour.

(b) Green colour.

(c) Yellow colour.

(2) The probability that the pointer does not stop at the red colour.

---

20 The opposite spinning game is divided into 8 sectors of the same area. $\frac{1}{8}$ of the sectors is coloured in red , and $\frac{1}{4}$ of the sectors is coloured in green , $\frac{3}{8}$ of the sectors is coloured in blue and the rest in yellow.

If the pointer of the spinner is spinned , what is the probability that the pointer stops at the yellow or the red colour ?

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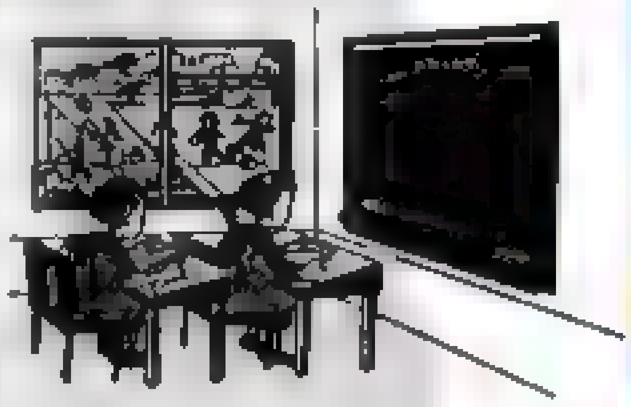
21 A class contains 40 students 30 of them succeeded in maths. 24 succeeded in science and 20 succeeded in both.

A student is chosen randomly. Find the probability that this student :



(1) Succeeded in maths.

(2) Succeeded in science.

(3) Failed in science.

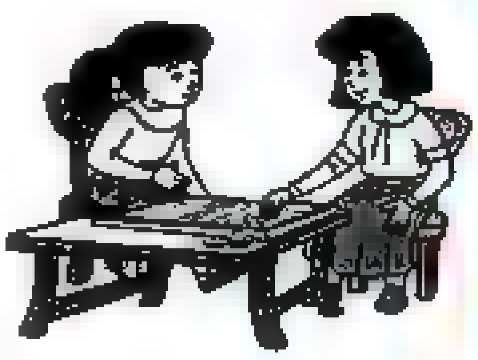
(4) Failed in both maths and science.

---

22 Two players play in a football team. During training , one of them shot 21 penalty kicks and he scored 18 goals and the other shot 32 penalty kicks and he scored 25 goals. which of them should you choose to shoot a penalty kick ? Why ?

---

23 Maryam and Souad played together with two dice. If the product of the two apparent numbers on the upper face is even , then Souad wins the game. If the product of those numbers is odd , then Maryam wins :

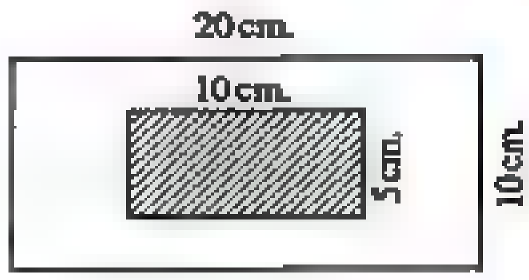


(1) On your opinion , is this system of the game fair ? Why ?

(2) If it is not fair , determine which one of the two girls has the greater chance to win ? Why ?

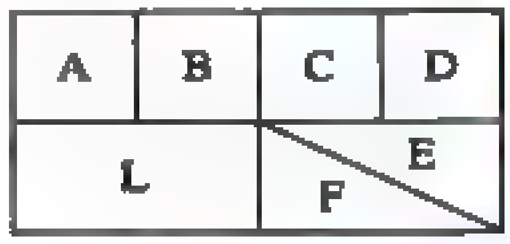
24 In the opposite figure :

If a person shot towards the drawn board ,
find the probability of shooting the shaded part.



For excellent pupils

25 In the game of the target and the arrow , the target was in the shape of a rectangle divided into parts as shown in the opposite figure :



(1) Find the probability of shooting the part E

(2) Find the probability of shooting the part formed from A , B and C together.

---

26 A bag contains a number of similar balls , 5 of them are white and the rest are red.
If the probability of drawing a red ball is $\frac{2}{3}$
Find the total number of balls.

---

27 A card is drawn from a group of cards numbered from 1 to n. If the probability that the drawn card carries a number greater than 8 is $\frac{1}{3}$, then find the value of n

---

28 When you throw a regular die two successive times and notice the upper face. Find the probability of appearance of the number 3 in the two times.

Worksheets

on

Algebra and Statistics

Worksheet 1 on lesson 1 unit 1

Answer the following questions :

1 Choose the correct answer from those given :

(1) $\left(\frac{1}{3}\right)^4 = \dots\dots$

(a) $\frac{1}{27}$ (b) $\frac{4}{81}$ (c) $\frac{1}{81}$ (d) $\frac{4}{27}$

(2) The multiplicative inverse of the number $\left(-\frac{3}{4}\right)^{zero}$ is $\dots\dots$

(a) -1 (b) $-\frac{4}{3}$ (c) $\frac{4}{3}$ (d) 1

(3) The additive inverse of the number $(-2)^3 = \dots\dots$

(a) 8 (b) -8 (c) -4 (d) 6

(4) $\left(-1\frac{1}{4}\right)^3 = \dots\dots$

(a) $\frac{125}{64}$ (b) $-\frac{125}{64}$ (c) $\frac{25}{16}$ (d) $-\frac{1}{64}$

(5) If $a = b$, then $\left(\frac{5}{7}\right)^{a-b} = \dots\dots$

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$ (c) 1 (d) zero

(6) If $x = -\frac{2}{3}$, $y = 2$, then $x^y = \dots\dots$

(a) $\frac{4}{9}$ (b) $-\frac{4}{9}$ (c) $\frac{8}{27}$ (d) $-\frac{8}{27}$

2 Complete the following :

(1) $\frac{64}{125} = \left(\frac{4}{5}\right)^{\dots\dots}$

(2) $\left(\frac{3}{5}\right)^2 \times \left(\frac{5}{3}\right)^{zero} = \dots\dots$

(3) $\left(-\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 = \dots\dots$

(4) $9 \times \left(\frac{1}{3}\right)^2 = \dots\dots$

(5) $\left(-\frac{1}{3}\right)^3 \times \left(\frac{3}{2}\right)^2 = \dots\dots$

(6) If $a = -3$, $b = -2$, then $\left(\frac{b}{a}\right)^3 = \dots\dots$

3 Find the value of each of the following putting the result in the simplest form :

(1) $\left(2\frac{1}{4}\right) \div \left(-1\frac{1}{2}\right)^2$

(2) $\left(-\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^3 \div \left(-\frac{2}{9}\right)^2$

4 If $x = -\frac{3}{2}$, $y = \frac{1}{2}$, $z = -\frac{3}{4}$,

find in the simplest form the numerical value of each of the following :

(1) $y^2 \div z^2$

(2) $\frac{x^2 + z}{y^2}$

The answer of worksheet 1

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

(2) ⓐ ⓑ ⓒ ⓓ

(3) ⓐ ⓑ ⓒ ⓓ

(4) ⓐ ⓑ ⓒ ⓓ

(5) ⓐ ⓑ ⓒ ⓓ

(6) ⓐ ⓑ ⓒ ⓓ

3

2 (1) ....

(2) ....

(3) ....

(4) ....

(5) ....

(6) ....

3

3 (1) ....

....

(2) ....

....

4

4 (1) ....

....

(2) ....

....

5

Worksheet 2 till lesson 2 unit 1

Answer the following questions :

1 Choose the correct answer from those given :

(1) $5^2 \times 5^3 = \dots\dots$

(a) 5^6 (b) 5^5 (c) 5 (d) 5^{32}

(2) $(a^2)^4 = \dots\dots$

(a) a^6 (b) a^8 (c) a^2 (d) a^4

(3) $\frac{(y^5)^2}{y^3} = \dots\dots$, $y \neq 0$

(a) y^4 (b) y^{13} (c) y^{10} (d) y^7

(4) The additive inverse of the number $\left(-\frac{3}{4}\right)^2 = \dots\dots$

(a) $\frac{9}{16}$ (b) $-\frac{9}{16}$ (c) $-\frac{3}{4}$ (d) $\frac{3}{4}$

(5) The quarter of the number $4^{20} = \dots\dots$

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

(6) $2^5 + 2^5 + 2^5 + 2^5 = \dots\dots$

(a) 8^5 (b) 2^{10} (c) 2^7 (d) 2^{20}

2 Calculate each of the following putting the result in the simplest form :

(1) $\left(\frac{3}{5}\right)^7 \div \left(\frac{3}{5}\right)^5 \times \frac{3}{5}$

(2) $\frac{x^5 \times x^8}{x^3 \times x^2 \times x^4}$ where $x \neq 0$

(3) $\left(-\frac{c^2}{d}\right)^3$

(4) $\left(\left(-\frac{2}{3}\right)^2\right)^3$

3 If $a = -\frac{1}{2}$, $b = 2$, $c = \frac{3}{4}$,

find the numerical value of the expression : $a^3 b^2 + b^2 c - 8 abc$

4 Reduce to the simplest form : $\frac{(-2x^2 y)^3}{(-4xy)^2}$ where $xy \neq 0$:

, then find the numerical value of the result if : $x = 1$, $y = 4$

The answer of worksheet 2

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

(2) ⓐ ⓑ ⓒ ⓓ

(3) ⓐ ⓑ ⓒ ⓓ

(4) ⓐ ⓑ ⓒ ⓓ

(5) ⓐ ⓑ ⓒ ⓓ

(6) ⓐ ⓑ ⓒ ⓓ

3

2 (1) ....

....

(2) ....

....

(3) ....

....

(4) ....

....

4

3 ....

....

....

4

4 ....

....

....

4

Worksheet 3 till lesson 3 unit 1

Answer the following questions :

1 Choose the correct answer from those given :

(1) If $x^{-1} = \frac{1}{2}$, then $x =$ ....

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) -2

(2) $\frac{(-2x^2y)^3}{(-4xy^2)^2} =$ .... , $xy \neq 0$

(a) $\frac{x^3}{2y}$ (b) $-\frac{x^4}{2y}$ (c) $\frac{x^5}{2y^2}$ (d) $\frac{x}{y}$

(3) $\frac{(7xy^{-2})^{zero}}{5^{zero}x^{-3}y^2} =$ .... , $xy \neq 0$

(a) $\frac{y^2}{x^3}$ (b) x^3y^2 (c) $\frac{1}{x^3y^2}$ (d) $\frac{x^3}{y^2}$

(4) $(3^2)^5 =$ ....

(a) 3^5 (b) 3^3 (c) 3^{10} (d) 3^7

(5) If $x = \frac{1}{2}$, $y = \frac{1}{4}$, then $x^2 + y =$ ....

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{9}{16}$ (d) 1

(6) $\left(\frac{m^2}{n^{-3}}\right)^{-1} \times \left(\frac{3m^{-2}}{n^{-2}}\right)^{-2} =$ .... , $mn \neq 0$

(a) $\frac{9m^2}{n^7}$ (b) $\frac{m^2}{9n^7}$ (c) $\frac{m^2}{9n}$ (d) $\frac{9m^6}{n}$

2 Complete the following :

(1) $2x^{-4} = \frac{2}{\cdots\cdots}$

(2) $2\frac{1}{4} = \left(\frac{3}{2}\right)^{\cdots}$

(3) $5^6 \times 5^{-6} = 7^{\cdots}$

(4) If $x = \frac{1}{4}$, $y = \frac{1}{8}$, then : $(x - y)^{-1} =$ ....

(5) $5^{-3}\left(\frac{3}{2}\right)^{zero} =$ ....

(6) $(3a^2)^{-1} = \frac{1}{\cdots\cdots}$

3 Calculate the value of each of the following :

(1) $\frac{5^{-2} \times 5^5}{5^3}$ (2) $\left(\frac{3^4 \times 7^2}{7^3 \times 3^2}\right)^{-1}$ (3) $\frac{x^2y^2 \times x^2y \times y^2}{x^2 \times y^2}$ (4) $\left(-\frac{x^3}{y^2}\right)^{-2}$

4 If $x = \frac{2}{3}$, $y = 6$, $z = \frac{1}{3}$, find the value of the expression : $\frac{x^{-1}y}{z^{-1}} + y$

The answer of worksheet 3

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) (a) (b) (c) (d)

(2) (a) (b) (c) (d)

(3) (a) (b) (c) (d)

(4) (a) (b) (c) (d)

(5) (a) (b) (c) (d)

(6) (a) (b) (c) (d)

3

2 (1) ....

(2) ....

(3) ....

(4) ....

(5) ....

(6) ....

3

3 (1) ....

(2) ....

(3) ....

(4) ....

4

4 ....

....

....

5

Worksheet 4 till lesson 4 unit 1

Answer the following questions :

1 Choose the correct answer from those given :

(1) The additive inverse of the number 3^{-1} is ....

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) 3 (d) -3

(2) $(2\ a)^3 =$ ....

(a) $2\ a^3$ (b) 8 a (c) $8\ a^3$ (d) 32 a

(3) Which of the following $= \frac{1}{4}$ million ?

(a) 25×10^5 (b) 0.25×10^5 (c) 0.25×10^6 (d) 0.25×10^7

(4) $\left(\frac{2}{5}\right)^{-1} + \frac{5}{2} =$ ....

(a) 1 (b) $\frac{5}{2}$ (c) $\frac{25}{4}$ (d) $\frac{4}{25}$

(5) $2.37 \times 10^{-4} =$ ....

(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

(6) $3\ y^{-1} =$ .... , $y \neq 0$

(a) $\frac{1}{3}\ y$ (b) $\frac{3}{y}$ (c) y^{-3} (d) y^3

2 Write each of the following numbers on its standard form (scientific form) :

(1) -2540000 (2) 0.000046

(3) 0.7×10^{-7} (4) 0.0435×10^9

3 [a] If $a = -\frac{1}{2}$, $b = 2$, $c = \frac{3}{2}$, find the numerical value of the expression : $a^2\ b^3 + (a + c)^5$

[b] Calculate the value of each of the following :

(1) $\frac{(3^{-2})^4}{3^{-5} \times 3^{-2}}$ (2) $(3^{zero} \times 2^{-2})^{-2}$ (3) $\frac{(-3)^7 \times (-3)^{-2}}{(-3)^5}$

4 Write the result of each of the following on the standard form :

(1) $(5.8 \times 10^7) + (3.2 \times 10^5)$ (2) $(65.5 \times 10^{-2}) \div (5 \times 10^2)$

(3) 60000×5000

The answer of worksheet 4

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) (a) (b) (c) (d)

(2) (a) (b) (c) (d)

(3) (a) (b) (c) (d)

(4) (a) (b) (c) (d)

(5) (a) (b) (c) (d)

(6) (a) (b) (c) (d)

3

2 (1) ....

(2) ....

(3) ....

(4) ....

4

3 [a] ....

....

....

[b] (1) ....

(2) ....

(3) ....

5

4 (1) ....

(2) ....

(3) ....

3

Worksheet 5 till lesson 5 unit 1

Answer the following questions :

1 Choose the correct answer from those given :

(1) If $0.000237 = 2.37 \times 10^n$, then $n =$ ....

(a) 4 (b) 2 (c) -4 (d) -2

(2) $(x^{-2})^3 =$ .... , $x \neq 0$

(a) x^{-6} (b) x^{-5} (c) x (d) x^6

(3) $-11 + 3 \times 7 =$ ....

(a) 11 (b) 10 (c) -56 (d) -1

(4) $3^2 + 3^2 + 3^2 =$ ....

(a) 3^3 (b) 3^8 (c) 1^2 (d) 3^6

(5) The number $1\frac{9}{16} = (\text{..........})^2$

(a) $1\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$

(6) Twice the number $2^{18} =$ ....

(a) 2^9 (b) 2^{36} (c) 2^{10} (d) 2^{19}

2 Calculate the value of each of the following :

(1) $8 \times 2^2 - 7 \times (4 + 1)$

(2) $2\left((5^2 + 1) - (4^2 - 1)\right)$

(3) $\frac{5 + 2 \times 5}{2^2 + 1} + 5^2 - 5$

(4) $16 \div \frac{1}{4} - \frac{3}{4} \times 10^2 + 25$

3 **[a] Find the result in the standard form :** $(4.4 \times 10^3) \times (2 \times 10^5)$

[b] Find the value of : $\frac{7^{-2} \times 7^5}{7^3}$

4 **[a] If** $x = -\frac{1}{2}$, $y = \frac{3}{4}$ **, find the numerical value of :** $x^3 y^2$

[b] If $x = 5$, **find the value of the expression :** $2\left(\frac{3x + 6}{4x - 13}\right)$

The answer of worksheet 5

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) (a) (b) (c) (d)

(2) (a) (b) (c) (d)

(3) (a) (b) (c) (d)

(4) (a) (b) (c) (d)

(5) (a) (b) (c) (d)

(6) (a) (b) (c) (d)

3

2 (1) ....

(2) ....

(3) ....

(4) ....

4

3 [a] ....

....

[b] ....

....

4

4 [a] ....

....

[b] ....

....

4

Worksheet 6 till lesson 6 unit 1

Answer the following questions :

1 Choose the correct answer from those given :

(1) $\sqrt{(-7)^2} = \dots\dots$

(a) 49 (b) 7 (c) -7 (d) ± 7

(2) $\sqrt{10^2 - 8^2} = \dots\dots$

(a) 2 (b) 6 (c) ± 2 (d) ± 6

(3) $4 + 2 \times 3 = \dots\dots$

(a) 18 (b) 10 (c) 14 (d) 24

(4) The multiplicative inverse of the number 3^{-2} is $\dots\dots$

(a) $\frac{1}{3}$ (b) 9 (c) $\frac{1}{9}$ (d) 3

(5) If the thickness of a piece of paper = 0.012 cm. , then which of the following is the height of a ream of 600 pieces of paper ?

(a) (72×10^{-3}) cm. (b) (72×10^{-2}) cm.
(c) 72 cm. (d) (7.2×10^{zero}) cm.

(6) $\sqrt{\sqrt{16}} = \dots\dots$

(a) 16 (b) 8 (c) 4 (d) 2

2 Complete the following :

(1) $\frac{5^{-2} \times 5^{3}}{5^{4}} = 5^{\dots}$

(2) $-\sqrt{9+16} = -3 + \dots\dots$

(3) $(4\frac{1}{2})^3 \div (2\frac{1}{4})^3 = \dots\dots$

(4) A square is of side length $\frac{2x}{3}$ length unit , then its area is $\dots\dots$ square unit.

(5) If $x = \frac{1}{2}$, $y = \frac{1}{4}$, then $(x - y)^{-1} = \dots\dots$

(6) $2^5 \times \dots\dots = 2^{10}$

3 [a] A circle whose area is 154 cm^2. Find the length of its radius

[b] Reduce to the simplest form : $(-\frac{1}{2})^3 \times \sqrt{\frac{25}{9}} \times \sqrt{(\frac{8}{5})^2} \times 3^{-1}$

4 [a] If $a = \frac{1}{3}$, $b = -\frac{2}{3}$, **find the value of :** $|(a^3 \div b^3)^{-1}|$

[b] Calculate the value of : $5\left((2^2 - 1) - (2^2 - 2)\right)$

The answer of worksheet 6

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

3

(1) (a) (b) (c) (d)

(2) (a) (b) (c) (d)

(3) (a) (b) (c) (d)

(4) (a) (b) (c) (d)

(5) (a) (b) (c) (d)

(6) (a) (b) (c) (d)

2 3

(1) ....

(2) ....

(3) ....

(4) ....

(5) ....

(6) ....

3 4

[a] ....

....

[b] ....

....

3 5

[a] ....

....

[b] ....

....

Worksheet 7 till lesson 7 unit 1

Answer the following questions :

1 Choose the correct answer from those given :

(1) $\sqrt{10^2 - 6^2} = \dots\dots$

(a) 4 (b) 8 (c) ± 4 (d) ± 8

(2) The number 0.0000014 = ...

(a) 1.4×10^{-5} (b) 1.4×10^{5} (c) 1.4×10^{-6} (d) 1.4×10^{6}

(3) $\frac{9}{x^{-2}} \times \frac{x^{-4}}{3} = \dots\dots$

(a) $3\,x^{-6}$ (b) $\frac{3}{x^2}$ (c) $\frac{3}{x^6}$ (d) $\frac{3}{x^8}$

(4) The S.S. of the equation : $2\,x = -6$ in $\mathbb{Z}$ is ....

(a) $\{-3\}$ (b) $\varnothing$ (c) $\{-6\}$ (d) $\{0\}$

(5) $(a^3)^4 = \dots\dots$

(a) a^{43} (b) a^7 (c) a^{-1} (d) a^{12}

(6) If : $3\,y = 6$, then $6\,y = \dots\dots$

(a) 2 (b) 3 (c) 6 (d) 12

2 Complete the following :

(1) $\sqrt{36} + \sqrt{16} = \sqrt{\dots\dots}$

(2) If the middle number of three consecutive natural numbers is x , then the sum of these three numbers in the simplest form = ...

(3) $(2\,b^{-1})^{-1} = \dots\dots$

(4) $2 \times 6 - 4 + 2$

3 [a] Two natural numbers , the difference between them is 5 and their sum is 15 find the two numbers.

[b] Reduce to the simplest form : $\sqrt{\frac{49\,x^2\,y^2}{25}}$

4 **[a] Find the S.S. of the following equation in $\mathbb{Q}$:** $3\,(2\,x - 1) = 15$

[b] Find the value of : $(-\frac{1}{3})^2 + \sqrt{\frac{64}{81}} - (\frac{3}{7})^{zero}$

The answer of worksheet 7

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

(2) ⓐ ⓑ ⓒ ⓓ

(3) ⓐ ⓑ ⓒ ⓓ

(4) ⓐ ⓑ ⓒ ⓓ

(5) ⓐ ⓑ ⓒ ⓓ

(6) ⓐ ⓑ ⓒ ⓓ

3

2 (1) ....

(2) ....

(3) ....

(4) ....

2

3 [a] ....

....

[b] ....

....

5

4 [a] ....

....

[b] ....

....

5

Worksheets on Algebra & Statistics

The answer of worksheet 8

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ	ⓑ	ⓒ	ⓓ
(2) ⓐ	ⓑ	ⓒ	ⓓ
(3) ⓐ	ⓑ	ⓒ	ⓓ
(4) ⓐ	ⓑ	ⓒ	ⓓ
(5) ⓐ	ⓑ	ⓒ	ⓓ
(6) ⓐ	ⓑ	ⓒ	ⓓ

3

2 (1) .... (2) ....

(3) .... (4) ....

2

3 [a] ....

....

[b] ....

....

5

4 [a] ....

....

[b] ....

....

5

Worksheet 9 till lesson 2 unit 2

Answer the following questions :

1 Choose the correct answer from those given :

(1) If $2x = 3x$, then $x =$ ....
(a) 2 (b) -2 (c) zero (d) 1

(2) If $x = 28 \div 4 + 3 - 2 \times 5$, then $x =$ ....
(a) -6 (b) 12 (c) 10 (d) zero

(3) Twice the number $2^{18} =$ ....
(a) 2^9 (b) 2^{26} (c) 2^{10} (d) 2^{19}

(4) $x^4 \div x^{-2} =$ ....
(a) x^2 (b) x^{-6} (c) x^{-8} (d) x^6

(5) As flipping a fair coin once , the probability of appearing a head is ....
(a) 5 (b) 5 % (c) 50 % (d) $\frac{1}{5}$

(6) The probability of the impossible event = ...
(a) 1 (b) 2 (c) zero (d) $\frac{1}{2}$

2 Complete the following :

(1) $2^3 \times 2^{-3} = 5^{\cdots\cdots}$

(2) If $x > 4$, then $-x$ .... (-4)

(3) Half a million = ... (in the standard form).

(4) If the probability of occuring an event is $\frac{5}{8}$, then the probability of not occuring of the same event = ...

3 [a] The sum of three consecutive natural numbers is 33. Find these numbers.

[b] **Reduce to the simplest form** $\frac{x^2 \times x^7}{x^3 \times x^2}$

, then find the value of the result when : $x = -3$

4 [a] Half of the area of a square is 18 cm^2. Find its side length.

[b] A card is drawn randomly from 8 cards numbered from 1 to 8 Write the sample space , then find the probability of each of the following :

(1) Appearing an even number. (2) Appearing a number divisible by 3

The answer of worksheet 9

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

(2) ⓐ ⓑ ⓒ ⓓ

(3) ⓐ ⓑ ⓒ ⓓ

(4) ⓐ ⓑ ⓒ ⓓ

(5) ⓐ ⓑ ⓒ ⓓ

(6) ⓐ ⓑ ⓒ ⓓ

3

2 (1) ....

(2) ....

(3) ....

(4) ....

2

3 [a] ....

....

[b] ....

....

5

4 [a] ....

....

[b] (1) ....

(2) ....

5

Model Examinations of the school book

Model 1

Answer the following questions :

1 Complete :

(1) $2 \times 6 - 4 \div 2 =$ ....

(2) If : $7 - 2x = 3$, then $x =$ ....

(3) If : $3x + 1 \geq 10$, then $x \geq$ ....

(4) The standard form of the number $0.7 \times 0.005 =$ ....

(5) A class has 36 students , the number of boys are 20 , if a student is chosen randomly , then the probability that the student is a girl = ...

2 Choose the correct answer :

(1) The sum of the probabilities for all possible outcomes of a randomly experiment is ....

(a) zero (b) 1 (c) > 1 (d) < 1

(2) If : $3a = \sqrt{4}\, b$, then $\frac{a}{b} =$ ....

(a) 2 : 3 (b) 3 : 2 (c) 3 : 4 (d) 4 : 3

(3) $\left(\frac{-2}{3}\right)^{-3}$ equals ....

(a) $\frac{-27}{8}$ (b) $\frac{-8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{27}{8}$

(4) There are 21 boys and 15 girls in a classroom , one pupil is chosen randomly , the probability that the chosen pupil is a girl = ...

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{4}{7}$ (d) $\frac{5}{6}$

3 Simplify to the simplest form : $\left(-\frac{3}{7}\right)^0 \times \left(\frac{-2}{5}\right)^2 \times \sqrt{6\frac{1}{4}}$

4 [a] Find in Q the S.S. of the following :

(1) $3x + 1 = 25$ (2) $2x + 5 < 16$

[b] The population of a city has been growing according to the rule :

$y = 3\,(1.02)^n$ million. Calculate the population that will be in 2 years in the standard form.

5 **A factory of a tire record the distance that traveled by a certain type of them before damage for 800 units of this type as following.**

The distance in thousand (km.)	Less than 50	50 to 100	More than 100 till 150	More than 150
The number of damage tire	80	120	280	320

If you bought the type of this tyre , what is the probability of change it :

(1) Before traveled 50 thousand km.

(2) After traveled more than 100 thousand km.

Model (2)

Answer the following questions :

1 **Complete :**

(1) $\left(\frac{-2}{3}\right)^0 =$ .... (2) $\sqrt{\frac{16}{49}} =$ ....

(3) The probability of impossible event = ...

(4) 1 , 2 , 3 , 5 , 8 , ... , ... (In the same pattern)

(5) If the probability that the student is absent in a school is 0.15 , if the number of students of this school is 600 , then the number of the present student that day is ....

2 **Choose the correct answer :**

(1) $2^3 \times 2^5 =$ ....

(a) 2^2 (b) 2^8 (c) 2^{15} (d) 2^{53}

(2) Which of the following the greatest ? ....

(a) 2.3×10^4 (b) 2.3×10^5 (c) 3.2×10^4 (d) 3.2×10^5

(3) The side length of a square whose area $9x^2$ cm^2 is .... cm.

(a) $3x$ (b) $3x^2$ (c) $9x$ (d) $9x^2$

(4) Which of the following may be probability of an event ? ....

(a) – 0.25 (b) 87 % (c) 1.05 (d) 130 %

3 Two integers number the smaller one is $2x$ and the greater is $5x$, if the difference between them is 30 Find the two numbers.

4 **[a] Find in $\mathbb{Q}$ the S.S. of each of the following :**

(1) $(3x+2)+5=13$ (2) $2x+15<19$

[b] Find the value of the expression in simplest form :

$$\left(\frac{-2}{3}\right)^2+\sqrt{\frac{64}{81}}-\left(\frac{3}{7}\right)^0$$

5 If a regular die is thrown once and observed the number on upper face , find the probability of each of the following :

(1) Getting prime even number.

(2) Getting odd number less than 4

Model 3

Answer the following questions :

1 **Complete :**

(1) The probability of the certain event = ...

(2) $\frac{1}{9}$, $\frac{1}{8}$, $\frac{1}{7}$, $\frac{1}{6}$, ... , ... (In the same pattern)

(3) The S.S. in $\mathbb{Q}$ for the equation : $2x+3=4$ is ....

(4) $0.00037=3.7\times10^n$, the value of n = ...

2 **Choose the correct answer :**

(1) The multiplicative inverse of the number : $\sqrt{\frac{9}{16}}$ is ....

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

(2) $\frac{x}{2}<5$ equivalent ....

(a) $x<\frac{5}{2}$ (b) $x>\frac{5}{2}$ (c) $x<10$ (d) $x>10$

(3) $3^x+3^x+3^x$ equals : ....

(a) 3^x (b) 3^{x+1} (c) 27^x (d) $3x^3$

(4) There are 480 pupils in a school , 120 of them failed. A pupil is chosen at random , then the probability that the pupil is succeeded ....

(a) 0.25 % (b) 0.75 (c) 0.8 (d) 0.667

3 What is the number which if we add it to its three times , the result is 28 ?

4 **[a] Find in $\mathbb{Q}$ the solution set of the following :**

(1) $3x+5=11$ (2) $2x+3\leq 7$

[b] If the distance between the sun and the earth is 1.44×10^8 km. and the light velocity is 3×10^8 m/sec. **Calculate the elapsed time that the light takes to reach from the sun to the earth.**

5 **[a] Find the result of the expression :** $(5.4\times 10^4)+(3.7\times 10^5)$ in the form $a\times 10^n$ where n is integer number.

[b] A coin is tossed twice calculate the probability :

(1) The two faces are similar. (2) Appearance only one tail.

Model 4

Answer the following questions :

1 **Complete :**

(1) When a coin is tossed once then the probability of appearance of a head is ....

(2) $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, ... (In the same pattern)

(3) The S.S. of the inequality $2<x\leq 4$ in $\mathbb{N}$ is ....

(4) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, ... , ...

2 **Choose the correct answer :**

(1) The S.S. of the equation : $x+3=3$ in $\mathbb{N}$ is ....

(a) $\varnothing$ (b) $\{0\}$ (c) $\{3\}$ (d) $\{6\}$

(2) The number which in the standard form between the following numbers is ....

(a) 11×10^8 (b) 9.7×10^{-5} (c) 10.3×10^{-3} (d) 0.87×10^8

(3) If a coin is tossed 160 times then the approximate expected number of the appearance of a head is ....

(a) 60 (b) 78 (c) 90 (d) 159

(4) The number $\sqrt{0.09}$ is ....

(a) natural. (b) positive integer. (c) negative integer. (d) rational.

(5) If : $\frac{6x}{5}=-2$, then $x^2=$ ....

(a) $\frac{-25}{9}$ (b) $\frac{5}{9}$ (c) $\frac{25}{9}$ (d) $\frac{25}{3}$

3 [a] If : $x = \frac{3}{4}$, $y = \frac{-3}{2}$ Find the numerical value of the expression $\left(\frac{x^2}{y^3}\right)^2$

[b] The sum of two natural number is 15 and the difference between them is 5

Find the two number.

4 **Find in $\mathbb{Q}$ the solution set for each of the following :**

(1) $3x + 2 = 8$ (2) $4x - 3 < 7$

5 **A coin is tossed twice Calculate the probabilities :**

(1) The appearance of at least one head.

(2) The appearance of at most one head.

Model 5

Answer the following questions :

1 **Complete :**

(1) In the experiment of tossing a die once then the probability of appearance even number is ....

(2) If : $\frac{x}{y} = \frac{7}{2}$, then $\frac{2x}{7y} = $ ....

(3) If : $a = 0.000625$, then $\sqrt{a} = 2.5 \times 10^{\cdots\cdots}$

(4) The result of the expression : $\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right)^3 = $ ....

(5) Quarter of 4^{20} equals ....

2 **Choose the correct answer :**

(1) The S.S. of the inequality $x < 2$ in $\mathbb{N}$ is ....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0 , 1\}$ (d) $\varnothing$

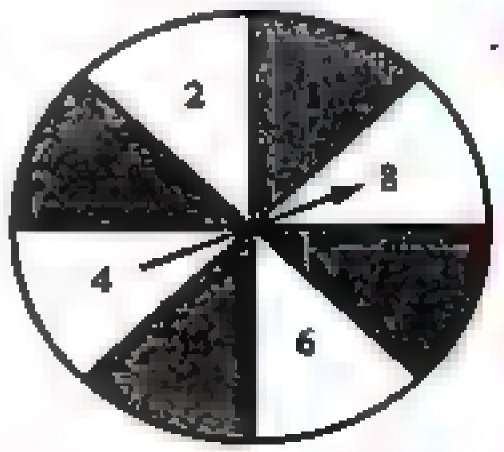
(2) If : $\frac{26}{x} + 1 = 14$, then $x = $ ....

(a) 2 (b) 10 (c) 13 (d) 20

(3) If : $5x = 35$, then $2x + 1 = $ ....

(a) 7 (b) 8 (c) 15 (d) 71

(4) **In the opposite figure :**



The probability that the pointer stop at a number greater than 6 equals ....

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{3}{4}$

3 **Find the value of the expression :** $12 \times (2)^2 \div 24 + 3^2$

4 **[a] Find in $\mathbb{Q}$ the solution set of each of the following :**

(1) $3 - 4X = -5$ (2) $2X - 1 \geq 5$

[b] Simplify : $\frac{n}{2}[3n - 6] + \frac{1}{2}[6 - 2n]$ **, then find its value when** $n = 1$

5 **[a]** The sum of the age of 3 sisters now is 25 years. If the eldest was born before the middle by 3 years , and the middle was born before the youngest by 2 years.

Find the age of each of them now.

[b] A box contains 4 white , 5 red and 6 blue balls. A ball is drawn randomly from the box. **Calculate the probabilities of the following events.**

(1) The ball is red.

(2) The ball is white or red.

School Examinations

1 Cairo Governorate — Nasr City Educational Zone

St. Fatima Language School

Answer the following questions :

1 Choose the correct answer from those between brackets :

(1) Half the number 2^{20} = ...

(a) 2^{10} (b) 2^{21} (c) 2^{19} (d) 40

(2) $\left(\frac{2}{3}\right)^{-2}$ = ...

(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{-4}{9}$ (d) $-\frac{9}{4}$

(3) If : $5x = 20$, then $x + 3$ = ...

(a) 16 (b) 12 (c) 17 (d) 7

(4) * $\sqrt{\left(-\frac{5}{6}\right)^2}$ = ...

(a) $\frac{5}{6}$ (b) $-\frac{5}{6}$ (c) $\pm\frac{5}{6}$ (d) $\frac{25}{36}$

(5) In an experiment of tossing a metal coin once , the probability of appearance of a head is ....

(a) $\frac{1}{2}$ (b) 1 (c) 0 (d) $\frac{1}{3}$

2 Complete :

(1) * $\sqrt{16+9}$ = ...

(2) If : $a - 3 < 0$, then .... > ....

(3) If : $0.00052 = 5.2 \times 10^n$, then n = ...

(4) The additive inverse of $\left(-\frac{2}{5}\right)^2$ is ....

(5) The probability of the impossible event is ....

3 [a] Solve each of the following inequalities and represent the solution set on the number line where $x \in \mathbb{Z}$:

(1) $8x - 3x - 1 \leq 29$ (2) $x + 4 > 1$

[b] Find each of the following :

(1) $\left(-\frac{2}{3}\right)^2 \times \frac{9}{4} \times \left(-\frac{1}{4}\right)^0$ (2) $\left(-\frac{2}{3}\right)^6 \div \left(-\frac{2}{3}\right)^7$

4 [a] Two natural numbers , one of them is twice the other and their sum is 45

Find the two numbers.

[b] If : $x = \left(-\frac{1}{2}\right)$, $y = \left(\frac{4}{3}\right)$, $z = \left(\frac{1}{5}\right)$, then find the numerical value of $x^3 y^2 z$

5 [a] ✻ **Arrange in an ascending order :** 2.7×10^{-4} , 7.3×10^{-5} , 10^{-4} , 0.25×10^{-2}

[b] One card is selected randomly from 8 cards number from 1 to 8 Write down the sample space , then find the probability of each of following event :

(1) Getting an even number.

(2) Getting a number greater than or equal to 6

2 Cairo Governorate — El-Zeitoun Education Zone

El-Salam Experimental Language School

Answer the following questions :

1 Choose the correct answer :

(1) The third of 3^{20} = ...

(a) 3^{10} (b) 3^{19} (c) 3^{18} (d) 3^{17}

(2) If a die is tossed once then the probability of getting a number satisfies the inequality $2 < x < 3$ equals ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) zero

(3) $\sqrt{\frac{25}{36}}$ = ...

(a) $\frac{\pm 5}{6}$ (b) $\frac{-5}{6}$ (c) $\frac{5}{6}$ (d) otherwise

(4) $\left(\frac{-1}{2}\right)^0$ = ...

(a) -1 (b) $\frac{-1}{2}$ (c) 1 (d) 0

(5) The solution set of the equation : $-2x + 1 = -3$ in $\mathbb{N}$ is ....

(a) {1} (b) {2} (c) {3} (d) {4}

2 Complete the following to get a right statement :

(1) If the age of a man now is x years , then his age 3 years ago is ....

(2) $\sqrt{(91)^2 - 91 \times 2 + 1}$ = ...

(3) The probability of the impossible event is ....

(4) If : $0.000024 = 2.4 \times 10^n$, then n = ...

(5) If : $\frac{x}{5} = \frac{8}{5}$, then $\frac{1}{4}x =$ ....

3 **[a] If : $x = -\frac{3}{2}$, y = $\frac{1}{2}$ and z = $-\frac{4}{3}$, find the numerical value of : $x^2 - yz^2$**

[b] Solve the inequality in $\mathbb{Q}$: $3(x + 2) < -x + 4$

4 **[a] Solve the equation : $5x + 8 = 13 - 2x$, where $x \in \mathbb{Q}$**

[b] A card is chosen at random from ten cards numbered from 1 to 10 , what is the probability that the selected card shows :

(1) An odd number. (2) A prime number.

(3) An odd number greater than 3

5 **[a] Calculate : $\frac{7^{-2} \times 7^5}{7^3}$**

[b] The length of a rectangle exceeds its width by 4 metres and its perimeter is 68 metres. Find the dimenions of the rectangle.

3 Giza Governorate — Omrania Directorate

El-Sadat Exp. Lang.Sch.

Answer the following questions :

1 **Choose the correct answer from the given ones :**

(1) 0.00000027 = ... (In the standard form)

(a) 2.7×10^{-6} (b) 2.7×10^{6} (c) 2.7×10^{-7} (d) 2.7×10^{7}

(2) ✱ $\frac{5a^0}{b^{-2}} =$ .... , $a \neq 0$, $b \neq 0$

(a) $5b^2$ (b) $\frac{b^2}{5}$ (c) 0 (d) 1

(3) If the age of Ahmed now is x years then this age 10 years ago is .... years.

(a) $10x$ (b) $x + 10$ (c) $10 - x$ (d) $x - 10$

(4) As throwing a fair die once , the probability of appearance of prime number on the upper face is ....

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) $\frac{1}{3}$

(5) If : $5x = 20$, then $x + 5 =$ ....

(a) 10 (b) 15 (c) 9 (d) 20

2 **Complete each of the following :**

(1) The multiplicative inverse of $\sqrt{6\frac{1}{4}}$ is ....

(2) $* -(2)^3 \times (-2)^4 = (\text{..........})^7$

(3) The probability of the impossible event = ...

(4) $2 \times 6 - 4 \div 2 =$ ....

(5) If : $x < y$ and z is negative then x z .... yz

3 **[a] Simplify each of the following :**

(1) $\left(\frac{-2}{3}\right)^2 \times \sqrt{\frac{9}{4}} \times \left(\frac{2}{7}\right)^0$ (2) $\frac{(2)^5 \times (-2)^4}{(2)^9}$

[b] Three consecutive natural numbers their sum is 33 Find these numbers.

4 **Find the solution set of each of the following where $x \in \mathbb{Q}$:**

(1) $2x + 1 = 13$ (2) $3x - 1 \le 2x + 4$

5 **[a] If : $x = \frac{-2}{3}$, $y = \frac{1}{2}$ and $z = \frac{-4}{3}$ Find the value of : $x^2 - y^2 z$**

[b] A box contains 5 white , 4 black and 7 red balls. A ball is drawn randomly from the box find the probabilities of the following events :

(1) The ball is black. (2) The ball is not white.

4 Giza Governorate — Kerdasa E. Directorate

Mathematics directing

Answer the following questions :

1 **Choose the correct answer from the given ones :**

(1) $\left(\frac{x}{y}\right)^n \div \left(\frac{x}{y}\right)^m =$ ....

(a) $\left(\frac{x}{y}\right)^{n+m}$ (b) $\left(\frac{x}{y}\right)^{n-m}$ (c) $\left(\frac{x}{y}\right)^{m-n}$ (d) $\left(\frac{x}{y}\right)^{\frac{n}{m}}$

(2) $2.37 \times 10^{-4} =$ ....

(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

(3) $\sqrt{\left(\frac{-5}{6}\right)^2} =$ ....

(a) $\frac{-5}{6}$ (b) $\frac{5}{6}$ (c) $\pm\frac{5}{6}$ (d) $\frac{2}{6}$

(4) ✱ If : $-2x < 6$, then ....

(a) $x < 3$ (b) $x < 4$ (c) $x > 3$ (d) $x > -3$

(5) A class contains 20 boys and 15 girls. If a pupil is chosen randomly then the probability that the pupil is a boy = ...

(a) $\frac{1}{20}$ (b) $\frac{1}{15}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

2 **Complete each of the following :**

(1) $4 + 2 \times 3 =$ ....

(2) The additive inverse of the number $\left(\frac{-1}{2}\right)^2$ is ....

(3) ✱ If your age now is x years then your age 5 years ago was .... years.

(4) ✱ $2x = 9$, then $4x - 1 =$ ....

(5) $\sqrt{16 + 9} =$ ....

3 **[a] Find the S.S. of each of the following :**

(1) $8x + 4 = 12$ where $x \in \mathbb{Z}$ (2) $2x + 3 \leq 7$ where $x \in \mathbb{N}$

[b] ✱ **If :** $a = \frac{1}{3}$, $b = -\frac{2}{3}$ **Find the value of :** $(a^3 + b^3)^{-1}$

4 **[a] Simplify to it's simplest form :**

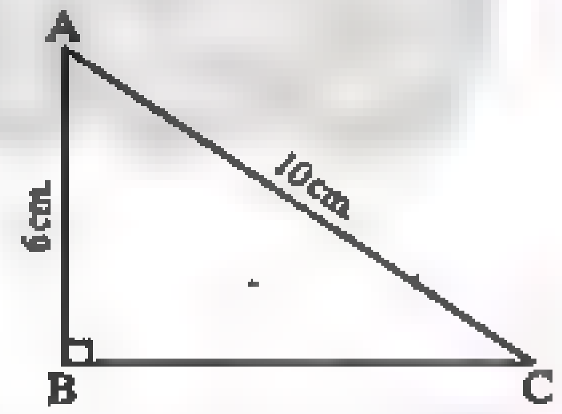
(1) $\frac{(-3)^3 \times (-3)^5}{(-3)^6}$ (2) $\left(\frac{-5}{4}\right)^2 \times \sqrt{\frac{25}{16}} \times \left(\frac{-4}{5}\right)^3$

[b] In the opposite figure :

m ($\angle$ B) = 90° , AB = 6 cm.

and AC = 10 cm.

Find the length of $\overline{BC}$



5 [a] Three consecutive odd natural numbers are with sum 117 **Find this numbers.**

[b] A card is drawn randomly from 8 cards numbered from 1 to 8 write down the sample space then find the probability of each of the following events :

(1) A = event of getting an even number.

(2) B = event of getting a prime number.

(3) C = event of getting a number divisible by 3

5 Alexandria Governorate — El-Montaza Education Zone

Victory College

Answer the following questions :

1 Choose the correct answer :

(1) ✱ The S.S. of : $-x < 0$ in $\mathbb{Z}$ is ....

(a) $\mathbb{Z}$ (b) $\mathbb{Z}_+$ (c) $\mathbb{Z}_-$ (d) $\mathbb{Z}^*$

(2) If : $a = -3$, then $a^{-2} =$ ....

(a) $-\frac{1}{9}$ (b) $\frac{1}{9}$ (c) 9 (d) -9

(3) If : $3x = 5$, then the value of $12x =$ ....

(a) 4 (b) 36 (c) 20 (d) 60

(4) A regular die is rolled , the probability of getting a number less than 3 is ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

(5) Third of $3^{12} =$ ....

(a) 3^{-1} (b) 3^{-11} (c) 3^{11} (d) 1

2 Complete the following :

(1) $\sqrt{\frac{81}{25}} + \left(\frac{3}{5}\right)^2 =$ ....

(2) If : $0.000028 = 2.8 \times 10^n$, then $n =$ ....

(3) If : $x \in \mathbb{N}$, then the solution set of the equation : $x + 7 = 1$ is ....

(4) $x = \frac{1}{2}$ and $y = \frac{3}{4}$, then the numerical value of : $x^2 + y =$ ....

(5) $2^3 \div 4 \times 3 + (2 - 1) =$ ....

3 [a] ✱ **Find the result of :** $\frac{2^3 \times 2^{-4}}{2^{-2} \times 2^5}$

[b] Find the solution set of the following ($x \in \mathbb{Q}$) :

(1) $2x - 5 = 1$ (2) $4x + 9 \le 1$

4 [a] The age of a man is three times the age of his son , if the sum of their ages is 60 years. **Find the age of each of the man and his son.**

[b] If : $x = \frac{1}{2}$, $y = \frac{4}{3}$ and $z = \frac{3}{2}$ **Find the numerical value of :** $8x^3 y z^2$

5 [a] **Calculate :** $\left(\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{2}{5}\right)^0$

[b] A ball was drawn randomly from a sack containing 8 balls numbered from 1 to 8

Find the probabilities of drawing a balls numbered with :

(1) A number that is divisible by 3 (2) A prime number.

6 Kalyoubia Governorate — West Shubra El-Kema Educational Zone

Answer the following questions :

1 Choose the correct answer :

(1) $2^6 \div 2^3 = \dots\dots$

(a) 3^9 (b) 2^2 (c) 3^2 (d) 8

(2) $3^3 \times 3^{zero} = \dots\dots$

(a) 9 (b) 27 (c) 1 (d) zero

(3) $1\frac{9}{16} = (\dots\dots)^2$

(a) $1\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $1\frac{1}{4}$

(4) A class contain 15 girls and 10 boys if a pupil is go out , then the probability that the pupil is a girl is ……….

(a) $\frac{1}{15}$ (b) $\frac{1}{25}$ (c) $\frac{10}{25}$ (d) $\frac{3}{5}$

(5) ✱ If : $x = y$, then $3^{x-y} = \dots\dots$

(a) 1 (b) 3 (c) 3^x (d) $3xy$

2 Complete :

(1) If the probability of success of a students is 90% , then the probability of his failure is ……….

(2) The multiplicative inverse of $\sqrt{\frac{16}{25}}$ is ……….

(3) If : $-x < 5$, then x ………. (-5)

(4) ✱ If : $x + 9 = 11$, then the value of $7x = \dots\dots$

(5) The standard form of the number 7 millions is ……….

3 [a] ✱ **Simplify to the simplest form :** $\frac{x^2 \times x^7}{x^3 \times x^2}$, then find its numerical value when $x = -3$

[b] **Calculate the value of :** $20 \div 5 + 8 - (4 - 1)$

4 **[a] Find the solution set of the inequality :** $3x - 2 > x + 4$, where $x \in \mathbb{Q}$

[b] Find the solution set of the equation : $3x - 1 = 5$ in $\mathbb{Q}$

5 [a] ✱ The sum of three consecutive even numbers is 966 Find them.

[b] A bag contains 5 red balls , 3 white balls and 2 blue balls. If all balls are a like and a ball is drawn from the bag randomly , **find :**

(1) The probability that the drawn ball is red.

(2) The probability that the drawn ball is white.

(3) The probability that the drawn ball is not blue.

7 El-Kalyoubia Governorate — Kalyoub Educational Zone

Answer the following questions :

1 **Choose the correcte answer :**

(1) $5^{-1} = \cdots\cdots$

(a) -1 (b) 5 (c) -5 (d) $\frac{1}{5}$

(2) $2 \times 5 - 6 \div 2 = \cdots\cdots$

(a) 2 (b) 8 (c) 7 (d) $\frac{1}{2}$

(3) The multiplicative inverse of $\left(\frac{2}{3}\right)^0$ is $\cdots\cdots$

(a) $\frac{3}{2}$ (b) $-\frac{2}{3}$ (c) 1 (d) 0

(4) The probability of the impossible event = $\cdots\cdots$

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(5) If : $x + 3 > 5$, then x may be equal $\cdots\cdots$

(a) 5 (b) 1 (c) -5 (d) -2

2 **Complete :**

(1) ✱ $\sqrt{6^2 + 8^2} = \cdots\cdots$

(2) If : $2700 = 2.7 \times 10^n$, then n = $\cdots\cdots$

(3) If : $x + 9 = 11$, then the value of $7x = \cdots\cdots$

(4) ✱ $\left(\frac{-2}{5}\right)^6 \div \left(\frac{2}{5}\right)^4 = \cdots\cdots$

(5) When a coin is tossing once then the probability of getting a head = $\cdots\cdots$

3 [a] Find the result in the simplest form : $\frac{4}{3} \times \sqrt{\frac{9}{25}} \times \left(-\frac{1}{2}\right)^2$

[b] ✱ The length of a rectangle exceeds its width by 4 metres and its perimeter is 68 metres. Find the dimensions of the rectangle.

4 [a] Simplify : $\frac{x^7 \times x^5}{x^8}$, then find the numerical value of the result at $x = -2$

[b] Find in $\mathbb{Q}$ the S.S. of the equation : $3x + 7 = 13$

5 [a] Find in $\mathbb{Z}$ the S.S. of the inequality : $x - 1 < 3$, then represent the solution on number line.

[b] When a die is tossing once then find the probability of the event A appearing an odd number on the upper face.

8 El-Sharkia Governorate

Omar El-Farouk E.L.S.

Answer the following questions :

1 Choose the correct answer :

(1) $\sqrt{36 + 64} = 6 + \cdots\cdots$

(a) 10 (b) 4 (c) 8 (d) 6

(2) The solution set of : $-2x + 1 = -3$ in $\mathbb{Z}$ is $\cdots\cdots$

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{-4\}$ (d) $\varnothing$

(3) The probability of the possible event could be $\cdots\cdots$

(a) -0.9 (b) 1.2 (c) 16 % (d) 5

(4) The multiplicative inverse of $\left(-\frac{3}{4}\right)^2$ is $\cdots\cdots$

(a) $\left(\frac{4}{3}\right)^2$ (b) $\left(-\frac{4}{3}\right)^2$ (c) $\left(\frac{3}{4}\right)^2$ (d) $-\left(\frac{3}{4}\right)^2$

(5) Ali's age 2 years ago was x , then his age now is $\cdots\cdots$ years.

(a) $x + 2$ (b) $x - 2$ (c) $2 - x$ (d) $2x$

2 Complete :

(1) ✱ $2 \times 6 - 4 \div 2 = \cdots\cdots$

(2) The probability of the sure event is $\cdots\cdots$

(3) The solution set of : $1 > x \geq -2$ in $\mathbb{N}$ is $\cdots\cdots$

(4) $0.000237 = \cdots\cdots \times (10)^{\cdots\cdots}$ (in the standard form)

(5) $(2\,a^{-1})^{-1} = \cdots\cdots$ (In the simplest form).

3 [a] A box contains 2 red balls , 6 white balls and 4 green balls. A ball is drawn randomly.

Find the probability that the drawn ball is :

(1) Black. (2) Not red.

[b] If : $\frac{a}{b} \in \mathbb{Q}$, $\frac{a^2}{b^2} = 0.16$, **Find the value of :** $\left(\frac{a}{b}\right)^3$

4 **[a] Simplify :** $\frac{y^3 \times y^{-4}}{y^{-2} \times y^5}$, then find its numerical value at $y = -1$

[b] Find S.S. of : $3\,x - 2 \leq 3 - 2\,x$ in $\mathbb{Q}$

5 [a] Two consecutive even numbers , their sum is 150 Find them

[b] ✱ The population of city has been growing according to the rule :

$y = 3\,(1.02)^n$ million. **Calculate the population that will be in 2 years in the standard form.**

9 El-Beheira Governorate — Damanhour Educational

Safwa Language Schools

Answer the following questions :

1 **Complete :**

(1) $\left(\frac{-3}{7}\right)^7 \div \left(\frac{3}{7}\right)^5 = \cdots\cdots$ (In the simplest form).

(2) If : $A = 0.000625$, then : $\sqrt{A} = 2.5 \times 10^{\cdots\cdots}$

(3) If : $z > y$ and $y > x$, then $z > \cdots\cdots$

(4) ✱ The multiplicative inverse of $\sqrt{\frac{4}{9}}$ is $\cdots\cdots$

(5) A bag contains 36 marbles , Noha draw one randomly it was found red colour. If the probability of getting red marble is $\frac{1}{9}$, then there are $\cdots\cdots$ red marbles in that bag.

2 **Choose the correct answer :**

(1) $\left(\frac{2\,ab^{-2}}{3^0\,a^{-2}\,b}\right)^0 = \cdots\cdots$

(a) $\frac{a^3}{3\,b^3}$ (b) a^2 (c) 1 (d) $\frac{a^2}{b}$

(2) $12 (2^2) \div 24 + 3^2 =$ ....

(a) 4 (b) 5 (c) 11 (d) 13

(3) ✱ The third of the number $3^{18} =$ ....

(a) 3^6 (b) 3^9 (c) 3^{17} (d) 3^{18}

(4) The solution set of the equation : $3x - 13 = 26$ in $\mathbb{Q}$ is ....

(a) {36} (b) {13} (c) {3} (d) {31}

(5) Ebrahim is in a grade 7 class of 36 students and 16 of them are girls. If a student is selected at random from the class , what is the probability that the student is a boy ?

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) $\frac{5}{9}$ (d) $\frac{1}{36}$

3 Two natural numbers , one of them is twice the other and their sum is 108

Find the two numbers.

4 [a] If : $x = \frac{-1}{2}$, $y = \frac{3}{4}$ and $z = \frac{-3}{2}$, then find : $x^3 \div y^2 z^2$

[b] Solve the following inequality in $\mathbb{Q}$: $1 - (4x - 1) > 2(x - 3)$

5 **[a] Without using calculator find in form $a \times 10^n$, $n \in \mathbb{Z}$:**

$(3.8 \times 10^8) \div (1.9 \times 10^6)$

[b] A sample consists of 100 persons who watch T.V. , if some of was selected at random.

What is the probability of that person's preference :

Programs	Documentaries	Drama	News	Sport
Viewers	12	31	21	36

(1) Sport (2) Not watch news

(3) Documentaries (4) all of them

10 El-Dakahlia Governorate

M.L.S

Answer the following questions :

1 **Complete :**

(1) The additive inverse of $\left(\frac{-2}{3}\right)^2$ is ....

(2) $\sqrt{6^2 + 8^2} = 6 +$ ....

(3) ✱ The probability of the impossible event = ...

(4) 0.00000027 = ... (In the standard form)

(5) A cubic die with numbers 1 to 6 is rolled once. The probability of rolling an even number is ... .

2 **Choose the correct answer :**

(1) If : X > Y , then – X .... – Y

(a) = (b) ≥ (c) > (d) <

(2) ✱ $\sqrt{(-5)^2}$ = ...

(a) – 5 (b) 25 (c) 10 (d) $|-5|$

(3) $\left(\frac{1}{2}\right)^5 \div \left(\frac{1}{2}\right)^3$ = ...

(a) $\frac{1}{32}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

(4) If the area of a square is 100 cm^2 , then its side length = ... cm.

(a) 10 (b) 12 (c) 13 (d) 25

(5) The solution set of the equation : $2x + 1 = -3$ in $\mathbb{N}$ is ....

(a) {1} (b) {2} (c) {4} (d) ∅

3 **[a] Evaluate :** $\left(\frac{5}{4}\right)^2 \times \sqrt{\frac{16}{25}} \times \left(\frac{-4}{5}\right)^0$

[b] Solve the inequality : $5x - 7 \leq 3$, $x \in \mathbb{Q}$

4 **[a] Find the result of :** $\frac{(-2)^6 \times (-2)^4}{(-2)^8}$

[b] The sum of three consecutive numbers is 42 **Find them.**

5 **[a]** ✱ If : $x = \frac{3}{4}$ and $y = -\frac{3}{2}$ **Find the numerical value of the expression :** $\left(\frac{x^2}{y^3}\right)^2$

[b] A box contains 3 white , 5 black and 7 red balls. A ball is drawn randomly from the box. **Calculate the probabilities of the following events :**

(1) The ball is red. (2) The ball is not white.

11 El-Monoufia Governorate — Al-Shohada Directorate

Danasor Experimental Language School

Answer the following questions :

1 **Choose the correct answer :**

(1) 0.00000052 = ...

(a) 5.2×10^{-6} (b) 5.2×10^{-7} (c) 5.2×10^{6} (d) 5.2×10^{2}

(2) $\left(-\frac{1}{3}\right)^{-1}$ = ...

(a) $\frac{-1}{3}$ (b) – 3 (c) 3 (d) $\frac{1}{3}$

(3) $\sqrt{\left(\frac{3}{4}\right)^2} = \dots\dots$

(a) $\frac{3}{4}$ (b) $-\frac{6}{16}$ (c) $\pm\frac{3}{4}$ (d) $-\frac{3}{4}$

(4) If : $-x < 3$, then $x \dots\dots -3$

(a) $<$ (b) $=$ (c) $>$ (d) $\leq$

(5) ✱ The side length of a square whose area $9x^2$ cm² is $\dots\dots$

(a) $3x$ (b) $3x^2$ (c) $9x$ (d) $9x^2$

2 **Complete with the correct answer :**

(1) ✱ If : $3x = 6$, then $5x = \dots\dots$

(2) ✱ $2^x \times 2^{1-x} \times \frac{1}{2} = \dots\dots$

(3) The probability of the impossible event $= \dots\dots$

(4) $3x^0 = \dots\dots$

(5) $2 \times 6 - 4 \div 2 = \dots\dots$

3 **[a] Find in the simplest form :**

(1) $\left(\frac{-2}{3}\right)^2 \times \sqrt{\frac{9}{4}} \times \left(\frac{1}{4}\right)^0$ (2) $\left(\frac{-2}{3}\right)^6 \div \left(\frac{-2}{3}\right)^4$

[b] Find the solution set of the inequality , where $x \in \mathbb{Z}$: $4x + 5 \geq 1$

4 **[a] Solve the equation : $3x - 3 = 9$, where $x \in \mathbb{Q}$**

[b] ✱ The sum of the age of 3 sisters now is 25 years. If the eldest was born before the middle by 3 years , and the middle was born before the youngest by 2 years. **Find the age of each of them now.**

5 **[a] Find the result of : $\frac{3^3 \times 3^{-4}}{3^{-2} \times 3^5}$**

[b] One card is selected randomly from 8 cards numbered from 1 to 8 , write down the sample space , then find the probability of each of the following events :

(1) Getting an odd number.

(2) Getting a number divisible by 3

12 El-Gharbia Governorate — East-Tanta Educational Directorate

Al-Salam Language School

Answer the following questions :

1 Complete each of the following :

(1) The probability of the impossible event is ....

(2) $\sqrt{(-9)^2} =$ ....

(3) If the age of Ahmed now is x years , then his age after three years is .... years.

(4) ✱ $\frac{(-2)^6 \times 3^5}{3^4 \times 2^7} =$ ....

(5) If : $3x = 6$, then $6x =$ ....

2 Choose the correct answer :

(1) ✱ Half the number $2^{20} =$ ....

(a) 2^{10} (b) 2^{21} (c) 2^{19} (d) 40

(2) If : $-2x < 4$, then ....

(a) $x < -2$ (b) $x < -6$ (c) $x < 2$ (d) $x > -2$

(3) $2.37 \times 10^{-4} =$ ....

(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

(4) As tossing a fair die once , the probability of appearance of an even number is ....

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{5}{6}$

(5) $3^7 \times 3^{-5} =$ ....

(a) -9 (b) 9 (c) 3 (d) -3

3 [a] The length of a rectangle exceeds its width by 4 meters and its perimeter is 68 meters

Find the dimensions of the rectangle.

[b] Solve the equation : $3x - 3 = 9$, where $x \in \mathbb{Q}$

4 **[a] Solve the inequality :** $4x + 5 \geq -3$ and represent the solution set on the number line where $x \in \mathbb{N}$

[b] Simplify : $2^3 + \left[4 + (2^2 \div 2)\right]$

5 [a] A card is drawn randomly from 10 cards numbered from 1 to 10

Find the probability of that card is numbered :

(1) An even number. (2) A number divisible by 5

[b] Simplify : $\sqrt{3 \times 7 - 15 \div 3}$

13 El-Ismailia Governorate — Directorate of Education

El-Manar language School

Answer the following questions :

1 Choose the correct answer :

(1) $\sqrt{(-5)^2} = \dots\dots$

(a) -5 (b) 25 (c) 10 (d) 5

(2) If the probability of success of a student is $\frac{7}{10}$, then the probability of failure is ………

(a) 0.7 (b) $\frac{1}{10}$ (c) 1 (d) $\frac{3}{10}$

(3) $12 \div 3 + 3 \times 2 = \dots\dots$

(a) 10 (b) 4 (c) 24 (d) 1

(4) ✱ $7.5 \times 10^{-3} = \dots\dots$

(a) 0.075 (b) 0.0075 (c) 7500 (d) 0.00075

(5) The S.S. of $X + 1 = 9$ in $\mathbb{N}$ is ………

(a) $\{9\}$ (b) $\{10\}$ (c) $\{8\}$ (d) $\varnothing$

2 Complete each of the following :

(1) The probability of the impossible event = ………

(2) The multiplicative inverse of $\left(\frac{-2}{3}\right)^2$ ………

(3) $\sqrt{16+9} = 4 + \dots\dots$

(4) If : $3X = 6$, then $5X = \dots\dots$

(5) ✱ The S.S. of the inequality : $-2X > 4$ in $\mathbb{N}$ is ………

3 [a] Find the result of : $\left(\frac{-2}{3}\right)^2 \times \sqrt{\frac{9}{4}} \times \left(\frac{-1}{4}\right)^{\text{zero}}$

[b] Answer the following :

If : $a = \frac{-1}{2}$, $b = 2$, $c = \frac{3}{4}$, then find the numerical value of : $a^2 b^3 + b^2 c$

4 Find in $\mathbb{Q}$ the S.S of :

(1) $4X - 1 \geq 7$ (2) $2(X + 5) = 16$

5 [a] ✱ Two complementary angles , whose measures are $2X$ and $(2X - 18°)$

Find the measure of each of them.

[b] A box contains 2 red balls , 6 white balls and 4 green balls. A ball is drawn randomly. Find the probability that the drawn ball is :

(1) Red (2) Green (3) Black

14 Assiut Governorate — Assiut Educational Zone

Al-Tahreer Language School

Answer the following questions :

1 Complete :

(1) The multiplicative inverse of $\sqrt{\frac{10}{2.5}}$ is ....

(2) The probability of the impossible event = ...

(3) * If : $x \in \mathbb{Z}$, then the solution set of the inequality : $20 < 5x < 25$ is ....

(4) If the age of Mona now is x years , then her age 5 years ago is .... years.

(5) $3 \times 7 - 15 \div 3 =$ ....

2 Choose the correct answer from the given ones :

(1) If : $(0.0005)^2 = 25 \times 10^n$, then n = ...

(a) 4 (b) 8 (c) -8 (d) -6

(2) $\left(\frac{2}{3}\right)^{-2} =$ ....

(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{-4}{9}$ (d) $\frac{-9}{4}$

(3) Half the number $2^{20} =$ ....

(a) 2^{10} (b) 2^{21} (c) 2^{19} (d) 40

(4) A letter is selected at random from the word "Nora" the probability of selecting the letter N is ....

(a) $\frac{1}{4}$ (b) $\frac{2}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{4}$

(5) If x is an odd natural number then the next odd number directly is ....

(a) $x+1$ (b) $x+2$ (c) $2x+1$ (d) $2x$

3 [a] **Find :** (1) $\left(\frac{-5}{8}\right)^0 \times \sqrt{6\frac{1}{4}} \times \left(\frac{-2}{5}\right)^2$ (2) $\frac{3^4 \times 3^3}{3^6}$

[b] Three consecutive even numbers are of sum 78 **Find these numbers.**

4 [a] **Solve the equation :** $3x + 5 = 26$, where $x \in \mathbb{Z}$

[b] A box contains 5 white , 4 black and 7 red balls , a ball is drawn randomly from the box. **Calculate the probabilities of the following events :**

(1) The drawn ball is white. (2) The drawn ball is red.

5 [a] **Solve the inequality :** $4x - 7 \geq 1$, where $x \in \mathbb{Q}$

[b] If : $a = \frac{1}{2}$, $b = 2$ and $c = \frac{3}{4}$, then find the numerical value of : $a^2 b^3 + b^2 c$

15 Qena Governorate — Qena Directorate of Education

Answer the following questions :

1 Choose the correct answer :

(1) ✱ The S.S. of the inequality : $-2X <$ zero in $\mathbb{Q}$ is ....

(a) $\varnothing$ (b) $\mathbb{Q}_+$ (c) $\mathbb{Q}_-$ (d) $\mathbb{Z}_+$

(2) 0.00000032 = ...

(a) 3.2×10^{-6} (b) 3.2×10^{6} (c) 3.2×10^{-7} (d) 3.2×10^{7}

(3) If the age of Mohamed now is X years then his age after five years is .... years.

(a) $5X$ (b) $X+5$ (c) $X-5$ (d) X^5

(4) $\frac{1}{2}(2^{20}) =$ ....

(a) 2^5 (b) 2^{10} (c) 2^{19} (d) 2^{18}

(5) $-\sqrt{\frac{4}{9}} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

2 Complete each of the following :

(1) $2 + 3 \times 6 + 3 =$ .... (2) $\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^4 =$ ....

(3) The probability of the impossible event = ...

(4) ✱ If : $2X + 3 = 27$, then the value of $\frac{1}{3}X =$ ....

(5) The multiplicative inverse of $\sqrt{\frac{25}{64}}$ is ....

3 [a] Find the solution set of each of the following in $\mathbb{Z}$:

(1) $2X + 14 = 12$ (2) $3X + 1 \leq 7$

[b] Simplify : $\frac{5^6 \times 5^2 \times 5^3}{5^4 \times 5^5}$

4 [a] If : $a = -\frac{2}{3}$, $b = \frac{3}{4}$ and $c = \frac{1}{2}$ Find the value of : $(ab)^2 - c^2$

[b] A card is drawn randomly from 10 cards numbered from 1 to 10

Find the probability of that card numbered by :

(1) An even number. (2) A number divisible by 5

5 [a] Find the value of : $\left(\frac{5}{8}\right)^0 \times \sqrt{6\frac{1}{4}} \times \left(-\frac{2}{5}\right)^2$

[b] The sum of three consecutive numbers is 33 Find the three numbers.

Model Examinations of the School Book

Model 1

Answer the following questions :

1 Complete :

① $2 \times 6 - 4 \div 2 =$ ………. ② If : $7 - 2x = 3$, then $x =$ ……….

③ If : $3x + 1 \geq 10$, then $x \geq$ ……….

④ The standard form of the number $0.7 \times 0.005 =$ ……….

⑤ A class has 36 students , the number of boys are 20 , if a student is chosen randomly , then the probability that the student is a girl = ……….

2 Choose the correct answer :

① The sum of the probabilities for all possible outcomes of a randomly experiment is ……….

(a) zero (b) 1 (c) > 1 (d) < 1

② If : $3a = \sqrt{4}\,b$, then $\frac{a}{b} =$ ……….

(a) 2 : 3 (b) 3 : 2 (c) 3 : 4 (d) 4 : 3

③ $\left(\frac{-2}{3}\right)^{-3}$ equals ……….

(a) $\frac{-27}{8}$ (b) $\frac{-8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{27}{8}$

④ There are 21 boys and 15 girls in a classroom , one pupil is chosen randomly , the probability that the chosen pupil is a girl = ……….

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{4}{7}$ (d) $\frac{5}{6}$

⑤ $\sqrt{(-8)^2 + (-6)^2} =$ ……….

(a) $|-10|$ (b) ± 10 (c) 14 (d) -14

3 [a] Simplify to the simplest form : $\left(-\frac{3}{7}\right)^0 \times \left(\frac{-2}{5}\right)^2 \times \sqrt{6\frac{1}{4}}$

[b] Find the numerical value of the expression :

$3ab + 8a \div (4b)$ when $a = 4$, $b = -2$

4 [a] Find in $\mathbb{Q}$ the S.S. of the following :

① $3x + 1 = 25$ ② $2x + 5 < 16$

[b] The population of a city has been growing according to the rule :

$y = 3\,(1.02)^n$ million. Calculate the population that will be in 2 years in the standard form.

5 **A factory of a tire record the distance that traveled by a certain type of them before damage for 800 units of this type as following.**

The distance in thousand (km)	Less than 50	50 to 100	More than 100 till 150	More than 150
The number of damage tire	80	120	280	320

If you bought the type of this tyre , what is the probability of change it :

① Before traveled 50 thousand km.

② After traveled more than 100 thousand km.

Model 2

Answer the following questions :

1 **Complete :**

① $\left(\frac{-2}{3}\right)^0 = \dots\dots$ ② $\sqrt{\frac{16}{49}} = \dots\dots$

③ The probability of impossible event = ……….

④ 1 , 2 , 3 , 5 , 8 , ………. , ………. (In the same pattern)

⑤ If the probability that the student is absent in a school is 0.15 , if the number of students of this school is 600 , then the number of the present student that day is ……….

2 **Choose the correct answer :**

① $2^3 \times 2^5 = \dots\dots$

(a) 2^2 (b) 2^8 (c) 2^{15} (d) 2^{53}

② Which of the following the greatest ? ……….

(a) 2.3×10^4 (b) 2.3×10^5 (c) 3.2×10^4 (d) 3.2×10^5

③ The side length of a square whose area $9x^2$ cm^2 is …… …… cm.

(a) $3x$ (b) $3x^2$ (c) $9x$ (d) $9x^2$

④ Which of the following may be probability of an event ? ……..

(a) -0.25 (b) 87 % (c) 1.05 (d) 130 %

⑤ If : $-x > 4$, then :

(a) $x > -4$ (b) $x > 4$ (c) $x < -4$ (d) $x < 4$

3 [a] Two integers number the smaller one is $2x$ and the greater is $5x$, if the difference between them is 30 Find the two numbers.

[b] Find the value of $\frac{5^{-4} \times 5^7}{5^3}$ in the simplest form.

4 **[a] Find in $\mathbb{Q}$ the S.S. of each of the following :**

① $(3x+2)+5=13$ ② $2x+15<19$

[b] Find the value of the expression in simplest form :

$(\frac{-1}{3})^2+\sqrt{\frac{64}{81}}-(\frac{3}{7})^0$

5 [a] If a regular die is thrown once and observed the number on upper face , find the probability of each of the following :

① Getting prime even number. ② Getting odd number less than 4

[b] If the length of a rectangle is twice its width , its area is 12.5 cm^2. Calculate its length , its width.

Model 3

Answer the following questions :

1 **Complete :**

① The probability of the certain event = ...

② $\frac{1}{9}$, $\frac{1}{8}$, $\frac{1}{7}$, $\frac{1}{6}$, ... , ... (In the same pattern)

③ The S.S. in $\mathbb{Q}$ for the equation : $2x+3=4$ is ....

④ If : $x=\frac{1}{2}$, $y=\frac{1}{4}$, then $(x+y)^{-1}=$ ....

⑤ $0.00037=3.7\times10^n$, the value of n = ...

2 **Choose the correct answer :**

① The multiplicative inverse of the number : $\sqrt{\frac{9}{16}}$ is ....

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

② $\frac{x}{2}<5$ equivalent ....

(a) $x<\frac{5}{2}$ (b) $x>\frac{5}{2}$ (c) $x<10$ (d) $x>10$

③ $3^x+3^x+3^x$ equals ....

(a) 3^x (b) 3^{x+1} (c) 27^x (d) $3x^3$

④ There are 480 pupils in a school , 120 of them failed. A pupil is chosen at random , then the probability that the pupil is succeeded ....

(a) 0.25 % (b) 0.75 (c) 0.8 (d) 0.667

⑤ If : $x=y$, then $(\frac{3}{5})^{x-y}=$ ....

(a) 0 (b) 1 (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

3 [a] What is the number which if we add it to its three times , the result is 28 ?

[b] If the area of a Square equals the area of a triangle whose base length is 9 cm, its height is 8 cm. Find the Side length of the square.

4 **[a] Find in $\mathbb{Q}$ the solution set of the following :**

① $3x+5=11$ ② $2x+3\leq 7$

[b] If the distance (s) between the sun and the earth is 1.44×10^8 km. and the light velocity (v) is 3×10^8 m/sec. **Calculate the elapsed time (t) that the light takes to reach from the sun to the earth.** (given that ; $s = v \times t$)

5 **[a] Find the result of the expression :** $(5.4\times 10^4)+(3.7\times 10^5)$ in the form $a\times 10^n$ where n is integer number.

[b] A coin is tossed twice calculate the probability :

① The two faces are similar. ② Appearance only one tail.

Model 4

Answer the following questions :

1 **Complete :**

① When a coin is tossed once then the probability of appearance of a head is ....

② $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, ... (In the same pattern)

③ The S.S. of the inequality $2 < x \leq 4$ in $\mathbb{N}$ is ....

④ The additive inverse of $\sqrt{\left(-\frac{2}{5}\right)^2}$ is ....

⑤ $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, ... , ...

2 **Choose the correct answer :**

① The S.S. of the equation : $x+3=3$ in $\mathbb{N}$ is ....

(a) $\varnothing$ (b) {0} (c) {3} (d) {6}

② The number which in the standard form between the following numbers is ....

(a) 11×10^8 (b) 9.7×10^{-5} (c) 10.3×10^{-3} (d) 0.87×10^8

③ If a coin is tossed 160 times then the approximate expected number of the appearance of a head is ....

(a) 60 (b) 78 (c) 90 (d) 159

④ The number $\sqrt{0.09}$ is ....

(a) natural. (b) positive integer. (c) negative integer. (d) rational.

⑤ If : $\frac{6x}{5} = -2$, then $x^2 =$ ....

(a) $\frac{-25}{9}$ (b) $\frac{5}{9}$ (c) $\frac{25}{9}$ (d) $\frac{25}{3}$

3 [a] If : $x = \frac{3}{4}$, $y = \frac{-3}{2}$ Find the numerical value of the expression $\left(\frac{x^2}{y^3}\right)^2$

[b] The sum of two natural number is 15 and the difference between them is 5
Find the two number.

4 **[a] Find in $\mathbb{Q}$ the solution set for each of the following :**

① $3x + 2 = 8$ ② $4x - 3 < 7$

[b] If $\frac{3}{4}$ of the area of a square is $1\frac{11}{64}$ m^2. Find its side length.

5 **[a] A coin is tossed twice Calculate the probabilities :**

① The appearance of at least one head. ② The appearance of at most one head.

[b] Find the value of $\left(\frac{7^4 \times 7^{-2}}{7^3}\right)^{-2}$

Model 5

Answer the following questions :

1 **Complete :**

① In the experiment of tossing a die once then the probability of appearance even number is ....

② If : $\frac{x}{y} = \frac{7}{2}$, then $\frac{2x}{7y} =$ ....

③ If : $a = 0.000625$, then $\sqrt{a} = 2.5 \times 10^{\text{..........}}$

④ The result of the expression : $\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right)^3 =$ ....

⑤ Quarter of 4^{20} equals ....

2 Choose the correct answer :

① The S.S. of the inequality $x < 2$ in $\mathbb{N}$ is ....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0 , 1\}$ (d) $\varnothing$

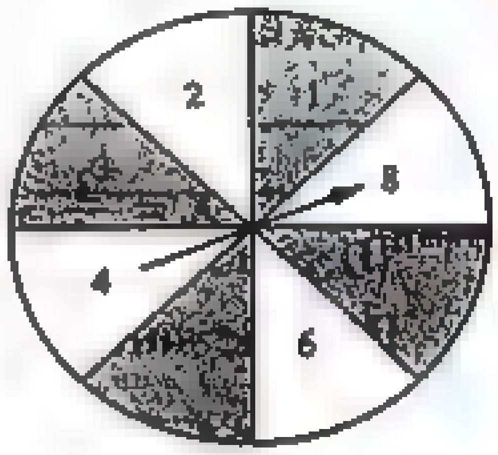
② If : $\frac{26}{x} + 1 = 14$, then $x =$ ....

(a) 2 (b) 10 (c) 13 (d) 20

③ If : $5x = 35$, then $2x + 1 =$ ....

(a) 7 (b) 8 (c) 15 (d) 71

④ **In the opposite figure :**



The probability that the pointer stop at a number greater than 6 equals ....

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{3}{4}$

⑤ $\sqrt{100 - (-6)^2} =$ ....

(a) 4 (b) 8 (c) -8 (d) 16

3 **[a] Find the value of the expression :** $12 \times (2)^2 \div 24 + 3^2$

[b] If : $x = -\frac{1}{2}$, $y = \left|\frac{-3}{4}\right|$, find the numerical value of $\left(\frac{y}{x^2}\right)^{-2}$

4 **[a] Find in $\mathbb{Q}$ the solution set of each of the following :**

① $3 - 4x = -5$ ② $2x - 1 \geq 5$

[b] Simplify : $\frac{n}{2}[3n - 6] + \frac{1}{2}[6 - 2n]$, then find its value when $n = 1$

5 [a] The sum of the age of 3 sisters now is 25 years. If the eldest was born before the middle by 3 years , and the middle was born before the youngest by 2 years.

Find the age of each of them now.

[b] A box contains 4 white , 5 red and 6 blue balls. A ball is drawn randomly from the box. **Calculate the probabilities of the following events.**

① The ball is red. ② The ball is white or red.

School Examinations

1 Cairo Governorate — Maadi Educational Zone

Victory College Maadi

Answer the following questions :

1 Choose the correct answer :

① The multiplicative inverse of $\left(\frac{-3}{4}\right)^2$ is ....

(a) $\left(\frac{4}{3}\right)^2$ (b) $-\left(\frac{4}{3}\right)^2$ (c) $\left(\frac{3}{4}\right)^2$ (d) $-\left(\frac{3}{4}\right)^2$

② Ahmed's age 3 years ago was x, then his age now is .... years.

(a) $x+3$ (b) $x-3$ (c) $3-x$ (d) $3x$

③ $\left(\frac{2}{5}\right)^0 =$ ....

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) 1 (d) 0

④ A regular die is rolled once, the probability of getting a number less than 3 is ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

⑤ The solution set of : $-2x+1=-3$ in $\mathbb{Z}$ is ....

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{-4\}$ (d) $\varnothing$

2 Complete each of the following :

① The additive inverse of $\left(\frac{-5}{7}\right)^2$ is ....

② The probability of the impossible event = ...

③ $\sqrt{16+9} = 4 +$ ....

④ If : $x+3=5$, then $3x =$ ....

⑤ If : $0.000035 = 3.5 \times 10^k$, then $k =$ ....

3 **[a] Find the S.S. of : $5x+3 > 15+x$, where $x \in \mathbb{Q}$**

[b] Simplify : $\frac{a^5 \times a^7}{a^8}$

4 **[a] If : $a = \frac{-1}{2}$, $b = \frac{3}{4}$, $c = \frac{-3}{2}$**

Find the value of : $a^3 \div b^2 c^2$

[b] A box contains 8 white, 7 black and 5 red balls. A ball is drawn randomly from the box.

Find the probability of the following events :

① The ball is red. ② The ball is not black.

5 [a] Find the solution set of : $3x + 17 = 38$, $x \in \mathbb{Q}$

[b] Find the result in the simplest form : $\frac{4}{3} \times \sqrt{\frac{9}{25}} \times \left(\frac{-1}{2}\right)^2$

2 Cairo Governorate — El-Zeitoun Directorate

Answer the following questions :

1 Choose the correct answer :

① Third of 3^{20} is ....

(a) 3^{10} (b) 3^{19} (c) 3^{18} (d) 60

② $\sqrt{\frac{25}{36}} =$ ....

(a) $\frac{\pm 5}{6}$ (b) $\frac{-5}{6}$ (c) $\frac{5}{6}$ (d) otherwise.

③ As throwing a fair die once, the probability of appearance of prime number on the upper face is ....

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) $\frac{1}{3}$

④ If : $2x = 12$, then $3x =$ ....

(a) 6 (b) 4 (c) 3 (d) 18

⑤ If : $(0.0005)^2 = 25 \times 10^n$, then n = ...

(a) 4 (b) 8 (c) -8 (d) -6

2 Complete each of the following :

① The probability of impossible event = ...

② The multiplicative inverse of $\left(\frac{-2}{3}\right)^2$ is ....

③ If the age of a man now is x years , then his age after 5 years is ....

④ $3 \times 7 - 15 \div 3 =$ ....

⑤ The solution set of : $2x = 10$ in $\mathbb{N}$ is ....

3 Simplify each of the following :

① $\left(\frac{-2}{3}\right)^2 \times \sqrt{\frac{9}{4}} \times \left(\frac{2}{7}\right)^0$ ② $\frac{(2)^5 \times (-2)^4}{(2)^9}$

4 **Find the solution set in $\mathbb{Z}$:**

(1) $2x + 14 = 12$ (2) $3x + 1 \le 7$

5 [a] If : $a = \frac{-2}{3}$, $b = \frac{3}{4}$, $c = \frac{1}{2}$

Find the numerical value of $(ab)^2 - c^2$

[b] A box contains 3 white , 5 black and 7 red balls. A ball is drawn randomly.

Find the probability of getting :

(1) red ball. (2) not white ball.

3 Cairo Governorate — Hel. Educ. Administration

St. Joseph's School

Answer the following questions :

1 **Choose the correct answer :**

(1) If a coin is flipped once , then the probability of appearance of a tail = ...

(a) 1 (b) 0.5 (c) 0 (d) 2

(2) If : $x + 8 = 12$, then : $9x =$ ....

(a) 1 (b) 9 (c) 36 (d) 72

(3) If : $-2x \ge 1$, then : x ....

(a) $\ge -\frac{1}{2}$ (b) $\le -\frac{1}{2}$ (c) ≥ -2 (d) ≥ -1

(4) If : $(AB)^2 = 9$, $(BC)^2 = 25$ and $B \in \overline{AC}$, then the length of $\overline{AC}$ = ... cm.

(a) 34 (b) 2 (c) 8 (d) 16

(5) If : $x = y$, then $\left(\frac{4}{9}\right)^{x-y}$ equals ....

(a) 1 (b) 0 (c) $\frac{4}{9}$ (d) $\frac{9}{4}$

2 **Complete :**

(1) The set $\{2 , 3 , 5\}$ is used in writing a 2-digit number ,
the probability of the event : both of the two digits are even = ...

(2) 0.0007 in the standard form ($a \times 10^n$, $n \in \mathbb{Z}$) is $7 \times$ ....

(3) $2 \times 6 - 4 \div 2 =$ ....

(4) $(a^{-1})^{-3} = a$ ....

(5) If : $x^2 + y^2 = 7$, then the value of $\frac{x^3 y + x y^3}{x y} =$ ....

3 **[a] Solve the equation :** $5x + 8 = 13 - 2x$, where $x \in \mathbb{Q}$

[b] Put the result of : $\left(-\frac{1}{2}\right)^3 \div \left[8 \times \left(-\frac{1}{2}\right) \times \frac{3}{4}\right]$ in the simplest form.

4 [a] If : $a = -\frac{1}{2}$, $b = 2$, and $c = \frac{3}{4}$, **Find the numerical value of :** $a^3 b^2 + b^2 c - 8 abc$

[b] If : $x = 0.000625$, find $\sqrt{x}$ in the standard form ($a \times 10^n$, $n \in \mathbb{Z}$)

5 **[a] Solve the inequality :** $3x - 1 \le 2x + 3$, $x \in \mathbb{Q}$

[b] A bag contains 40 cards numbered from 1 to 40 , Mirna picked one randomly , it was an even number. She picked another one without replacing the first card. Find the probability to get a card with an odd number.

4 Giza Governorate — Experimental Language Schools

Answer the following questions :

1 **Choose the correct answer :**

① $2x^{-1} = \dots\dots\dots$

(a) $-2x$ (b) $2x$ (c) $\frac{1}{2x}$ (d) $\frac{2}{x}$

② The additive inverse of $\left(-\frac{2}{5}\right)^{zero}$ is $\dots\dots\dots$

(a) $\frac{2}{5}$ (b) $-\frac{2}{5}$ (c) -1 (d) 1

③ The probability of the impossible event = $\dots\dots\dots$

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) -1

④ If : $x - 2 = 3$, then the value of $5x = \dots\dots\dots$

(a) 5 (b) 3 (c) 25 (d) 10

⑤ $3 \times 2 - 16 \div 8 = \dots\dots\dots$

(a) 4 (b) 6 (c) 2 (d) 3

2 **Complete :**

① $\sqrt{3^2 + 4^2} = 3 + \dots\dots\dots$

② $\left(-\frac{3}{2}\right)^4 \div \left(\frac{3}{2}\right)^2 = \dots\dots\dots$

③ If : $x + 1 > 3$, then the solution set in $\mathbb{N} = \dots\dots\dots$

④ When a coin is tossing once , then the probability of getting a head = $\dots\dots\dots$

⑤ If : $0.00032 = 3.2 \times 10^n$, then $n = \dots\dots\dots$

3 [a] Simplify : $\frac{x^3 \times x^4}{x^5}$

, then find the numerical value of the result at $x = -3$

[b] Find in $\mathbb{Q}$ the solution set of the inequality : $2x - 3 \leq 1$

4 [a] Find the result in the simplest form : $\left(\frac{3}{4}\right)^{zero} \times \sqrt{\frac{81}{64}} \times \left(-\frac{2}{3}\right)^3$

[b] Find in $\mathbb{Q}$ the solution set of the equation : $2x - 3 = 6$

5 [a] Two integers the smaller one is $3x$ and the greater is $5x$. If the difference between them is 20 find the two numbers .

[b] A fair die rolled once , find sample space and calculate the probability of rolling

(1) An even number. (2) A number greater than 1

5 Giza Governorate — Inspection of mathematice

Experimental Directorate

Answer the following questions :

1 Choose the correct answer :

(1) Half the number 2^{20} = ...

(a) 2^{10} (b) 2^{21} (c) 2^{19} (d) 40

(2) If : $-2x < 4$, then ....

(a) $x < -2$ (b) $x < -6$ (c) $x < 2$ (d) $x > -2$

(3) 2.37×10^{-4} = ...

(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

(4) As tossing a fair die once , the probability of appearance of an even number is ....

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{5}{6}$

(5) $3^7 \times 3^{-5}$ = ...

(a) -9 (b) 9 (c) 3 (d) -3

2 Complete :

(1) The multiplicative inverse of $\sqrt{\frac{10}{2.5}}$ is ....

(2) The probability of the impossible event = ...

(3) If : $x \in \mathbb{Z}$, then the solution set of the inequality : $20 < 5x < 25$ is ....

(4) If the age of Mona now is x years , then her age 5 years ago is .... years.

(5) $3 \times 7 - 15 \div 3 =$ ....

3 **[a] Find :**

(1) $\left(\frac{-5}{8}\right)^0 \times \sqrt{6\frac{1}{4}} \times \left(\frac{-2}{5}\right)^2$ (2) $\frac{3^4 \times 3^3}{3^6}$

[b] Three consecutive even numbers their sum is 78 **Find these numbers.**

4 **[a] Solve the equation :** $3x + 5 = 26$, where $x \in \mathbb{Z}$

[b] A box contains 5 white , 4 black and 7 red balls. A ball is drawn randomly from the box. **Calculate the probabilities of the following events :**

(1) The drawn ball is white. (2) The drawn ball is red

5 **[a] Solve the inequality :** $4x - 7 \geq 1$, where $x \in \mathbb{Q}$

[b] If : $a = \frac{1}{2}$, $b = 2$ and $c = \frac{3}{4}$, then find the numerical value of : $a^2 b^3 + b^2 c$

6 Alexandria Governorate — East Education Zone

Inspectorate of Math

Answer the following questions :

1 **Choose the correct answer :**

(1) The S.S. of equation : $x + 5 = 5$ in $\mathbb{N}$ is ....

(a) $\varnothing$ (b) $\{0\}$ (c) $\{5\}$ (d) $\{10\}$

(2) The S.S. of inequality $1 < x \leq 3$ is ....

(a) $\varnothing$ (b) $\{3\}$ (c) $\{2 , 3\}$ (d) $\{1 , 3\}$

(3) The number which is in the standard form between the following numbers is ....

(a) 1.1×10^8 (b) 27×10^{-5} (c) 10×10^{-3} (d) 0.87×10^8

(4) The number $\sqrt{0.09}$ is ....

(a) natural. (b) positive integer. (c) negative integer. (d) rational.

(5) The additive inverse of $\sqrt{\frac{9}{25}}$ is ....

(a) $\frac{-3}{5}$ (b) $\frac{3}{5}$ (c) $\frac{9}{25}$ (d) $\frac{-9}{25}$

2 **Complete :**

(1) The probability of impossible event = ...

(2) $\left(\frac{-4}{5}\right)^{zero}$ = ...

(3) If : $0.0027 = 2.7 \times 10^n$, then n = ...

(4) $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, ... , ... (in the same pattern).

(5) When a coin is tossed once , then the probability of appearance of a head is ....

3 [a] If : $x = \frac{3}{4}$, $y = \frac{-3}{2}$, then find the numerical value of $\left(\frac{x^2}{y^3}\right)^2$

[b] Find the value of expression $12 \times 2^2 \div 24 + 3^2$

4 **If : $x \in \mathbb{Q}$ Find the S.S. of each of the following :**

(1) $4x + 3 = 11$

(2) $6x - 2 < 7$

5 **A card is drawn randomly from 8 cards are numbered from 1 to 8 , find the probability of each of the following event :**

(1) Getting even number greater or equal to 6

(2) Getting number = 2^n ($n \in \mathbb{Z}$, $n < 4$)

7 Alexandria Governorate — Borg Elarab Admin.

Answer the following questions :

1 **Complete each of the following :**

(1) If : $x = \frac{1}{4}$, $y = \frac{1}{8}$, then $(x - y)^{-1}$ = ...

(2) The probability of the certain event = ...

(3) $0.00037 = 3.7 \times 10^n$, the value of n = ...

(4) $3x^0$ = ...

(5) $\sqrt{(-9)^2}$ = ...

2 **Choose the correct answer :**

(1) If : $2^4 \times 3^4$ = ...

(a) 5^4 (b) 6^4 (c) 6^8 (d) 6^{16}

(2) The side length of square whose area $\sqrt{9x^2}$ cm^2 is .... cm.

(a) $3x$ (b) $3x^2$ (c) $9x$ (d) $9x^2$

(3) The multiplicative inverse of the number $\sqrt{\frac{9}{16}}$ is ....

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

(4) The S.S. of : $-2x+1=-3$ in $\mathbb{Z}$ is ....

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{-4\}$ (d) $\varnothing$

(5) $\left(\frac{-1}{3}\right)^{-1} =$ ....

(a) $\frac{-1}{3}$ (b) -3 (c) 3 (d) $\frac{1}{3}$

3 **[a] Find the result of :** $\frac{2^3 \times 2^{-4}}{2^{-2} \times 2^5}$

[b] Find the S.S. of the inequality : $3x-2 > x+4$, where $x \in \mathbb{Q}$

4 [a] Simplify : $2^3 + [4 + (2^2 \div 2)]$

[b] A die is tossing once , find the probability of the event (A) appearing an odd number on the upper face.

5 **[a] Find the value of :** $\left(\frac{5}{8}\right)^0 \times \sqrt{6\frac{1}{4}} \times \left(-\frac{2}{5}\right)^2$

[b] If : $x = -\frac{1}{2}$, $y = \frac{3}{4}$, Find the numerical value of : $x^3 y^2$

8 El-Kalyoubia Governorate — Inspectorate of Math

Answer the following questions :

1 **Choose the correct answer :**

(1) Quarter of the number $4^{20} =$ ....

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

(2) $2.37 \times 10^{-4} =$ ....

(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

(3) The age of Ahmed now is x years , then his age 5 years ago is .... years.

(a) $5x$ (b) $5+x$ (c) $5-x$ (d) $x-5$

(4) If : $-x < 3$, then ....

(a) $x > 3$ (b) $x > -3$ (c) $x < 3$ (d) $x < -3$

(5) $\sqrt{\left(\frac{-2}{3}\right)^2} =$ ....

(a) $\frac{-4}{9}$ (b) $\frac{-2}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{9}$

2 Complete :

① The probability of the impossible event = ...

② $3 \times 4 - 21 \div 3 =$ ....

③ The additive inverse of the number $\left(\frac{-4}{5}\right)^2 =$ ....

④ If a coin is tossed once the probability of appearance of a head = ...

⑤ $\frac{1}{9}$, $\frac{1}{8}$, $\frac{1}{7}$, ... , ... (in the same pattern).

3 [a] Find in the simplest form : $\frac{7^{-3} \times 7^5}{7^2}$

[b] If the regular die is thrown once and observed the number on upper face , find the probability of each of the following :

① Getting even number ② Getting odd number less than 4

4 [a] Find the solution set of the following equation in $\mathbb{Q}$: $3X + 9 = 11$

[b] If : $X = \frac{-3}{2}$ and $y = \frac{-4}{3}$, then find in the simplest form $\left(\frac{X}{y}\right)^2$

5 [a] Find the solution set of the following inequality in $\mathbb{Z}$: $2X + 3 \le 7$

[b] If : $\overline{XY}$ is a line segment such that $(XY)^2 = 25 \text{ cm}^2$ and z is a midpoint of $\overline{XY}$

Calculate the length of $\overline{XZ}$

9 El-Monofia Governorate — El-Bagor Educational zone

Answer the following questions :

1 Choose the correct answer :

① $8 + 14 \div 2 - 5 =$ ....

(a) 6 (b) 8 (c) 10 (d) 12

② $\sqrt{10^2 - 6^2} =$ ....

(a) 4 (b) 8 (c) ± 4 (d) ± 8

③ $3^7 + 3^7 + 3^7 =$ ....

(a) 3^7 (b) 3^{21} (c) $(27)^7$ (d) 3^8

④ In an experiment of throwing a regular die once , the probability of appearance a number greater than 6 is ....

(a) zero (b) 1 (c) $\frac{1}{6}$ (d) $\frac{1}{4}$

⑤ The multiplicative inverse of $\left(\frac{-3}{5}\right)^2$ is ....

(a) $\frac{9}{25}$ (b) $\frac{-5}{3}$ (c) $\frac{25}{9}$ (d) 1

2 **Complete the following :**

(1) $\frac{1}{8}$, $\frac{1}{7}$, $\frac{1}{6}$, $\frac{1}{5}$, ... , ... in the same pattern.

(2) If : $\frac{x}{y} = \frac{8}{5}$, then $\frac{5x}{8y} =$ ....

(3) If the standard form of the number $0.00057 = 5.7 \times 10^n$, then n = ...

(4) The probability of the certain event = ...

(5) The S.S. of : $-x > 1$ in $\mathbb{Z}$ is ....

3 **[a] Put the expression :** $\left(\frac{-2}{5}\right)^2 \times \sqrt{6\frac{1}{4}} \times \left(\frac{-5}{8}\right)^{zero}$ in the simplest form.

[b] Find in $\mathbb{Q}$ the solution set of the following inequality : $3x + 8 \leq 1$

4 **[a] Find in $\mathbb{Q}$ the solution set of the following equation :** $2x + 9 = 17$

[b] A man's age now is three times son's age and after five years , the sum of their ages will be 58 years , what the age of each now ?

5 **[a]** If : $x = \frac{-3}{2}$ and $y = \frac{3}{4}$. Find the numerical value of expression $\left(\frac{x^3}{y^2}\right)^2$

[b] A box contains 4 white , 5 red and 6 blue balls. A ball is drown randomly from the box. Calculate the probabilities of the following events :

(1) The ball is red. (2) The ball isn't blue. (3) The ball is green.

10 Port Said Governorate — Education Directorate

South Education Administration

Answer the following questions :

1 **Complete :**

(1) $\sqrt{9 + 40} =$ ....

(2) The probability of the impossible event = ...

(3) The additive inverse of $\left(\frac{-2}{3}\right)^0$ is ....

(4) $2 \times 6 - 4 \div 2 =$ .... (5) If : $x + 9 = 11$, then $7x =$ ....

2 **Choose the correct answer :**

(1) $2^3 \times 2^5 =$ ....

(a) 2^2 (b) 2^8 (c) 2^{15} (d) 2^{35}

(2) The multiplicative inverse of the number $\sqrt{\frac{4}{25}}$ is : ....

(a) $\frac{2}{25}$ (b) $\frac{25}{2}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$

③ $10^{-3} = \frac{1}{\dots\dots\dots}$

(a) 30 (b) 1000 (c) 10^2 (d) 31

④ The number 600000 in the standard form $a \times 10^n$, $n \in \mathbb{Z}$ is ....

(a) 6×10^5 (b) 6×10^4 (c) 6×10^6 (d) 6×10^{10}

⑤ The S.S. of the inequality : $x < 2$ in $\mathbb{N}$ is ....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0 , 1\}$ (d) $\{0 , 1 , -1 , ...\}$

3 **Find in $\mathbb{Q}$ the solution set of each of the following in :**

① $-3 + x = 5$ ② $2x - 1 \geq 5$

4 [a] If : $x = \frac{1}{4}$, $y = \frac{1}{8}$ Find the numerical value of : $(x + y)^{-1}$

[b] Simplify to the simplest form : $(\frac{-3}{7})^0 \times (\frac{-2}{5})^2 \times \sqrt{\frac{25}{4}}$

5 **[a] Find the product : $3\,ab^2 \times 2a^2b^3$**

[b] A card is drawn randomly from 10 cards numbered from 1 to 10 , then calculate the probability of drawing.

① Card carries an odd number greater than 10

② Card carries an even number and less than 10

③ Card carries a prime number.

11 Kafr El-Sheikh Governorate General Math Supervision ELS

Answer the following questions :

1 **Choose the correct answer :**

① $0.00000092 = \dots\dots\dots$

(a) 9.2×10^{-6} (b) 9.2×10^6 (c) 9.2×10^{-7} (d) 9.2×10^7

② $\frac{1}{3}$ of $3^{30} = \dots\dots\dots$

(a) 3^5 (b) 3^{10} (c) 3^{27} (d) 3^{29}

③ $\sqrt{16 + 9} = 4 + \dots\dots\dots$

(a) 1 (b) 3 (c) 5 (d) 25

④ As tossing a coin once. The probability of appearance of a head is ....

(a) $\frac{1}{2}$ (b) $\frac{5}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

⑤ If : $x = y$, then $5^{y-x} = \dots\dots\dots$

(a) zero (b) 9 (c) 5 (d) 1

2 **Complete each of the following :**

(1) The S.S. of the inequality : $-x \leq -4$ in $\mathbb{N}$ is x 4

(2) The multiplicative inverse of $\sqrt{81}$ = ...

(3) If : $x + 7 = 10$, then the value of $4x$ = ...

(4) $2 \times 6 - 4 \div 2$ = ...

(5) The probability of success of a students is 85 % , then the probability of his failure is ....

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3 **[a] Solve the equation :** $3x - 5 = 4$ (Where $x \in \mathbb{Z}$)

[b] Solve the inequality : $-2x - 3 \geq 1$ (Where $x \in \mathbb{Z}$) and represent the solution on number line.

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4 **Find each of the following :**

(1) $\left(\frac{4}{5}\right)^2 \times \sqrt{\frac{25}{16}} \times \left(\frac{1}{4}\right)^0$

(2) $\frac{(-3)^6 \times (3)^{-3}}{(3)^5 \times (3)^{-4}}$

---

5 [a] If : $x = \frac{1}{2}$, $y = \frac{4}{5}$, $z = \frac{5}{2}$, then find $x^2 y z$

[b] A box contain 10 balls numbered from 1 to 10 If a ball is drawn randomly , then find :

(1) The probability of getting a number is divisible by 7

(2) The probability of getting an even number.

(3) The probability of getting a number less than 8

12 Beni Suef Governorate — Directorate Language School

Education Administration

Answer the following questions :

1 **Choose the correct answer :**

(1) $2^7 \times 3^7$ = ...

(a) 5^7 (b) 6^7 (c) 6^{14} (d) 6^{49}

(2) $\left(\frac{1}{3}\right)^{-2}$ = ...

(a) $\frac{1}{9}$ (b) -9 (c) $-\frac{1}{9}$ (d) 9

③ $\left(-\frac{3}{7}\right)^7 \div \left(\frac{3}{7}\right)^5 = \dots\dots\dots$

(a) $\frac{9}{49}$ (b) $-\frac{9}{49}$ (c) $\pm\frac{9}{49}$ (d) $\frac{3}{7}$

④ $\pm\sqrt{\frac{4}{9}} = \dots\dots\dots$

(a) $-\frac{4}{9}$ (b) $-\frac{2}{3}$ (c) $\pm\frac{2}{3}$ (d) $\frac{2}{3}$

⑤ If : $-x < 3$, then $\dots\dots\dots$

(a) $x > 3$ (b) $x < -3$ (c) $x < 3$ (d) $x > -3$

2 **Complete :**

① If : $0.00037 = 3.7 \times 10^n$, then the value of n = $\dots\dots\dots$

② If : $x + 9 = 11$, then the value of $7x = \dots\dots\dots$

③ The multiplicative inverse of $\sqrt{\frac{25}{36}}$ is $\dots\dots\dots$ in its simplest form.

④ The probability of the impossible event = $\dots\dots\dots$

⑤ If a coin is tossed once , then the probability of appearance of a head = $\dots\dots\dots$

3 **Find in $\mathbb{Q}$ the solution set of each of the following :**

① $4x - 1 \geq 7$ ② $2(x + 4) = 15$

4 [a] If : $a = \frac{-1}{2}$, $b = 2$ and $c = \frac{3}{4}$, then find in simplest form the numerical value of :

$a^2 b^3 + b^2 c$

[b] Find the value of the following expression in simplest form : $144 - 8 \div 2^3$

5 [a] Find the value of the expression in simplest form : $\frac{7^{-2} \times 7^5}{7^3}$

[b] One card is selected randomly from 10 cards numbered from 1 to 10

Find the probability of the following events :

① Getting an even number. ② Getting a number divisible by 5

13 El-Menia Governorate — El-Minia Educational Directorate

Answer the following questions :

1 **Choose the correct answer :**

① If $\left(\frac{4}{9}\right)^x = \left(\frac{9}{4}\right)^4$, then $x = \dots\dots\dots$

(a) zero (b) 1 (c) 4 (d) -4

(2) The probability of the impossible event = ...

(a) zero (b) 1 (c) $\varnothing$ (d) $\frac{1}{2}$

(3) The half of the number 2^{20} = ...

(a) 2^{40} (b) 4^{20} (c) 2^{19} (d) 2^{21}

(4) $(7x)^{zero}$ = ... where $x \neq$ zero.

(a) 1 (b) zero (c) – 5 (d) 5

(5) The number 750000 is written in its scientific notation = 7.5×10^{n} , n = ...

(a) 4 (b) 5 (c) – 4 (d) – 5

2 Complete the following :

(1) If the probability of success of a student is 0.7 , then the probability of his failure = ...

(2) $\sqrt{25-9}$ = ...

(3) $7 - 3 \times 2 + 6$ = ...

(4) If : $-2x > -8$, then $x <$ ....

(5) The S.S of the equation $3x = 6$ in N is ....

3 [a] Find the solution set of the inequality : $3x - 5 > 13$, where $x \in N$

[b] Find in simplest form : $\left(\frac{4}{3}\right)^2 \times \sqrt{\frac{81}{16}} \times \left(\frac{3}{5}\right)^{zero}$

4 [a] Find the value of : $12 \times 2^2 + 24 + 3^2$

[b] Find the S.S of the equation : $3x - 13 = 26$, where $x \in N$

5 [a] Find the value : $\frac{(2)^7 \times (2)^5}{(2)^6 \times (2)^3}$

[b] For one roll of a fair die , Calculate the probability of :

(1) Getting a prime number. (2) Getting a number more than 6

14 Assiut Governorate Educational Expirementals unit

Answer the following questions :

1 Complete each of the following :

(1) If : $x = y$, then 5^{x-y} = ...

(2) $\sqrt{6^2 + 8^2}$ = ...

③ The probability of the impossible event = ...

④ If : $3x = 4$, then $12x - 2 =$ ....

⑤ 1 , 2 , 3 , 5 , 8 , ... , ... in the same pattern.

2 **Choose the correct answer from the given :**

① $3^5 \times 2^5 =$ ....

(a) 5^5 (b) 6^5 (c) 6^{10} (d) 5^{25}

② $\frac{4a^2b^4}{2a^3b^3} =$ .... where $ab \neq 0$

(a) $2ab$ (b) $2a^5b^7$ (c) $\frac{2b}{a}$ (d) $\frac{2}{ab}$

③ If the probability that a pupil succeed is 75 % , then the probability of his failure is ....

(a) -0.75 (b) 0.25 (c) 0.75 (d) 1.25

④ $x < \frac{5}{2}$ equivalent ....

(a) $x < \frac{5}{2}$ (b) $x > \frac{5}{2}$ (c) $x < 10$ (d) $x > 10$

⑤ If : $-x < 5$, then x .... -5

(a) $<$ (b) $>$ (c) $=$ (d) $\leq$

3 **[a] Find the solution set of each of the following where $x \in \mathbb{Q}$:**

① $3x - 1 \leq 2x + 3$ ② $(3x + 2) + 5 = 13$

[b] Find the value of the expression in the simplest form : $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

4 [a] Two integers the smaller one is $2x$ and the greater is $5x$, if the difference between them is 30 find the two numbers.

[b] Simplify : $\left(-\frac{4}{5}\right)^6 \div \left(\frac{4}{5}\right)^4$

5 **[a] Find the result of the expression :** $(5.4 \times 10^4) + (3.7 \times 10^5)$ in the form $a \times 10^n$ where n is integer number.

[b] A box contains 4 white , 5 red and 6 blue balls. A ball is drawn randomly from the box. Calculate the probabilities of the following events :

① The ball is red. ② The ball is white or red.

15 Qena Governorate — Math's S.V

Answer the following questions :

1 Choose the correct answer :

① $37 \times 10^3 =$ .... (In the standard form)

(a) 0.37×10^5 (b) 37×10^3 (c) 3.7×10^4 (d) 0.37×10^{-3}

② The additive inverse of $\left(-\frac{2}{5}\right)^2$ is ....

(a) $\frac{2}{5}$ (b) $\frac{4}{25}$ (c) $\frac{-4}{25}$ (d) otherwise.

③ $\sqrt{10^2 - 8^2} =$ ....

(a) 36 (b) $10 - 8$ (c) 6 (d) 100

④ If : $a > b$, c is a negative number , then ac .... bc

(a) $<$ (b) $\geq$ (c) $>$ (d) $=$

⑤ Which of the following number is the probability of occurrence of an event ?

(a) 1.2 (b) -0.4 (c) 3.15 (d) 75 %

2 Complete each of the following :

① $\sqrt{(-4)^2} =$ ....

② The age of boy now is x years , then his age after 5 years is .... years.

③ The probability of the impossible event = ...

④ If : $2x = 8$, then $x + 1 =$ .... ⑤ $\left(\frac{2}{7}\right)^{-1} =$ ....

3 [a] Find the S.S. for the equation : $5x - 6 = 34$, $x \in \mathbb{Q}$

[b] Find : $\left(\frac{4}{3}\right)^2 \times \sqrt{\frac{9}{16}} \times \left(\frac{4}{3}\right)^{-1}$

4 [a] Find the S.S. for the inequality in $\mathbb{N}$:

$5x - 2 \geq 2x + 1$

[b] If : $x = -\frac{1}{2}$, $y = \frac{3}{4}$. Find the value of : $y + x^2$

5 [a] Find the value of : $\frac{7^3 \times 7^{-4}}{(-7)^4 \times 7^{-3}}$

[b] A card is selected randomly from 10 cards numbered from 1 to 10

What is the probability that elected card shows :

① An odd number. ② An even number. ③ A prime number.

Answers of school book models in algebra and statistics

Model 1

1

(1) 10 (2) 2 (3) 3 (4) 3.5×10^{-3} (5) $\frac{4}{9}$

2

(1) (b) (2) (a) (3) (a) (4) (a)

3 $1 \times \frac{4}{25} \times \sqrt{\frac{25}{4}} = 1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$

4

[a] (1) $\because 3x + 1 = 25$ $\therefore 3x = 24$

$\therefore x = 8$ $\therefore$ The S.S. $= \{8\}$

(2) $\because 2x + 5 < 16$ $\therefore 2x < 11$

$\therefore x < \frac{11}{2}$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q}, x < \frac{11}{2}\}$

[b] The population that will be in 2 years

$= 3\,(1.02)^2 \times 70^6 = 3.1212 \times 10^6$ people

5

(1) The probability of change it before travelled 50 thousand km. $= \frac{80}{800} = \frac{1}{10}$

(2) The probability of change it after travelled more than 100 thousand km. $= \frac{600}{800} = \frac{3}{4}$

Model 2

1

(1) 1 (2) $\frac{4}{7}$ (3) zero (4) 13 , 21

(5) 510 students

2

(1) (b) (2) (d) (3) (a) (4) (b)

3 $\because 5x - 2x = 30$ $\therefore 3x = 30$

$\therefore x = 10$

$\therefore$ The two numbers are : 20 , 50

4

[a] (1) $\because 3x + 7 = 13$ $\therefore 3x = 6$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

(2) $\because 2x + 15 < 19$ $\therefore 2x < 4$

$\therefore x < 2$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q}, x < 2\}$

[b] $\frac{1}{9} + \frac{8}{9} - 1 = 1 - 1 =$ zero

5

(1) The probability of getting a prime even number $= \frac{1}{6}$

(2) The probability of getting an odd number less than 4 $= \frac{2}{6} = \frac{1}{3}$

Model 3

1

(1) 1 (2) $\frac{1}{5}$, $\frac{1}{4}$ (3) $\{\frac{1}{2}\}$ (4) -4

2

(1) (d) (2) (c) (3) (b) (4) (b)

3 Let the number be x

$\therefore$ Its three times $= 3x$

$\therefore x + 3x = 28$ $\therefore 4x = 28$

$\therefore x = 7$

$\therefore$ The number $= 7$

4

[a] (1) $\because 3x + 5 = 11$ $\therefore 3x = 6$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

(2) $\because 2x + 3 \le 7$ $\therefore 2x \le 4$

$\therefore x \le 2$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q}, x \le 2\}$

[b] The time elapsed

$= \frac{d}{v} = \frac{1.44 \times 10^8 \times 1000}{3 \times 10^8}$

$= 480$ seconds $= 8$ minutes

5

[a] $(5.4 \times 10^4) + (3.7 \times 10^5)$

$= 10^4\,(5.4 + 3.7 \times 10)$

$= 10^4\,(5.4 + 37)$

$= 10^4 \times 42.4 = 4.24 \times 10^5$

[b] (1) The probability of the two faces are similar

$= \frac{2}{4} = \frac{1}{2}$

(2) The probability of appearance only one tail

$= \frac{2}{4} = \frac{1}{2}$

Model 4

1

(1) $\frac{1}{2}$ (2) 1 (3) $\{3, 4\}$ (4) $\frac{31}{32}, \frac{63}{64}$

2

(1) (b) (2) (b) (3) (b) (4) (d) (5) (c)

3 [a] $\left(\frac{x^2}{y^3}\right)^2 = \left(\left(\frac{3}{4}\right)^2 \div \left(-\frac{3}{2}\right)^3\right)^2$

$= \left(\frac{9}{16} \div \left(-\frac{27}{8}\right)\right)^2$

$= \left(\frac{9}{16} \times \left(-\frac{8}{27}\right)\right)^2 = \left(-\frac{1}{6}\right)^2 = \frac{1}{36}$

[b] Let the great number be X

$\therefore$ Th small number $= X - 5$

$\therefore X + X - 5 = 15$ $\therefore 2X - 5 = 15$

$\therefore 2X = 20$ $\therefore X = 10$

$\therefore$ The two numbers are : 10 , 5

4

(1) $\because 3X + 2 = 8$ $\therefore 3X = 6$

$\therefore X = 2$ $\therefore$ The S.S. $= \{2\}$

(2) $\because 4X - 3 < 7$ $\therefore 4X < 10$

$\therefore X < \frac{10}{4}$ $\therefore X < \frac{5}{2}$

$\therefore$ The S.S. $= \{X : X \in \mathbb{Q} , X < \frac{5}{2}\}$

5

(1) The probability of the appearance of at least one head $= \frac{3}{4}$

(2) The probability of the appearance of at most one head $= \frac{3}{4}$

Model 5

1

(1) $\frac{1}{2}$ (2) 1 (3) -2 (4) $\frac{3}{8}$ (5) 4^{19}

2

(1) (c) (2) (a) (3) (c) (4) (b)

3 $12 \times 4 \div 24 + 9 = 48 \div 24 + 9 = 2 + 9 = 11$

4

[a] (1) $\because 3 - 4X = -5$

$\therefore 3 + 5 = 4X$ $\therefore 4X = 8$

$X = 2$ $\therefore$ The S.S. $= \{2\}$

(2) $\because 2X - 1 \geq 5$ $\therefore 2X \geq 6$

$\therefore X \geq 3$

$\therefore$ The S.S. $= \{X : X \in \mathbb{Q} , X \geq 3\}$

[b] $\frac{3n^2}{2} - 3n + 3 - n = \frac{3n^2}{2} - 4n + 3$

The value $= \frac{3 \times 1^2}{2} - 4 \times 1 + 3 = \frac{3}{2} - 4 + 3 = \frac{1}{2}$

5

[a] Let the age of the eldest sister be X years

$\therefore$ Let the age of the middle sister $= (X - 3)$ years

, Let the age of the youngest sister $= (X - 5)$ years

$\therefore X + X - 3 + X - 5 = 25$

$\therefore 3X - 8 = 25$

$\therefore 3X = 33$ $\therefore X = 11$

$\therefore$ The ages of three sisters are :

6 years , 8 years and 11 years

[b] (1) The probability the ball is red $= \frac{5}{15} = \frac{1}{3}$

(2) The probability the ball is white or red

$= \frac{9}{15} = \frac{3}{5}$

Answers of final examinations

Answers of school examinations in Algebra and Statistics

1 Cairo

1

(1) (c) (2) (b) (3) (d) (4) (a) (5) (a)

2

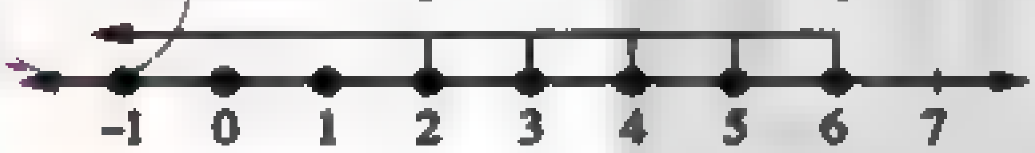
(1) 5 (2) $3 > a$ (3) -4 (4) $-\frac{4}{25}$ (5) zero

3

[a] (1) $\because 5x - 1 \le 29 \quad \therefore 5x \le 30$

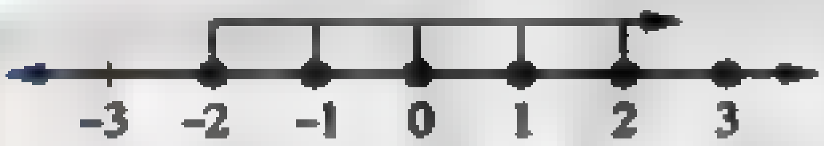
$\therefore x \le 6$

$\therefore$ The S.S. $= \{6, 5, 4, 3, 2, ...\}$



(2) $\because x + 4 > 1 \qquad \therefore x > -3$

$\therefore$ The S.S. $= \{-2, -1, 0, 1, ...\}$



[b] (1) $\frac{4}{9} \times \frac{9}{4} \times 1 = 1$

(2) $\left(-\frac{2}{3}\right)^6 \div \left(-\frac{2}{3}\right)^7 = \left(-\frac{2}{3}\right)^{6-7} = \left(-\frac{2}{3}\right)^{-1} = -\frac{3}{2}$

4

[a] Let the two numbers be x and $2x$

$\therefore x + 2x = 45 \qquad \therefore 3x = 45$

$\therefore x = 15$

$\therefore$ The two numbers are : 15 and 30

[b] $x^3 y^2 z = \left(-\frac{1}{2}\right)^3 \times \left(\frac{4}{3}\right)^2 \times \left(\frac{1}{5}\right)$

$= -\frac{1}{8} \times \frac{16}{9} \times \frac{1}{5} = -\frac{2}{45}$

5

[a] The order is : 7.3×10^{-5}, 10^{-4}, 2.7×10^{-4} and 0.25×10^{-2}

[b] $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(1) The probability of getting an even number $= \frac{4}{8} = \frac{1}{2}$

(2) The probability of getting a number greater than or equal to 6 $= \frac{3}{8}$

2 Cairo

1

(1) (b) (2) (d) (3) (c) (4) (c) (5) (b)

2

(1) $(x - 3)$ years (2) 90 (3) zero (4) -5 (5) 2

3

[a] $x^2 - y z^2 = \left(-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right) \times \left(-\frac{4}{3}\right)^2$

$= \frac{9}{4} - \frac{1}{2} \times \frac{16}{9} = \frac{9}{4} - \frac{8}{9} = \frac{49}{36}$

[b] $\because 3x + 6 < -x + 4$

$\therefore 3x + x < 4 - 6$

$\therefore 4x < -2 \qquad \therefore x < -\frac{2}{4} \qquad \therefore x < -\frac{1}{2}$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q}, x < -\frac{1}{2}\}$

4

[a] $\because 5x + 8 = 13 - 2x$

$\therefore 5x + 2x = 13 - 8 \quad \therefore 7x = 5$

$\therefore x = \frac{5}{7} \qquad \therefore$ The S.S. $= \left\{\frac{5}{7}\right\}$

[b] (1) The probability that the selected card shows an odd number $= \frac{5}{10} = \frac{1}{2}$

(2) The probability that the selected card shows a prime number $= \frac{4}{10} = \frac{2}{5}$

(3) The probability that the selected card shows an odd number greater than 3 $= \frac{3}{10}$

5

[a] $\frac{73}{73} = 1$

[b] Let the length of the rectangle $= x$ metres

$\therefore$ the width of the rectangle $= (x - 4)$ metres

, $\because$ the perimeter $=$ (length + width) $\times 2$

$\therefore 68 = (x + x - 4) \times 2 \qquad \therefore 34 = 2x - 4$

$\therefore 2x = 38 \qquad \therefore x = 38 \div 2 = 19$ metres

$\therefore$ the dimensions of the rectangle are 19 metres , 15 metres

3 Giza

1

(1) (c) (2) (a) (3) (d) (4) (a) (5) (c)

2

(1) $\frac{2}{5}$ (2) -2 (3) zero (4) 10 (5) $>$

3

[a] (1) $\frac{4}{9} \times \frac{3}{2} \times 1 = \frac{2}{3}$

(2) $\frac{(2)^5 \times (2)^4}{(2)^9} = 2^{5+4-9} = 2^0 = 1$

[b] Let the three consecutive natural numbers be x , $x + 1$ and $x + 2$

$\therefore x + x + 1 + x + 2 = 33$

$\therefore 3x + 3 = 33$ $\therefore 3x = 30$

$\therefore x = 10$

$\therefore$ The three numbers are : 10 , 11 , 12

4

(1) $\because 2x + 1 = 13$ $\therefore 2x = 12$

$\therefore x = 6$ $\therefore$ The S.S. $= \{6\}$

(2) $\because 3x - 1 \le 2x + 4$

$\therefore 3x - 2x \le 4 + 1$ $\therefore x \le 5$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x \le 5\}$

5

[a] $x^2 - y^2 z = \left(\frac{-2}{3}\right)^2 - \left(\frac{1}{2}\right)^2 \times \left(\frac{-4}{3}\right)$

$= \frac{4}{9} - \frac{1}{4} \times \left(\frac{-4}{3}\right) = \frac{4}{9} + \frac{1}{3} = \frac{7}{9}$

[b] (1) The probability of the ball is black $= \frac{4}{16} = \frac{1}{4}$

(2) The probability of the ball is not white

$= \frac{4 + 7}{16} = \frac{11}{16}$

4 Giza

1

(1) (b) (2) (b) (3) (b) (4) (d) (5) (d)

2

(1) 10 (2) $-\frac{1}{4}$ (3) $x - 5$ (4) 17 (5) 5

3

[a] (1) $\because 8x + 4 = 12$ $\therefore 8x = 8$

$\therefore x = 1$ $\therefore$ The S.S. $= \{1\}$

(2) $\because 2x + 3 \le 7$ $\therefore 2x \le 4$

$\therefore x \le 2$ $\therefore$ The S.S. $= \{2 , 1 , 0\}$

[b] $(a^3 + b^3)^{-1} = \left(\left(\frac{1}{3}\right)^3 + \left(-\frac{2}{3}\right)^3\right)^{-1}$

$= \left(\frac{1}{27} - \frac{8}{27}\right)^{-1} = \left(-\frac{7}{27}\right)^{-1} = -\frac{27}{7}$

4

[a] (1) $\frac{(-3)^3 \times (-3)^5}{(-3)^6} = (-3)^{3+5-6} = (-3)^2 = 9$

(2) $\left(\frac{-5}{4}\right)^2 \times \sqrt{\frac{25}{16}} \times \left(\frac{-4}{5}\right)^3 = \left(\frac{5}{4}\right)^2 \times \frac{5}{4} \times -\left(\frac{5}{4}\right)^{-3}$

$= -\left(\frac{5}{4}\right)^{2+1-3} = -1$

[b] $\because (BC)^2 = (AC)^2 - (AB)^2 = (10)^2 - (6)^2$

$= 100 - 36 = 64$

$\therefore BC = \sqrt{64} = 8$ cm.

5

[a] Let the three odd consecutive numbers : $x , x + 2 , x + 4$

$\therefore x + x + 2 + x + 4 = 117$

$\therefore 3x + 6 = 117$ $\therefore 3x = 111$

$\therefore x = 37$

$\therefore$ The three numbers are : 37 , 39 and 41

[b] $S = \{1 , 2 , 3 , 4 , 5 , 6 , 7 , 8\}$

(1) The probability of A $= \frac{4}{8} = \frac{1}{2}$

(2) The probability of B $= \frac{4}{8} = \frac{1}{2}$

(3) The probability of C $= \frac{2}{8} = \frac{1}{4}$

5 Alexandria

1

(1) (b) (2) (b) (3) (c) (4) (b) (5) (c)

2

(1) 5 (2) -5 (3) $\varnothing$ (4) 1 (5) 7

3

[a] $\frac{2^3 \times 2^{-4}}{2^{-2} \times 2^5} = 2^{3-4+2-5} = 2^{-4} = \frac{1}{16}$

[b] (1) $\because 2x - 5 = 1$ $\therefore 2x = 6$

$\therefore x = 3$ $\therefore$ The S.S. $= \{3\}$

(2) $\because 4x + 9 \le 1$ $\therefore 4x \le -8$

$\therefore x \le -2$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x \le -2\}$

4

[a] Let the age of the son be x years

$\therefore$ The age of the man $= 3x$ years

$\therefore x + 3x = 60$ $\therefore 4x = 60$ $\therefore x = 15$

$\therefore$ The age of man = 45 years

, The age of his son = 15 years

[b] $8x^3yz^2 = 8 \times \left(\frac{1}{2}\right)^3 \times \frac{4}{3} \times \left(\frac{3}{2}\right)^2$
$= 8 \times \frac{1}{8} \times \frac{4}{3} \times \frac{9}{4} = 3$

5

[a] $\frac{1}{9} + \frac{8}{9} - 1 = \frac{9}{9} - 1 = 1 - 1 = 0$

[b] (1) The probability of drawing a ball numbered with a number that is divisible by $3 = \frac{2}{8} = \frac{1}{4}$

(2) The probability of drawing a ball numbered with a prime number $= \frac{4}{8} = \frac{1}{2}$

6 El-Kalyoubia

1

(1) (d) (2) (b) (3) (d) (4) (d) (5) (a)

2

(1) 10 % (2) $\frac{5}{4}$ (3) > (4) 14 (5) 7×10^6

3

[a] $\frac{x^2 \times x^7}{x^3 \times x^2} = x^{2+7-3-2} = x^4$

The value $= (-3)^4 = 81$

[b] 9

4

[a] $\because 3x - 2 > x + 4$ $\therefore 3x - x > 4 + 2$

$\therefore 2x > 6$ $\therefore x > 3$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x > 3\}$

[b] $\because 3x - 1 = 5$ $\therefore 3x = 6$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

5

[a] Let the three consecutive even numbers be :

$x , x + 2 , x + 4$

$\therefore x + x + 2 + x + 4 = 966$

$\therefore 3x + 6 = 966$ $\therefore 3x = 960$

$\therefore x = 320$

$\therefore$ The three numbers : 320 , 322 , 324

[b] (1) The probability that the drawn ball is red $= \frac{5}{10} = \frac{1}{2}$

(2) The probability that the drawn ball is white $= \frac{3}{10}$

(3) The probability that the drawn ball is not blue $= \frac{5+3}{10} = \frac{8}{10} = \frac{4}{5}$

7 El-Kalyoubia

1

(1) (d) (2) (c) (3) (c) (4) (b) (5) (a)

2

(1) 10 (2) 3 (3) 14 (4) $\frac{4}{25}$ (5) $\frac{1}{2}$

3

[a] $\frac{4}{3} \times \frac{3}{5} \times \frac{1}{4} = \frac{1}{5}$

[b] Let length of the rectangle $= x$ metres

$\therefore$ the width of the rectangle $= (x - 4)$ metres

, $\because$ the perimeter = (length + width) × 2

$\therefore 68 = (x + x - 4) \times 2$ $\therefore 34 = 2x - 4$

$\therefore 2x = 38$ $\therefore x = 38 \div 2 = 19$ metres

$\therefore$ the dimensions of the rectangle are 19 metres , 15 metres

4

[a] $\frac{x^7 \times x^5}{x^8} = x^{7+5-8} = x^4$

The value $= (-2)^4 = 16$

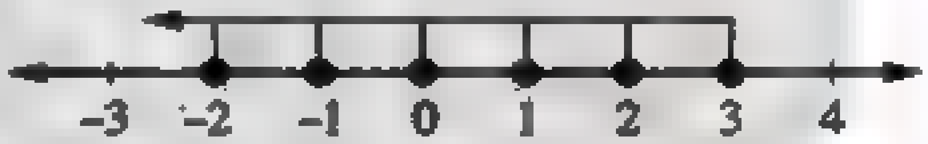
[b] $\because 3x + 7 = 13$ $\therefore 3x = 6$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

5

[a] $\because x - 1 < 3$ $\therefore x < 4$

$\therefore$ The S.S. $= \{3 , 2 , 1 , 0 , -1 , ...\}$



[b] The probability of the event A $= \frac{3}{6} = \frac{1}{2}$

8 El-Sharkia

1

(1) (b) (2) (a) (3) (c) (4) (a) (5) (a)

2

(1) 10 (2) 1 (3) {0} (4) 2.37 , −4 (5) $\frac{a}{2}$

3

[a] (1) The probability that the drawn ball is black $= \frac{0}{12} = 0$

(2) The probability that the drawn ball is not red $= \frac{6+4}{12} = \frac{10}{12} = \frac{5}{6}$

[b] $\because \frac{a^2}{b^2} = 0.16$ $\therefore \left(\frac{a}{b}\right)^2 = \frac{16}{100} = \frac{4}{25}$

$\therefore \frac{a}{b} = \pm \frac{2}{5}$ $\therefore \left(\frac{a}{b}\right)^3 = \left(\pm \frac{2}{5}\right)^3 = \pm \frac{8}{125}$

4

[a] $\frac{y^3 \times y^{-4}}{y^{-2} \times y^5} = y^{3-4+2-5} = y^{-4} = \frac{1}{y^4}$

The value $= \frac{1}{(-1)^4} = \frac{1}{1} = 1$

[b] $\because 3x - 2 \le 3 - 2x$ $\therefore 3x + 2x \le 3 + 2$

$\therefore 5x \le 5$ $\therefore x \le 1$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x \le 1\}$

5

[a] Let the two consecutive even numbers be : $x , x + 2$

$\therefore x + x + 2 = 150$ $\therefore 2x + 2 = 150$

$\therefore 2x = 148$ $\therefore x = 74$

$\therefore$ The two numbers are : 74 , 76

[b] In 2 years the population will be $3\,(1.02)^2 \times 10^6$

$= 3.1212 \times 10^6$

9 El-Beheira

1

(1) $-\frac{9}{49}$ (2) -2 (3) x (4) $\frac{3}{2}$ (5) 4

2

(1) (c) (2) (c) (3) (c) (4) (b) (5) (c)

3

Let the two natural numbers be $x , 2x$

$\therefore x + 2x = 108$ $\therefore 3x = 108$

$\therefore x = 36$

$\therefore$ The two numbers are : 36 and 72

4

[a] $\because x^3 + y^2 z^2 = \left(\frac{-1}{2}\right)^3 + \left[\left(\frac{3}{4}\right)^2 \times \left(-\frac{3}{2}\right)^2\right]$

$= -\frac{1}{8} + \left[\frac{9}{16} \times \frac{9}{4}\right] = -\frac{1}{8} + \frac{81}{64}$

$= -\frac{1}{8} \times \frac{64}{81} = -\frac{8}{81}$

[b] $1 - 4x + 1 > 2x - 6$

$\therefore 2 - 4x > 2x - 6$ $\therefore 2 + 6 > 2x + 4x$

$\therefore 8 > 6x$ $\therefore \frac{8}{6} > x$

$\therefore x < \frac{4}{3}$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x < \frac{4}{3}\}$

5

[a] $(3.8 \times 10^8) \div (1.9 \times 10^6)$

$= \frac{3.8}{1.9} \times \frac{10^8}{10^6} = 2 \times 10^2$

[b] (1) The probability of that person's preference sport $= \frac{36}{100} = 0.36$

(2) The probability of that person's preference not watch news $= \frac{12 + 31 + 36}{100} = \frac{79}{100} = 0.79$

(3) The probability of that person's preference documentaries $= \frac{12}{100} = 0.12$

(4) The probability of that person's preference all of them $= \frac{12 + 31 + 21 + 36}{100} = \frac{100}{100} = 1$

10 El-Dakahlia

1

(1) $\frac{-4}{9}$ (2) 4 (3) zero (4) 2.7×10^{-7} (5) $\frac{1}{2}$

2

(1) (d) (2) (d) (3) (c) (4) (a) (5) (d)

3

[a] $\frac{25}{16} \times \frac{4}{5} \times 1 = \frac{5}{4}$

[b] $\because 5x - 7 \le 3$ $\therefore 5x \le 10$

$\therefore x \le 2$

4

[a] $\frac{(-2)^6 \times (-2)^4}{(-2)^8} = (-2)^{6+4-8} = (-2)^2 = 4$

[b] Let the three consecutive numbers be $x , x + 1 , x + 2$

$\therefore x + x + 1 + x + 2 = 42$

$\therefore 3x + 3 = 42$ $\therefore 3x = 39$ $\therefore x = 13$

$\therefore$ The three numbers are : 13 , 14 , 15

5

[a] $\left(\frac{x^2}{y^3}\right)^2 = \left(\left(\frac{3}{4}\right)^2 \div \left(-\frac{3}{2}\right)^3\right)^2 = \left(\frac{9}{16} \div \frac{-27}{8}\right)^2$

$= \left(\frac{9}{16} \times \frac{-8}{27}\right)^2 = \left(-\frac{1}{6}\right)^2 = \frac{1}{36}$

[b] (1) The probability of the ball is red $= \frac{7}{15}$

(2) The probability of the ball is not white

$= \frac{5 + 7}{15} = \frac{12}{15} = \frac{4}{5}$

11 El-Monofia

1

(1) (b) (2) (b) (3) (a) (4) (c) (5) (a)

2

(1) 10 (2) 1 (3) zero (4) 3 (5) 10

3

[a] (1) $\frac{4}{9} \times \frac{3}{2} \times 1 = \frac{2}{3}$

(2) $\left(\frac{-2}{3}\right)^6 \div \left(\frac{-2}{3}\right)^4 = \left(\frac{-2}{3}\right)^{6-4} = \left(\frac{-2}{3}\right)^2 = \frac{4}{9}$

[b] $\because 4X + 5 \geq 1$ $\therefore 4X \geq -4$

$\therefore X \geq -1$

$\therefore$ The S.S. $= \{-1, 0, 1, 2, 3, ...\}$

4

[a] $\because 3X - 3 = 9$ $\therefore 3X = 12$

$\therefore X = 4$ $\therefore$ The S.S. $= \{4\}$

[b] Let the age of the eldest sister be X years

$\therefore$ Let the age of the middle sister be $= (X - 3)$ years

, Let the age of the youngest sister be $= (X - 5)$ years

$\therefore X + X - 3 + X - 5 = 25$

$\therefore 3X - 8 = 25$ $\therefore 3X = 33$ $\therefore X = 11$

$\therefore$ The ages of three sisters are : 6 years , 8 years and 11 years

5

[a] $\frac{3^3 \times 3^{-4}}{3^{-2} \times 3^5} = 3^{3-4+2-5} = 3^{-4} = \frac{1}{81}$

[b] (1) The probability of getting an odd number $= \frac{4}{8} = \frac{1}{2}$

(2) The probability of getting a number divisible by 3 $= \frac{2}{8} = \frac{1}{4}$

12 El-Gharbia

1

(1) zero (2) 9 (3) $X + 3$ (4) $\frac{3}{2}$ (5) 12

2

(1) (c) (2) (d) (3) (b) (4) (a) (5) (b)

3

[a] Let the length of the rectangle $= X$ metres

$\therefore$ the width of the rectangle $= (X - 4)$ metres

, $\because$ the perimeter $=$ (length + width) $\times 2$

$\therefore 68 = (X + X - 4) \times 2$ $\therefore 34 = 2X - 4$

$\therefore 2X = 38$ $\therefore X = 38 \div 2 = 19$ metres

$\therefore$ the dimensions of the rectangle are 19 metres , 15 metres

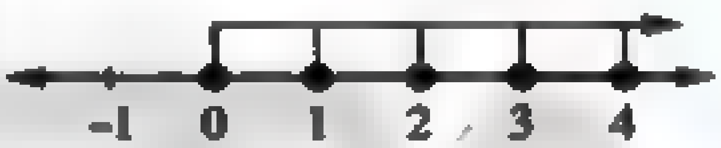
[b] $\because 3X - 3 = 9$ $\therefore 3X = 12$

$\therefore X = 4$ $\therefore$ The S.S. $= \{4\}$

4

[a] $\because 4X + 5 \geq -3$ $\therefore 4X \geq -8$ $\therefore X \geq -2$

$\therefore$ The S.S. $= \{0, 1, 2, 3, ...\}$



[b] $9 + [4 \div (4 \div 2)] = 9 + [4 \div 2] = 15$

5

[a] (1) The probability of that card is numbered an even number $= \frac{5}{10} = \frac{1}{2}$

(2) The probability of that card is numbered a number divisible by 5 $= \frac{2}{10} = \frac{1}{5}$

[b] $\sqrt{21 - 5} = \sqrt{16} = 4$

13 El Ismailia

1

(1) (d) (2) (d) (3) (a) (4) (b) (5) (c)

2

(1) zero (2) $\frac{9}{4}$ (3) 1 (4) 10 (5) $\varnothing$

3

[a] $\frac{4}{9} \times \frac{3}{2} \times 1 = \frac{2}{3}$

[b] $a^2 b^3 + b^2 c = \left(\frac{-1}{2}\right)^2 \times (2)^3 + (2)^2 \times \frac{3}{4}$

$= \frac{1}{4} \times 8 + 4 \times \frac{3}{4} = 2 + 3 = 5$

Answers of final examinations

4

(1) $\because 4x-1\geq 7$ $\quad\therefore 4x\geq 8$ $\quad\therefore x\geq 2$

$\therefore$ The S.S. $=\{x , x\in\mathbb{Q} , x\geq 2\}$

(2) $\because 2(x+5)=16$ $\quad\therefore 2x+10=16$

$\therefore 2x=16-10=6$ $\quad\therefore x=3$ $\quad\therefore$ The S.S. $=\{3\}$

5

[a] $\because 2x+2x-18^\circ=90^\circ$

$\therefore 4x-18^\circ=90^\circ$ $\quad\therefore 4x=108^\circ$

$\therefore x=27^\circ$

$\therefore$ The measures of the two angles : 54° and 36°

[b] (1) The probability that the drawn ball is red

$=\frac{2}{12}=\frac{1}{6}$

(2) The probability that the drawn ball is green

$=\frac{4}{12}=\frac{1}{3}$

(3) The probability that the drawn ball is black

$=\frac{0}{12}=0$

14 Assiut

1

(1) $\frac{1}{2}$ (2) zero (3) $\varnothing$ (4) $x-5$ (5) 16

2

(1) (c) (2) (b) (3) (c) (4) (a) (5) (b)

3

[a] (1) $\left(\frac{-5}{8}\right)^0\times\sqrt{\frac{25}{4}}\times\left(\frac{-2}{5}\right)^2=1\times\frac{5}{2}\times\frac{4}{25}=\frac{2}{5}$

(2) $\frac{3^4\times 3^3}{3^6}=3^{4+3-6}=3$

[b] Let the three consecutive even numbers be :

$x , x+2 , x+4$

$\therefore x+x+2+x+4=78$

$\therefore 3x+6=78$ $\quad\therefore 3x=72$ $\quad\therefore x=24$

$\therefore$ The three numbers are : 24 , 26 , 28

4

[a] $\because 3x+5=26$ $\quad\therefore 3x=21$

$\therefore x=7$ $\quad\therefore$ The S.S. $=\{7\}$

[b] (1) The probability of the drawn ball is white $=\frac{5}{16}$

(2) The probability of the drawn ball is red $=\frac{7}{16}$

5

[a] $\because 4x-7\geq 1$ $\quad\therefore 4x\geq 8$ $\quad\therefore x\geq 2$

$\therefore$ The S.S. $=\{x : x\in\mathbb{Q} , x\geq 2\}$

[b] $a^2b^3+b^2c=\left(\frac{1}{2}\right)^2\times(2)^3+(2)^2\times\frac{3}{4}$

$=\frac{1}{4}\times 8+4\times\frac{3}{4}=2+3=5$

15 Qena

1

(1) (b) (2) (c) (3) (b) (4) (c) (5) (d)

2

(1) 8 (2) $\frac{4}{9}$ (3) zero (4) 4 (5) $\frac{8}{5}$

3

[a] (1) $\because 2x+14=12$ $\quad\therefore 2x=-2$

$\therefore x=-1$ $\quad\therefore$ The S.S. $=\{-1\}$

(2) $\because 3x+1\leq 7$ $\quad\therefore 3x\leq 6$ $\quad\therefore x\leq 2$

$\therefore$ The S.S. $=\{2 , 1 , 0 , -1 , -2 , ...\}$

[b] $\frac{5^6\times 5^2\times 5^3}{5^4\times 5^5}=5^{6+2+3-4-5}=5^2=25$

4

[a] $(ab)^2-c^2=\left(-\frac{2}{3}\times\frac{3}{4}\right)^2-\left(\frac{1}{2}\right)^2$

$=\left(-\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2=\frac{1}{4}-\frac{1}{4}=0$

[b] (1) The probability of that card numbered by an even number $=\frac{5}{10}=\frac{1}{2}$

(2) The probability of that card numbered by a number divisible by $5=\frac{2}{10}=\frac{1}{5}$

5

[a] $\left(\frac{5}{8}\right)^0\times\sqrt{\frac{25}{4}}\times\frac{4}{25}=1\times\frac{5}{2}\times\frac{4}{25}=\frac{2}{5}$

[b] Let the three consecutive numbers be :

$x , x+1 , x+2$

$\therefore x+x+1+x+2=33$

$\therefore 3x+3=33$ $\quad\therefore 3x=30$

$\therefore x=10$

$\therefore$ The three numbers are : 10 , 11 , 12

Answers of school book models in algebra and statistics

Model 1

1

① 10 ② 2 ③ 3 ④ 3.5×10^{-3} ⑤ $\frac{4}{9}$

2

① (b) ② (a) ③ (a) ④ (a) ⑤ (a)

3 [a] $1 \times \frac{4}{25} \times \sqrt{\frac{25}{4}} = 1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$

[b] $3\,ab + 8\,a + (4\,b)$

$= 3 \times 4 \times (-2) + 8 \times 4 + (4 \times -2)$

$= -24 + 32 + (-8)$

$= -24 - 4 = -28$

4

[a] ① $\because 3x + 1 = 25$ $\therefore 3x = 24$

$\therefore x = 8$ $\therefore$ The S.S. = {8}

② $\because 2x + 5 < 16$ $\therefore 2x < 11$

$\therefore x < \frac{11}{2}$

$\therefore$ The S.S. = $\{x : x \in \mathbb{Q} , x < \frac{11}{2}\}$

[b] The population that will be in 2 years

$= 3\,(1.02)^2 \times 70^6 = 3.1212 \times 10^6$ people

5

① The probability of change it before travelled 50 thousand km. $= \frac{80}{800} = \frac{1}{10}$

② The probability of change it after travelled more than 100 thousand km. $= \frac{600}{800} = \frac{3}{4}$

Model 2

1

① 1 ② $\frac{4}{7}$ ③ zero ④ 13 , 21

⑤ 510 students

2

① (b) ② (d) ③ (a) ④ (b) ⑤ (c)

3 [a] $\because 5x - 2x = 30$ $\therefore 3x = 30$ $\therefore x = 10$

$\therefore$ The two numbers are : 20 , 50

[b] $\frac{5^{-4} \times 5^{7}}{5^{3}} = 5^{-4+7-3} = 5^{0} = 1$

4

[a] ① $\because 3x + 7 = 13$ $\therefore 3x = 6$

$\therefore x = 2$ $\therefore$ The S.S. = {2}

② $\because 2x + 15 < 19$ $\therefore 2x < 4$

$\therefore x < 2$

$\therefore$ The S.S. = $\{x : x \in \mathbb{Q} , x < 2\}$

[b] $\frac{1}{9} + \frac{8}{9} - 1 = 1 - 1 =$ zero

5

[a] ① The probability of getting a prime even number $= \frac{1}{6}$

② The probability of getting an odd number less than 4 $= \frac{2}{6} = \frac{1}{3}$

[b] Let the width of the rectangle = x cm.

$\therefore$ The length of the rectangle = $2x$ cm.

$\therefore$ The area = length × width

$\therefore 12.5 = x \times 2x$ $\therefore 12.5 = 2x^2$

$6.25 = x^2$ $\therefore x = \sqrt{6.25} = 2.5$ cm.

$\therefore$ The width = 2.5 cm.

, The length = 5 cm.

Model 3

1

① 1 ② $\frac{1}{5} , \frac{1}{4}$ ③ $\{\frac{1}{2}\}$ ④ $\frac{4}{3}$ ⑤ -4

2

① (d) ② (c) ③ (b) ④ (b) ⑤ (b)

3

[a] Let the number be x

$\therefore$ Its three times = $3x$

$\therefore x + 3x = 28$ $\therefore 4x = 28$

$\therefore x = 7$

$\therefore$ The number = 7

[b] $\because$ The area of the triangle = $\frac{1}{2}$ base × height

$\therefore$ The area of the triangle = $\frac{1}{2} \times 9 \times 8 = 36\ \text{cm}^2$

Let the side length of the square = ℓ

, $\because$ The area of the square = The area of the triangle

$\therefore 36 = \ell^2$ $\therefore \ell = \sqrt{36} = 6$ cm.

4

[a] ① $\because 3x + 5 = 11$ $\therefore 3x = 6$

$\therefore x = 2$ $\therefore$ The S.S. = {2}

② $\because 2x + 3 \le 7$ $\therefore 2x \le 4$

$\therefore x \le 2$

$\therefore$ The S.S. = $\{x : x \in \mathbb{Q} , x \le 2\}$

Answers of final examinations

[b] The time elapsed $= \frac{d}{v} = \frac{1.44 \times 10^{8} \times 1000}{3 \times 10^{8}}$

$= 480$ seconds $= 8$ minutes

5

[a] $(5.4 \times 10^{4}) + (3.7 \times 10^{5})$

$= 10^{4}\,(5.4 + 3.7 \times 10)$

$= 10^{4}\,(5.4 + 37)$

$= 10^{4} \times 42.4 = 4.24 \times 10^{5}$

[b] ① The probability of the two faces are similar $= \frac{2}{4} = \frac{1}{2}$

② The probability of appearance only one tail $= \frac{2}{4} = \frac{1}{2}$

Model 4

1

① $\frac{1}{2}$ ② 1 ③ $\{3 , 4\}$

④ $\frac{-2}{5}$ ⑤ $\frac{31}{32} , \frac{63}{64}$

2

① (b) ② (b) ③ (b) ④ (d) ⑤ (c)

3

[a] $\left(\frac{x^2}{y^3}\right)^2 = \left(\left(\frac{3}{4}\right)^2 \div \left(-\frac{3}{2}\right)^3\right)^2$

$= \left(\frac{9}{16} \div \left(-\frac{27}{8}\right)\right)^2$

$= \left(\frac{9}{16} \times \left(-\frac{8}{27}\right)\right)^2 = \left(-\frac{1}{6}\right)^2 = \frac{1}{36}$

[b] Let the great number be x

$\therefore$ The small number $= x - 5$

$\therefore x + x - 5 = 15$ $\therefore 2x - 5 = 15$

$\therefore 2x = 20$ $\therefore x = 10$

$\therefore$ The two numbers are : 10 , 5

4

[a] ① $\because 3x + 2 = 8$ $\therefore 3x = 6$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

② $\because 4x - 3 < 7$ $\therefore 4x < 10$

$\therefore x < \frac{10}{4}$ $\therefore x < \frac{5}{2}$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x < \frac{5}{2}\}$

[b] Let the side length of the square $= l$ m.

$\therefore \frac{3}{4} l^2 = \frac{75}{64}$ $\therefore l^2 = \frac{75 \times 4}{64 \times 3}$

$\therefore l^2 = \frac{25}{16}$ $\therefore l = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1\frac{1}{4}$ m.

5

[a] ① The probability of the appearance of at least one head $= \frac{3}{4}$

② The probability of the appearance of at most one head $= \frac{3}{4}$

[b] $\left(\frac{7^4 \times 7^{-2}}{7^3}\right)^{-2} = \frac{7^{-8} \times 7^{4}}{7^{-6}} = 7^{-8+4+6} = 7^2 = 49$

Model 5

1

① $\frac{1}{2}$ ② 1 ③ -2 ④ $\frac{3}{8}$ ⑤ 4^{19}

2

① (c) ② (a) ③ (c) ④ (b) ⑤ (b)

3

[a] $12 \times 4 \div 24 + 9 = 48 \div 24 + 9 = 2 + 9 = 11$

[b] $\left(\frac{y}{x^2}\right)^{-2} = \left(\frac{3}{4} \div \left(-\frac{1}{2}\right)^2\right)^{-2}$

$= \left(\frac{3}{4} \div \frac{1}{4}\right)^{-2}$

$= \left(\frac{3}{4} \times 4\right)^{-2} = 3^{-2} = \frac{1}{9}$

4

[a] ① $\because 3 - 4x = -5$

$\therefore 3 + 5 = 4x$ $\therefore 4x = 8$

$x = 2$ $\therefore$ The S.S. $= \{2\}$

② $\because 2x - 1 \geq 5$ $\therefore 2x \geq 6$ $\therefore x \geq 3$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x \geq 3\}$

[b] $\frac{3n^2}{2} - 3n + 3 - n = \frac{3n^2}{2} - 4n + 3$

The value $= \frac{3 \times 1^2}{2} - 4 \times 1 + 3 = \frac{3}{2} - 4 + 3 = \frac{1}{2}$

5

[a] Let the age of the eldest sister be x years

$\therefore$ Let the age of the middle sister $= (x - 3)$ years

, Let the age of the youngest sister $= (x - 5)$ years

$\therefore x + x - 3 + x - 5 = 25$ $\therefore 3x - 8 = 25$

$\therefore 3x = 33$ $\therefore x = 11$

$\therefore$ The ages of three sisters are :

6 years , 8 years and 11 years

[b] ① The probability the ball is red $= \frac{5}{15} = \frac{1}{3}$

② The probability the ball is white or red $= \frac{9}{15} = \frac{3}{5}$

Answers of final examinations

Answers of school examinations in Algebra and Statistics

1 Cairo

1

(1) (a) (2) (a) (3) (c) (4) (b) (5) (a)

2

(1) $-\frac{25}{49}$ (2) zero (3) 1 (4) 6 (5) -5

3

[a] $\because 5x+3>15+x$ $\therefore 5x-x>15-3$

$\therefore 4x>12$ $\therefore x>3$

$\therefore$ The S.S. $=\{x: x\in\mathbb{Q}, x>3\}$

[b] $\frac{a^5\times a^7}{a^8}=a^{5+7-8}=a^4$

4

[a] $a^3+b^2c^2=\left(\frac{-1}{2}\right)^3+\left[\left(\frac{3}{4}\right)^2\times\left(\frac{-3}{2}\right)^2\right]$

$=\frac{-1}{8}+\left(\frac{9}{16}\times\frac{9}{4}\right)=\frac{-1}{8}\div\frac{81}{64}$

$=\frac{-1}{8}\times\frac{64}{81}=-\frac{8}{81}$

[b] (1) The probability of the ball is red $=\frac{5}{20}=\frac{1}{4}$

(2) The probability of the ball is not black

$=\frac{8+5}{20}=\frac{13}{20}$

5

[a] $\because 3x+17=38$ $\therefore 3x=21$

$\therefore x=7$ $\therefore$ The S.S. $=\{7\}$

[b] $\frac{4}{3}\times\frac{3}{5}\times\frac{1}{4}=\frac{1}{5}$

2 Cairo

1

(1) (b) (2) (c) (3) (a) (4) (d) (5) (c)

2

(1) zero (2) $\frac{9}{4}$ (3) $x+5$ (4) 16 (5) $\{5\}$

3

(1) $\frac{4}{9}\times\frac{3}{2}\times 1=\frac{2}{3}$

(2) $\frac{2^5\times(-2)^4}{2^9}=\frac{2^5\times 2^4}{2^9}=2^{5+4-9}=2^0=1$

4

(1) $\because 2x+14=12$ $\therefore 2x=-2$ $\therefore x=-1$

$\therefore$ The S.S. $=\{-1\}$

(2) $\because 3x+1\le 7$ $\therefore 3x\le 6$ $\therefore x\le 2$

$\therefore$ The S.S. $=\{2, 1, 0, -1, ...\}$

5

[a] $(ab)^2-c^2=\left(\frac{-2}{3}\times\frac{3}{4}\right)^2-\left(\frac{1}{2}\right)^2=\left(\frac{-1}{2}\right)^2-\left(\frac{1}{2}\right)^2$

$=\frac{1}{4}-\frac{1}{4}=0$

[b] (1) The probability of getting red ball $=\frac{7}{15}$

(2) The probability of getting not white ball

$=\frac{5+7}{15}=\frac{12}{15}=\frac{4}{5}$

3 Cairo

1

(1) (b) (2) (c) (3) (b) (4) (c) (5) (a)

2

(1) $\frac{1}{9}$ (2) 10^{-4} (3) 10 (4) 3 (5) 7

3

[a] $\because 5x+8=13-2x$ $\therefore 5x+2x=13-8$

$\therefore 7x=5$ $\therefore x=\frac{5}{7}$

[b] $\frac{-1}{8}\div(-3)=\frac{-1}{8}\times\left(-\frac{1}{3}\right)=\frac{1}{24}$

4

[a] $a^3b^2+b^2c-8abc=\left(-\frac{1}{2}\right)^3\times 2^2+2^2\times\frac{3}{4}$

$-8\times\left(-\frac{1}{2}\right)\times 2\times\frac{3}{4}$

$=\frac{-1}{8}\times 4+4\times\frac{3}{4}+6=8\frac{1}{2}$

[b] $\because x=0.000625$

$\therefore \sqrt{x}=\sqrt{0.000625}=0.025=2.5\times 10^{-2}$

5

[a] $\because 3x-1\le 2x+3$ $\therefore 3x-2x\le 3+1$

$\therefore x\le 4$

[b] The probability of getting a card with an odd number $=\frac{20}{39}$

Answers of final examinations

4 Giza

1

(1) (d) (2) (c) (3) (b) (4) (c) (5) (a)

2

(1) 2 (2) $\frac{9}{4}$ (3) $\{3 , 4 , 5 , ...\}$

(4) $\frac{1}{2}$ (5) -4

3

[a] $\frac{x^3 \times x^4}{x^5} = x^{3+4-5} = x^2$

at $x = -3$ $\therefore (-3)^2 = 9$

[b] $\because 2x - 3 \le 1$ $\therefore 2x \le 4$ $\therefore x \le 2$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x \le 2\}$

4

[a] $1 \times \frac{9}{8} \times \left(\frac{-8}{27}\right) = -\frac{1}{3}$

[b] $\because 2x - 3 = 6$ $\therefore 2x = 9$ $\therefore x = \frac{9}{2}$

$\therefore$ The S.S. $= \left\{\frac{9}{2}\right\}$

5

[a] $\because 5x - 3x = 20$ $\therefore 2x = 20$ $\therefore x = 10$

$\therefore$ The two numbers are : 30 and 50

[b] $S = \{1 , 2 , 3 , 4 , 5 , 6\}$

(1) The probability of rolling an even number $= \frac{3}{6} = \frac{1}{2}$

(2) The probability of rolling a number greater than $1 = \frac{5}{6}$

5 Giza

1

(1) (c) (2) (d) (3) (b) (4) (a) (5) (b)

2

(1) $\frac{1}{2}$ (2) zero (3) $\varnothing$ (4) $x - 5$ (5) 16

3

[a] (1) $1 \times \frac{5}{2} \times \frac{4}{25} = \frac{2}{5}$ (2) $\frac{3^4 \times 3^3}{3^6} = 3^{4+3-6} = 3$

[b] Let the three numbers be : $x , x + 2 , x + 4$

$\therefore x + x + 2 + x + 4 = 78$

$\therefore 3x + 6 = 78$ $\therefore 3x = 72$

$\therefore x = \frac{72}{3} = 24$

$\therefore$ The three numbers are : 24 , 26 , 28

4

[a] $\because 3x + 5 = 26$ $\therefore 3x = 21$ $\therefore x = 7$

$\therefore$ The S.S. $= \{7\}$

[b] (1) The probability of the drawn ball is white $= \frac{5}{16}$

(2) The probability of the drawn ball is red $= \frac{7}{16}$

5

[a] $\because 4x - 7 \ge 1$ $\therefore 4x \ge 8$ $\therefore x \ge 2$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x \ge 2\}$

[b] $a^2b^3 + b^2c = \left(\frac{1}{2}\right)^2 \times (2)^3 + (2)^2 \times \frac{3}{4}$

$= \frac{1}{4} \times 8 + 4 \times \frac{3}{4} = 5$

6 Alexandria

1

(1) (b) (2) (c) (3) (a) (4) (d) (5) (a)

2

(1) zero (2) 1 (3) -3 (4) 1 , 10 (5) $\frac{1}{2}$

3

[a] $\left(\frac{x^2}{y^3}\right)^2 = \left(\frac{\left(\frac{3}{4}\right)^2}{\left(\frac{-3}{2}\right)^3}\right)^2 = \left(\frac{\frac{9}{16}}{-\frac{27}{8}}\right)^2$

$= \left(\frac{-1}{6}\right)^2 = \frac{1}{36}$

[b] $12 \times 4 \div 24 + 9 = 48 \div 24 + 9 = 2 + 9 = 11$

4

(1) $\because 4x + 3 = 11$ $\therefore 4x = 8$ $\therefore x = 2$

$\therefore$ The S.S. $= \{2\}$

(2) $\because 6x - 2 < 7$ $\therefore 6x < 9$ $\therefore x < \frac{3}{2}$

$\therefore$ The S.S. $= \left\{x : x \in \mathbb{Q} , x < \frac{3}{2}\right\}$

5

(1) The probability of getting an even number greater or equal to $6 = \frac{2}{8} = \frac{1}{4}$

(2) The probability of getting the number $= 2^n \ (n \in \mathbb{Z} < 4) = \frac{3}{8}$

Answers of final examinations

7 Alexandria

1

① 8 ② 1 ③ −4 ④ 3 ⑤ 9

2

① (b) ② (a) ③ (d) ④ (a) ⑤ (b)

3

[a] $\frac{2^3 \times 2^{-4}}{2^{-2} \times 2^5} = \frac{2^{-1}}{2^3} = 2^{-1-3} = 2^{-4} = \frac{1}{16}$

[b] $\because 3x - 2 > x + 4 \qquad \therefore 3x - x > 4 + 2$

$\therefore 2x > 6 \qquad \therefore x > 3$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x > 3\}$

4

[a] $8 + [4 + (4 + 2)] = 8 + 6 = 14$

[b] The probability of event A $= \frac{3}{6} = \frac{1}{2}$

5

[a] $1 \times \frac{5}{2} \times \frac{4}{25} = \frac{2}{5}$

[b] $x^3 y^2 = \left(-\frac{1}{2}\right)^3 \times \left(\frac{3}{4}\right)^2 = -\frac{1}{8} \times \frac{9}{16} = -\frac{9}{128}$

8 El-Kalyoubia

1

① (c) ② (b) ③ (d) ④ (b) ⑤ (c)

2

① zero ② 5 ③ $-\frac{16}{25}$ ④ $\frac{1}{2}$ ⑤ $\frac{1}{6} , \frac{1}{5}$

3

[a] $\frac{7^{-3} \times 7^5}{7^2} = 7^{-3+5-2} = 7^0 = 1$

[b] ① The probability of getting an even number $= \frac{3}{6} = \frac{1}{2}$

② The probability of getting an odd number less than 4 $= \frac{2}{6} = \frac{1}{3}$

4

[a] $\because 3x + 9 = 11 \qquad \therefore 3x = 2 \qquad \therefore x = \frac{2}{3}$

$\therefore$ The S.S. $= \left\{\frac{2}{3}\right\}$

[b] $\left(\frac{x}{y}\right)^2 = \left(\frac{\left(\frac{-3}{2}\right)}{\left(\frac{-4}{3}\right)}\right)^2 = \frac{81}{64}$

5

[a] $\because 2x + 3 \le 7 \qquad \therefore 2x \le 4 \qquad \therefore x \le 2$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x \le 2\}$

[b] $\because (XY)^2 = 25 \qquad \therefore XY = 5$ cm.

$\because$ Z is the midpoint of $\overline{XY}$

$\therefore XZ = \frac{5}{2} = 2.5$ cm.

9 El-Monofia

1

① (c) ② (b) ③ (d) ④ (a) ⑤ (c)

2

① $\frac{1}{4} , \frac{1}{3}$ ② 1 ③ −4

④ 1 ⑤ $\{-2 , -3 , -4 , ...\}$

3

[a] $\frac{4}{25} \times \frac{5}{2} \times 1 = \frac{2}{5}$

[b] $\because 3x + 8 \le 1 \qquad \therefore 3x \le -7 \qquad \therefore x \le -\frac{7}{3}$

$\therefore$ The S.S. $= \left\{x : x \in \mathbb{Q} , x \le -\frac{7}{3}\right\}$

4

[a] $\because 2x + 9 = 17 \qquad \therefore 2x = 8 \qquad \therefore x = 4$

$\therefore$ The S.S. $= \{4\}$

[b] Let the age of the son is x years.

$\therefore$ The age of the father is $3x$ years.

$\therefore x + 5 + 3x + 5 = 58 \qquad \therefore 4x + 10 = 58$

$\therefore 4x = 48 \qquad \therefore x = 12$

$\therefore$ The age of the son is 12 years

, The age of the father is 36 years.

5

[a] $\left[\frac{x^3}{y^2}\right]^2 = \left[\frac{\left(\frac{-3}{2}\right)^3}{\left(\frac{3}{4}\right)^2}\right]^2 = \left[\frac{\frac{-27}{8}}{\frac{9}{16}}\right]^2 = (-6)^2 = 36$

[b] ① The probability the ball is red $= \frac{5}{15} = \frac{1}{3}$

② The probability the ball isn't blue $= \frac{4+5}{15} = \frac{9}{15} = \frac{3}{5}$

③ The probability the ball is green $= \frac{0}{15} = 0$

10 Port Said

1

① 7 ② zero ③ −1 ④ 10 ⑤ 14

2

(1) (b) (2) (c) (3) (b) (4) (a) (5) (c)

3

(1) $\because -3 + x = 5$ $\therefore x = 8$

$\therefore$ The S.S. $= \{8\}$

(2) $\because 2x - 1 \geq 5$ $\therefore 2x \geq 6$ $\therefore x \geq 3$

$\therefore$ The S.S. $= \{3, 4, 5, ...\}$

1

[a] $(x + y)^{-1} = \left(\frac{1}{4} + \frac{1}{8}\right)^{-1} = \left(\frac{3}{8}\right)^{-1} = \frac{8}{3}$

[b] $1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$

4

[a] $3\,ab^2 \times 2\,a^2b^3 = 6\,a^3b^5$

[b] (1) The probability drawing a card carries an odd number greater than 10 $= \frac{0}{10} = 0$

(2) The probability drawing a card carries an even number less than 10 $= \frac{4}{10} = \frac{2}{5}$

(3) The probability drawing a card carries a prime number $= \frac{4}{10} = \frac{2}{5}$

11 Kafr El-Sheikh

1

(1) (c) (2) (d) (3) (a) (4) (a) (5) (d)

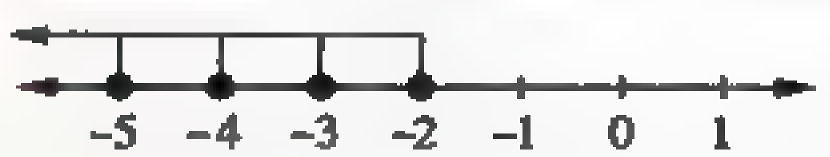
2

(1) $\geq$ (2) $\frac{1}{9}$ (3) 12 (4) 10 (5) 15 %

3

[a] $\because 3x - 5 = 4$ $\therefore 3x = 9$ $\therefore x = 3$

[b] $\because -2x - 3 \geq 1$ $\therefore -2x \geq 4$ $\therefore x \leq -2$



4

(1) $\frac{16}{25} \times \frac{5}{4} \times 1 = \frac{4}{5}$

(2) $\frac{(-3)^6 \times (3)^{-3}}{(3)^5 \times (3)^{-4}} = \frac{(3)^6 \times (3)^{-3}}{(3)^5 \times (3)^{-4}} = \frac{3^3}{3} = 3^{3-1} = 3^2 = 9$

5

[a] $x^2yz = \left(\frac{1}{2}\right)^2 \times \frac{4}{5} \times \frac{5}{2} = \frac{1}{4} \times \frac{4}{5} \times \frac{5}{2} = \frac{1}{2}$

[b] (1) The probability of getting a number divisible by 7 $= \frac{1}{10}$

(2) The probability of getting an even number $= \frac{5}{10} = \frac{1}{2}$

(3) The probability of getting a number less than 8 $= \frac{7}{10}$

12 Beni Suef

1

(1) (b) (2) (d) (3) (b) (4) (c) (5) (d)

2

(1) -4 (2) 14 (3) $\frac{6}{5}$ (4) zero (5) $\frac{1}{2}$

3

(1) $\because 4x - 1 \geq 7$ $\therefore 4x \geq 8$ $\therefore x \geq 2$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q}, x \geq 2\}$

(2) $\because 2(x + 4) = 15$ $\therefore 2x + 8 = 15$ $\therefore 2x = 7$

$\therefore x = \frac{7}{2}$ $\therefore$ The S.S. $= \left\{\frac{7}{2}\right\}$

4

[a] $a^2b^3 + b^2c = \left(\frac{-1}{2}\right)^2 \times 2^3 + 2^2 \times \frac{3}{4}$

$= \frac{1}{4} \times 8 + 4 \times \frac{3}{4} = 5$

[b] $144 - 8 + 8 = 144 - 1 = 143$

5

[a] $\frac{7^{-2} \times 7^5}{7^3} = 7^{-2+5-3} = 7^0 = 1$

[b] (1) The probability of getting an even number $= \frac{5}{10} = \frac{1}{2}$

(2) The probability of getting a number divisible by 5 $= \frac{2}{10} = \frac{1}{5}$

13 El-Menia

1

(1) (d) (2) (a) (3) (c) (4) (a) (5) (b)

2

(1) 0.3 (2) 4 (3) 7 (4) 4 (5) $\{2\}$

3

[a] $\because 3x-5>13 \quad \therefore 3x>18 \quad \therefore x>6$

$\therefore$ The S.S. $=\{7, 8, 9, \ldots\}$

[b] $\frac{16}{9}\times\frac{9}{4}\times 1=4$

4

[a] $12\times 4+24+9=48+24+9=81$

[b] $\because 3x-13=26 \quad \therefore 3x=39 \quad \therefore x=13$

$\therefore$ The S.S. $=\{13\}$

5

[a] $\frac{(2)^7\times(2)^5}{(2)^6\times(2)^3}=\frac{2^{12}}{2^9}=2^{12-9}=2^3=8$

[b] (1) The probability of getting a prime number $=\frac{0}{6}=\frac{1}{2}$

(2) The probability of getting a number more than 6 $=\frac{0}{6}=0$

14 Assiut

1

(1) 1 (2) 10 (3) zero (4) 14 (5) 13 , 21

2

(1) (b) (2) (c) (3) (b) (4) (a) (5) (b)

3

[a] (1) $\because 3x-1\le 2x+3 \quad \therefore 3x-2x\le 3+1$

$\therefore x\le 4$

$\therefore$ The S.S. $=\{x : x\in\mathbb{Q}, x\le 4\}$

(2) $\because (3x+2)+5=13 \quad \therefore 3x+7=13$

$\therefore 3x=6 \quad \therefore x=2 \quad \therefore$ The S.S. $=\{2\}$

[b] $\frac{1}{9}+\frac{8}{9}-1=0$

4

[a] $\because 5x-2x=30 \quad \therefore 3x=30 \quad \therefore x=10$

$\therefore$ The two numbers are : 20 and 50

[b] $\left(\frac{-4}{5}\right)^6\div\left(\frac{4}{5}\right)^4=\left(\frac{4}{5}\right)^6\div\left(\frac{4}{5}\right)^4=\left(\frac{4}{5}\right)^2=\frac{16}{25}$

5

[a] $(5.4\times 10^4)+(3.7\times 10^5)=(5.4+3.7\times 10)\times 10^4$

$=(5.4+37)\times 10^4=42.4\times 10^4=4.24\times 10^5$

[b] (1) The probability that the ball is red $=\frac{5}{15}=\frac{1}{3}$

(2) The probability that the ball is white or red

$=\frac{4+5}{15}=\frac{9}{15}=\frac{3}{5}$

15 Qena

1

(1) (c) (2) (c) (3) (c) (4) (a) (5) (d)

2

(1) 4 (2) $x+5$ (3) zero (4) 5 (5) $\frac{7}{2}$

3

[a] $\because 5x-6=34 \quad \therefore 5x=40 \quad \therefore x=8$

$\therefore$ The S.S. $=\{8\}$

[b] $\frac{16}{9}\times\frac{3}{4}\times\frac{3}{4}=1$

4

[a] $\because 5x-2\ge 2x+1 \quad \therefore 5x-2x\ge 1+2$

$\therefore 3x\ge 3 \quad \therefore x\ge 1$

$\therefore$ The S.S. $=\{1, 2, 3, \ldots\}$

[b] $y+x^2=\frac{3}{4}+\left(\frac{-1}{2}\right)^2=\frac{3}{4}+\frac{1}{4}=\frac{3}{4}\times 4=3$

5

[a] $\frac{7^3\times(7)^{-4}}{(-7)^4\times(7)^{-3}}=\frac{7^3\times(7)^{-4}}{7^4\times(7)^{-3}}=\frac{7^{-1}}{7}=7^{-1-1}=7^{-2}=\frac{1}{49}$

[b] (1) The probability of an odd number $=\frac{5}{10}=\frac{1}{2}$

(2) The probability of an even number $=\frac{5}{10}=\frac{1}{2}$

(3) The probability of a prime number $=\frac{4}{10}=\frac{2}{5}$

Second

Geometry and Measurement

Unit 3

Geometry and Measurement

Euclid (325 B.C. – 265 B.C.)

He was a Greek mathematician. He lived in Alexandria. Euclid introduced the system of axioms. Since this time, geometry of Euclid was considered a model of logical proof.

Euclid's Axioms (notations) :

- Things which are equal to one thing are equal to each other.
- If equals are added to equals, then the sums are equal.
- Things which coincide with one another are equal to each other.
- The whole is greater than the part.

Euclid
(325 BC - 265 BC)

Lesson Deductive proof

In the previous term , we had trained practically how to deduce some properties and theorems by measuring and using the geometric tools.

In this lesson , we will use the geometric properties that we had known before to deduce the solutions and proof for theorems and problems theoritically without going to the geometric tools in measure.

Also, in this lesson , we will give some examples to show how to use the deductive proof to prove some geometric theorems and problems.

But before that , we will give you some advices about how to write the proof in geometry.

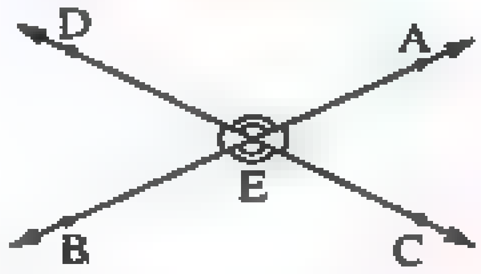
How to write the proof in geometry ?

1. Read the problem very good to understand all its parts. This may require to read it more than one time.
2. Write "The given" which are all information given in the problem.
3. Write "The required" which is the required to find or to prove.
4. Check the existence of a right mathematical figure expressing the problem. If there is no figure, draw it by yourself using the information given in the problem.
5. Put on the figure all the possible information given in the problem as : side lengths - angle measures - signs showing the equality of side lengths - signs showing the equality of angle measures - signs showing the parallelism - and so on.
6. Think how to use the given information, the facts and the theorems you studied before to get the required.

   Write mathematical sentences, knowing that you must write a reason to each writen sentence (if this sentence isn't one of the given information in the problem), for example : You can't write m ($\angle$ A) = m ($\angle$ B) without writting a mathematical reason as $\angle$ A and $\angle$ B are alternate angles resulted from cutting two parallel straight lines by another line.
7. Continue writting these sentences till getting one sentence expressing the required. Thus, you finish the proof.

In the following, some examples showing how to write the deductive proof :

1 **If two straight lines intersect , then the measures of each two vertically opposite angles are equal.**



Given $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two straight lines intersecting at E — the given information

Required to prove (R.T.P.) m (∠ AED) = m (∠ BEC) — what we want to find or prove

Proof

∵ ∠ AED and ∠ AEC are two adjacent angles where $\overrightarrow{EC} \cup \overrightarrow{ED} = \overleftrightarrow{CD}$

∴ m (∠ AED) + m (∠ AEC) = 180°

, ∵ ∠ AEC and ∠ BEC are two adjacent angles where $\overrightarrow{EA} \cup \overrightarrow{EB} = \overleftrightarrow{AB}$

The steps of proving the truth of the required.

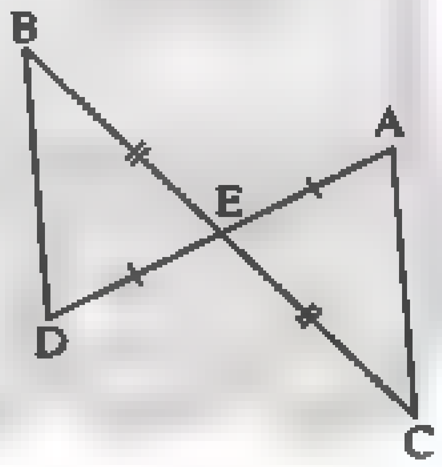
∴ m (∠ AEC) + m (∠ BEC) = 180°

∴ m (∠ AED) + m (∠ AEC) = m (∠ AEC) + m (∠ BEC)

∴ m (∠ AED) = m (∠ BEC) (Q.E.D.)*

Similarly , you can prove that m (∠ AEC) = m (∠ BED)

Example 1



In the opposite figure :

$\overline{AD} \cap \overline{BC} = \{E\}$ where AE = DE and BE = CE

Prove that : Δ AEC ≡ Δ DEB

Solution

Given $\overline{AD} \cap \overline{BC} = \{E\}$ where AE = DE , BE = CE

R.T.P. Δ AEC ≡ Δ DEB

Proof ∵ $\overline{AD} \cap \overline{BC} = \{E\}$ ∴ m (∠ AEC) = m (∠ DEB) (V.O.A)

∴ In ΔΔ AEC and DEB :

$\begin{cases} AE = DE \text{ (given)} \\ CE = BE \text{ (given)} \\ m(\angle AEC) = m(\angle DEB) \text{ (by proof)} \end{cases}$

∴ Δ AEC ≡ Δ DEB (Q.E.D.)

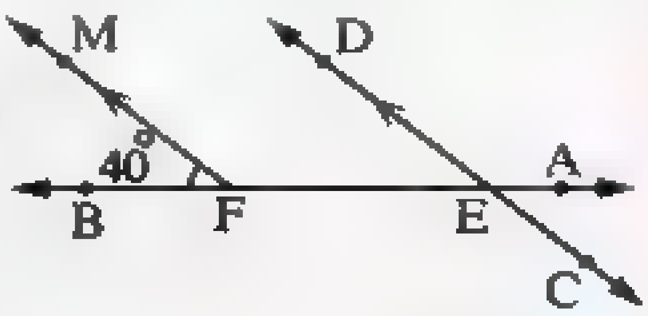
* Q.E.D. is an abbreviation for quod erat demonstrandum.
It is a Latin abbreviation which means to be demonstrated

Try by yourself

In the opposite figure :

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$, $\overleftrightarrow{CD} // \overrightarrow{FM}$, $F \in \overleftrightarrow{AB}$ and m (∠ MFB) = 40°

Find with proof : m (∠ AEC)



Solution

Given ....

Required to find (R.T.F.) ....

Proof

∵ $\overleftrightarrow{CD}$ // .... (given) and $\overleftrightarrow{AB}$ is a transversal.

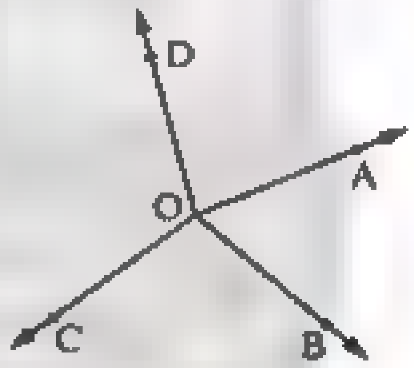
∴ m (∠ DEB) = m (∠ ....) = 40° (corresponding angles)

∵ $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$

∴ m (∠ AEC) = m (∠ ....) (V.O.A)

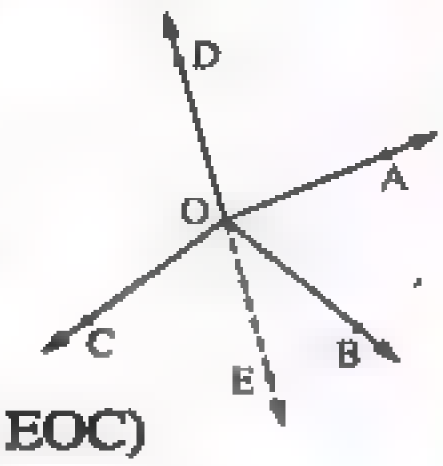
∴ m (∠ AEC) = ...° (The req.)

2 **The sum of the measures of the accumulative angles at a point is equal to 360°**



Given $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ are rays that start at O

R.T.P. The sum of the measures of the accumulative angles at O is 360°



Construction Draw $\overrightarrow{DO}$ and $E \in \overrightarrow{DO}$

Proof

∵ m (∠ EOB) + m (∠ BOA) + m (∠ AOD) = 180°

, m (∠ EOC) + m (∠ COD) = 180°

∴ m (∠ EOB) + m (∠ BOA) + m (∠ AOD) + m (∠ EOC) + m (∠ COD) = 180° + 180° = 360°

∴ m (∠ AOB) + m (∠ BOC) + m (∠ COD) + m (∠ DOA) = 360°

(Q.E.D.)

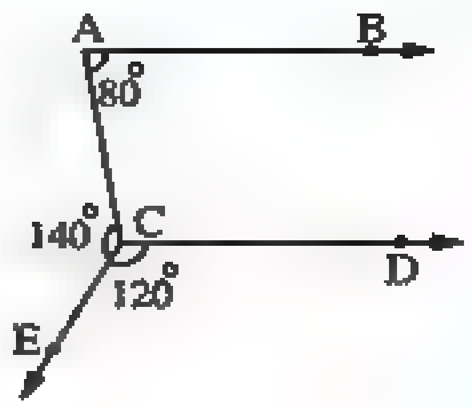
Example (2)

In the opposite figure :

m (∠ BAC) = 80° , m (∠ DCE) = 120°

and m (∠ ACE) = 140°

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{CD}$



Solution

Given | m (∠ BAC) = 80° , m (∠ DCE) = 120° , m (∠ ACE) = 140°

R.T.P. | $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Proof | ∵ m (∠ DCA) + m (∠ DCE) + m (∠ ACE) = 360°

(accumulative angles at C)

∴ m (∠ DCA) = 360° – (120° + 140°) = 100°

∴ m (∠ BAC) + m (∠ DCA) = 80° + 100° = 180°

And they are interior angles in the same side of the transversal $\overleftrightarrow{AC}$

∴ $\overrightarrow{AB} \parallel \overrightarrow{CD}$ (Q.E.D.)

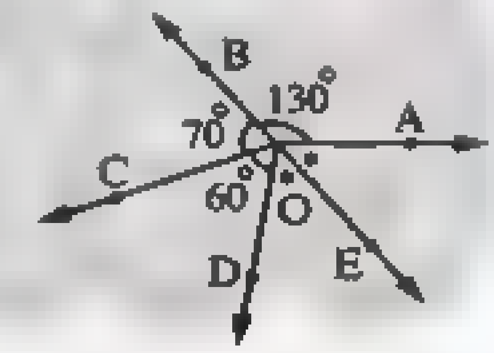
Try by yourself

In the opposite figure :

m (∠ AOB) = 130° , m (∠ BOC) = 70° ,

m (∠ COD) = 60° and $\overrightarrow{OE}$ bisects ∠ AOD

Prove that $\overrightarrow{OE}$ and $\overrightarrow{OB}$ are on one straight line.



Given | ....

R.T.P. | ....

Proof | ∵ m (∠ AOB) + m (∠ BOC) + m (∠ COD) + m (∠ AOD) = ... °

(Accumulative angles at O)

∴ m (∠ AOD) = ...° – .... ° = ... °

∵ $\overrightarrow{OE}$ bisects ∠ .... (given)

∴ m (∠ AOE) = $\frac{1}{2}$ m (∠ ....)

∴ m (∠ AOE) = $\frac{1}{2}$ × ....° = ...°

∴ m (∠ AOE) + m (∠ AOB) = ...° + ...° = ...°

∴ $\overrightarrow{OE}$ and $\overrightarrow{OB}$ are on one straight line. (Q.E.D.)

Lesson The polygon

The simple line and non-simple line :

The simple line : It is the line that does not cut itself.

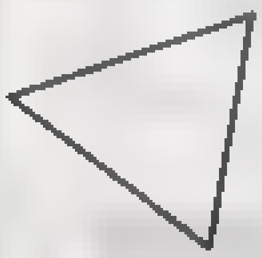
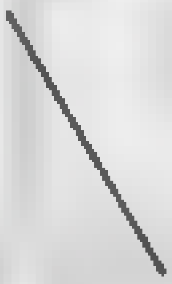
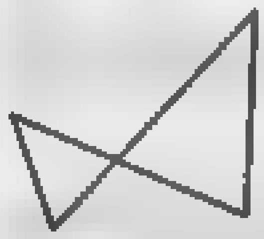
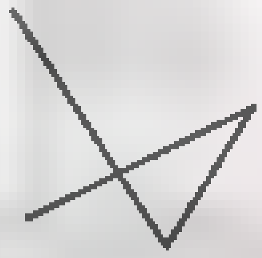
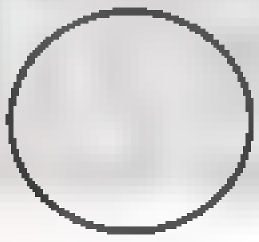
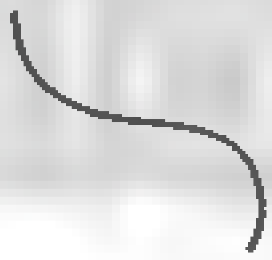
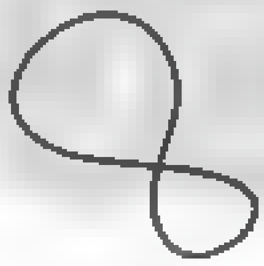
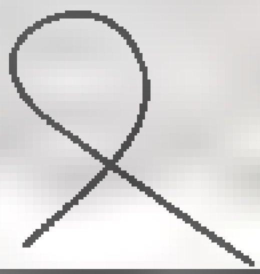
The non-simple line : It is the line that cuts itself once or more.

The closed line and the open line :

The closed line : It is the line that ends where it starts at the same point.
It may be simple or non-simple.

The open line : It is the line whose starting point is not the end point.
It may be simple or non-simple.

In the following , there are some figures which show simple lines and non-simple lines , closed lines or open lines :

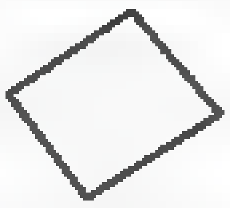
Simple line		Non-simple line	
Closed	Open	Closed	Open
			
			

The polygon

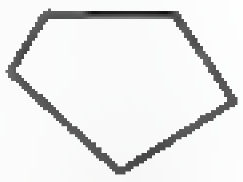
It is a simple closed line that consists of three line segments , or more. The polygon is named according to the number of its sides.

Examples for some polygons :

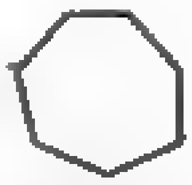
Triangle
(3 sides)



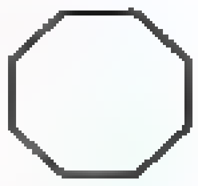
Quadrilateral
(4 sides)



Pentagon
(5 sides)



Heptagon
(7 sides)

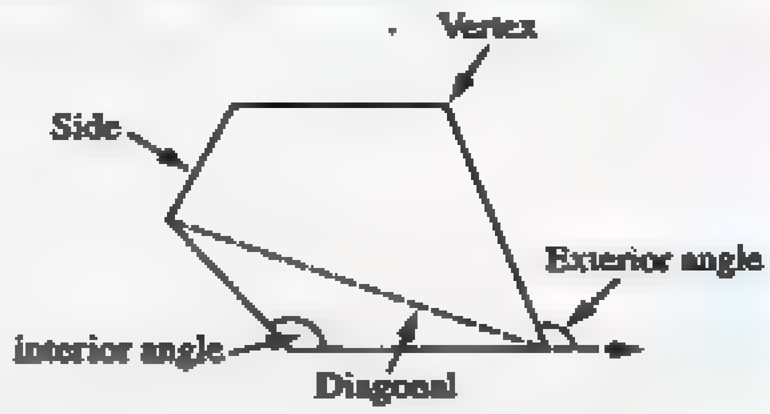


Octagon
(8 sides)

Unit 3

Remarks

1. Each line segment of the line segments forming the polygon is called a side.
2. Each point resulted from intersecting of two adjacent sides of the polygon is called a vertex.
3. The sum of the side lengths of the polygon is called the perimeter of the polygon.
4. Each line segment joining two non-adjacent vertices of the polygon is called a diagonal of the polygon.



5. The included angle between two adjacent sides of the polygon is called an interior angle.
6. The included angle between a side of the polygon and the extension of its adjacent side is called an exterior angle.

Convex polygon and concave polygon :

In the convex polygon :

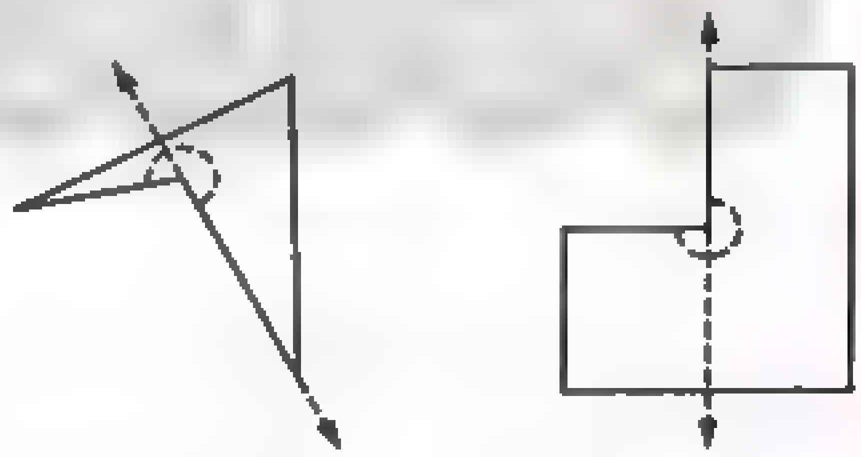
If a straight line is drawn to pass through any two consecutive vertices, then the remained vertices lie on one side of this straight line as shown in the two figures.

Notice that :

Any interior angle of the convex polygon has measure less than 180°

In the concave polygon :

There are straight lines (one at least) passing through two consecutive vertices and the remained vertices lie on two different sides of the straight line as shown in the two opposite figures.



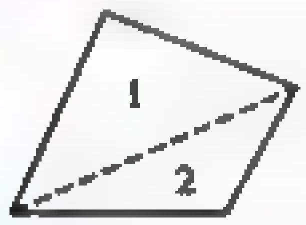
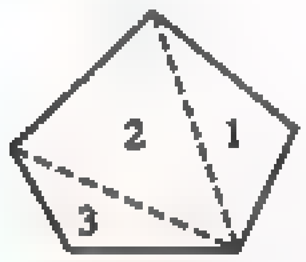
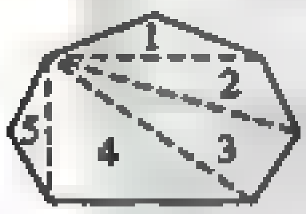
Notice that :

There is at least one interior angle of concave polygon of measure more than 180° (reflex angle).

The sum of measures of the interior angles of the polygon :

We knew before that : The sum of measures of the interior angles of the triangle equals 180° We can use that to deduce a general rule to find the sum of measures of the interior angles of any polygon whose number of sides is n as follows.

If we draw from any vertex of the polygon all diagonals that pass through this point , then the surface of this polygon will be divided into a number of triangles as shown in the following table :

The polygon	The number of its sides	The number of the resulting triangles	The sum of measures of the interior angles of the polygon
 quadrilateral	4	2	$2 \times 180° = 360°$
 pentagon	5	3	$3 \times 180° = 540°$
 hexagon	6	4	$4 \times 180° = 720°$
 heptagon	7	5	$5 \times 180° = 900°$

From the previous , we deduce that :

The number of triangles is less than the number of the sides by 2

Generally : If the number of sides of the polygon is "n" sides ,

then the number of triangles resulting from drawing all the diagonals that pass through a vertex of the polygon = (n – 2) triangles.

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore$ The sum of measures of the interior angles of a polygon of n sides equals $(n-2) \times 180°$

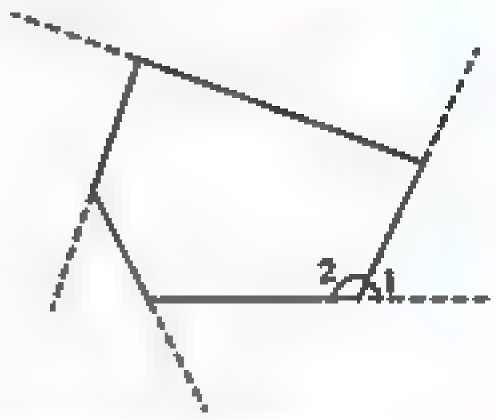
For example:

- The sum of measures of the interior angles of the octagon = $(8-2) \times 180° = 1080°$

- The sum of measures of the interior angles of the enneagon (nonagon) $= (9 - 2) \times 180° = 1260°$
- The sum of measures of the interior angles of the decagon $= (10 - 2) \times 180° = 1440°$

The sum of measures of the exterior angles of the convex polygon which has n sides :

- We mentioned that the exterior angle of the polygon is the angle included between one side and the extension of its adjacent side, and although we can draw two exterior angles equal in measure at each vertex of the polygon, but the rule of the sum of measures of the exterior angles use only one exterior angle at each vertex.



- At any vertex of the polygon , we find that the sum of measures of the interior angle and the exterior angle $= 180°$

In the previous figure : $m(\angle 1) + m(\angle 2) = 180°$

as an example.

In the previous pentagon , we find that the sum of measures of the five exterior and five interior angles of the pentagon $= 5 \times 180°$

Since the sum of measures of the interior angles equals $3 \times 180°$

$\therefore$ The sum of measures of the five exterior angles of the pentagon $= 2 \times 180° = 360°$

We can deduce that for any convex polygon of n sides as follows :

The sum of measures of the exterior angles + the sum of measures of the interior angles $= n \times 180°$

$\therefore$ The sum of measures of the exterior angles $+ (n - 2) \times 180° = n \times 180°$

$\therefore$ The sum of measures of the exterior angles $= n \times 180° - (n - 2) \times 180°$

$= 180°\, n - 180°\, n + 360° = 360°$

So we get :

> The sum of measures of the exterior angles of a convex polygon of n sides $= 360°$
> (taking into account one exterior angle at each vertex)

Example

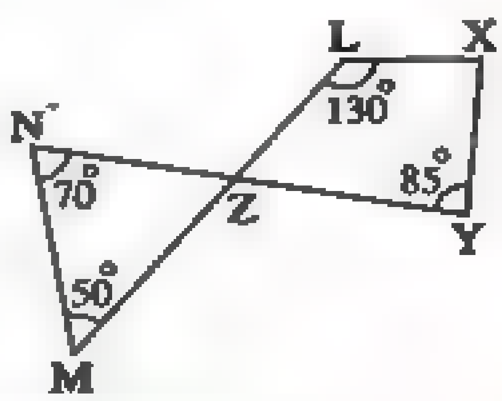
In the opposite figure :

$\overline{LM} \cap \overline{YN} = \{Z\}$, $m(\angle M) = 50°$,

$m(\angle N) = 70°$, $m(\angle Y) = 85°$ and

$m(\angle L) = 130°$

Find : $m(\angle X)$



Solution

Given | m (∠ M) = 50° , m (∠ N) = 70° , m (∠ Y) = 85° , m (∠ L) = 130°

R.T.F. | m (∠ X)

Proof | In Δ ZMN :

∵ m (∠ M) = 50° , m (∠ N) = 70°

∴ m (∠ NZM) = 180° – (50° + 70°) = 60°

∵ m (∠ LZY) = m (∠ NZM) (V.O.A.)

∴ m (∠ LZY) = 60°

∵ The figure XYZL is a quadrilateral.

∴ The sum of measures of its interior angles = 360°

∴ m (∠ X) = 360° – (130° + 85° + 60°) = 85° (The req.)

Example (2)

If the ratio among the measures of the interior angles of a quadrilateral is 2 : 3 : 3 : 4 , find the smallest measure of these angles of that quadrilateral.

Solution

Let the measures of the interior angles of the quadrilateral be $2x$, $3x$, $3x$ and $4x$

∵ The sum of measures of the interior angles of the quadrilateral

$= (4 - 2) \times 180° = 2 \times 180° = 360°$

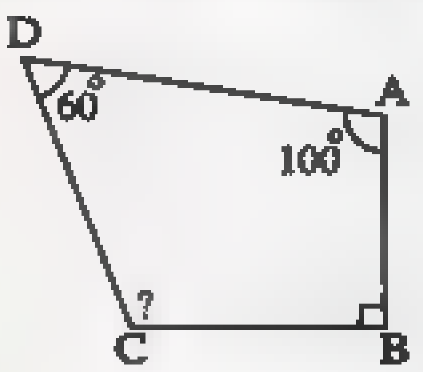
∴ $2x + 3x + 3x + 4x = 360°$

∴ $12x = 360°$ ∴ $x = \frac{360°}{12} = 30°$

∵ The smallest measure $= 2x$ ∴ The smallest measure of the angles $= 2 \times 30° = 60°$

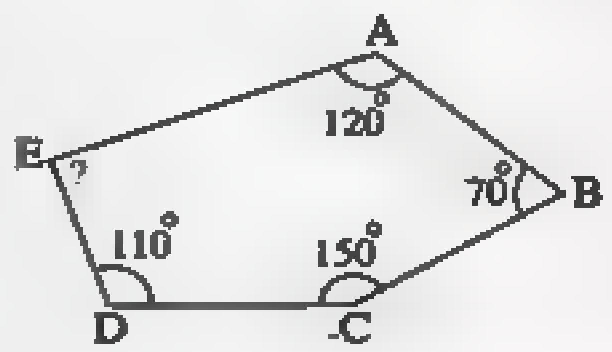
Try by yourself

Complete each of the following by using the given data for each figure :



∵ The sum of measures of the interior angles of the figure = ...°

∴ m (∠ C) = ...°



∵ The sum of measures of the interior angles of the figure = ...°

∴ m (∠ E) = ...°

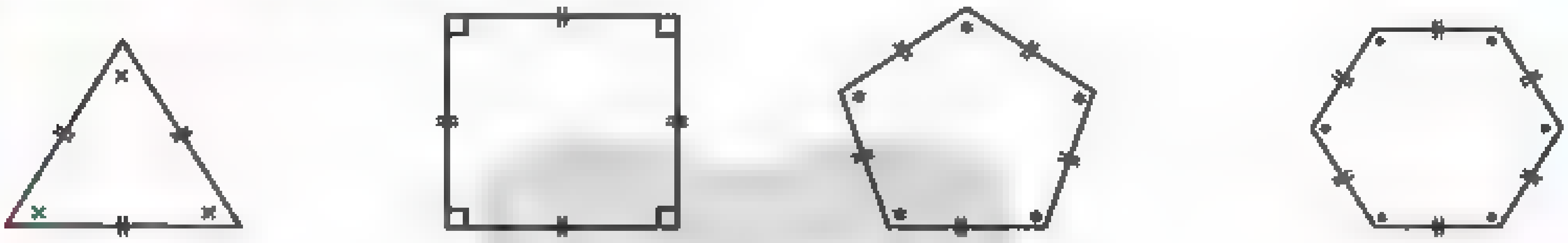
Unit 3

The regular polygon :

The polygon is regular if :

1. All its sides are equal in length.
2. All its angles are equal in measure.

As examples for the regular polygons :



Equilateral triangle | Square | Regular pentagon | Regular hexagon

The measure of the interior angle of a regular polygon :

We knew that the sum of measures of the interior angles of a polygon of n-sides $= (n-2) \times 180°$

Then :

If the polygon is regular , then its interior angles (whose number is n) are equal in measure.

$\therefore$ The measure of each interior angle of the regular polygon of n-sides $= \dfrac{(n-2) \times 180°}{n}$

For example:

- The measure of each interior angle of the equilateral triangle $= \dfrac{(3-2) \times 180°}{3} = 60°$
- The measure of each interior angle of the square $= \dfrac{(4-2) \times 180°}{4} = 90°$
- The measure of each interior angle of the regular pentagon $= \dfrac{(5-2) \times 180°}{5} = 108°$
- The measure of each interior angle of the regular hexagon $= \dfrac{(6-2) \times 180°}{6} = 120°$

Example (4)

The measure of one of the interior angles of a regular polygon is 144°
Find the number of its sides.

Solution

∵ The measure of each interior angle of the regular polygon of n-sides $= \frac{(n-2) \times 180°}{n}$

∴ $\frac{(n-2) \times 180°}{n} = 144°$ ∴ $(n-2) \times 180° = 144°\ n$

∴ $180°\ n - 360° = 144°\ n$ ∴ $180°\ n - 144°\ n = 360°$

∴ $36°\ n = 360°$ ∴ $n = 10$

∴ The number of sides = 10 sides.

Another solution

∵ The measure of the exterior angle of the polygon
= 180° – the measure of the interior angle
= 180° – 144° = 36°

∵ The sum of the measures of the exterior angles = 360°

∴ The number of exterior angles $= \frac{360°}{36°} = 10$ angles

∴ The number of sides = 10 sides.

Notice that :
The number of the polygon sides
= The number of its vertices
= The number of its interior angles
= The number of its exterior angles

Remark
The number of sides of the regular polygon in which the measure of one of its interior angles is $x° = \frac{360°}{180° - x}$

For example :

The number of sides of the regular polygon which the measure of one of its interior angles is $144° = \frac{360°}{180° - 144°} = 10$ sides.

Try by yourself

Complete the following table :

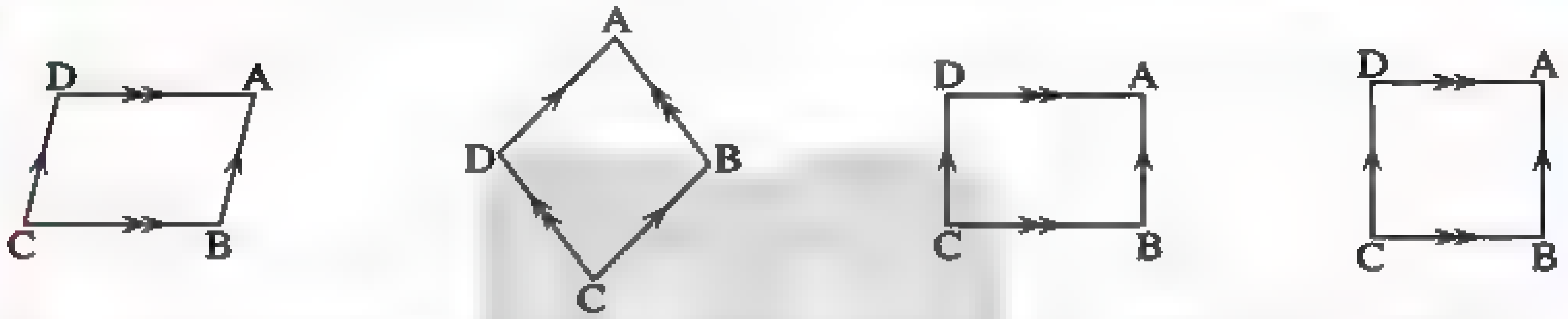
The number of sides of the regular polygon	3	5	12	....	....
The measure of one of its interior angles	..........°	..........°	..........°	135°	160°

Unit 3

Lesson The parallelogram and its properties

Definition :

A parallelogram is a quadrilateral , in which each two opposite sides are parallel.

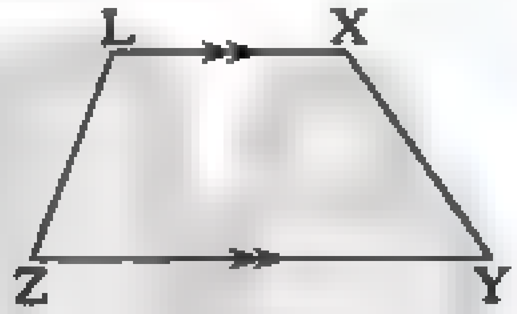


Each of the above figures is called a parallelogram for $\overline{AB} // \overline{DC}$ and $\overline{AD} // \overline{BC}$

Notice that :

A quadrilateral in which only two sides are parallel is called a trapezium , as shown in the opposite figure in which :

$\overline{XL} // \overline{YZ}$

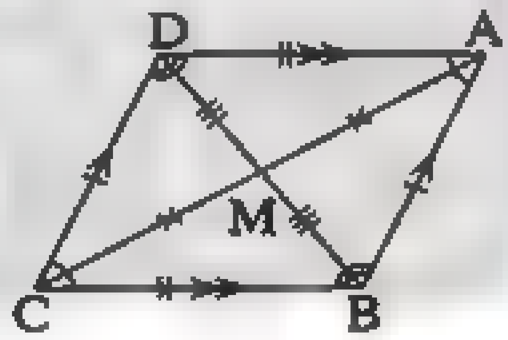


Properties of a parallelogram :

In the opposite figure :

ABCD is a parallelogram whose diagonals $\overline{AC}$ and $\overline{BD}$ intersect at M

We can deduce the following properties :



1 The sum of measures of each two consecutive angles in a parallelogram is 180°

i.e. • m (∠ A) + m (∠ B) = 180° • m (∠ B) + m (∠ C) = 180°
• m (∠ C) + m (∠ D) = 180° • m (∠ D) + m (∠ A) = 180°

2 In a parallelogram , each two opposite angles are equal in measure

i.e. • m (∠ A) = m (∠ C) • m (∠ B) = m (∠ D)

3 In a parallelogram , each two opposite sides are equal in length

i.e. • AB = CD • AD = BC

4 **The two diagonals in a parallelogram bisect each other**

i.e. • AM = CM • BM = DM

Remark

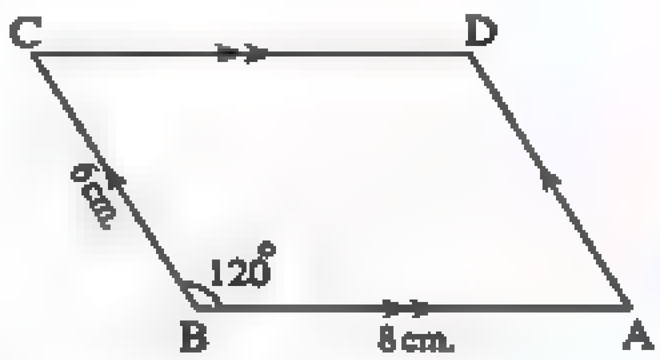
The perimeter of the parallelogram = The sum of two consecutive sides × 2

Example 1

In the opposite figure :

ABCD is a parallelogram in which :

AB = 8 cm. , BC = 6 cm. and m (∠ B) = 120°



Find : **1** The length of each of $\overline{CD}$ and $\overline{DA}$

2 The measure of each of ∠ D , ∠ A and ∠ C

3 The perimeter of ABCD

Solution

Given ABCD is a parallelogram , AB = 8 cm. , BC = 6 cm. and m (∠ B) = 120°

R.T.F. **1** CD and DA

2 m (∠ D) , m (∠ A) and m (∠ C)

3 The perimeter of ABCD

Proof ∵ ABCD is a parallelogram

∴ CD = AB = 8 cm. (Properties of a parallelogram)

and DA = CB = 6 cm. (Properties of a parallelogram) (First req.)

, m (∠ D) = m (∠ B) = 120° (Properties of a parallelogram)

, ∵ m (∠ A) + m (∠ B) = 180° (Properties of a parallelogram)

, m (∠ B) = 120°

∴ m (∠ A) = 180° − 120° = 60° and m (∠ C) = m (∠ A) = 60° (Second req.)

The perimeter of ABCD = (AB + BC) × 2

= (8 + 6) × 2 = 14 × 2 = 28 cm. (Third req.)

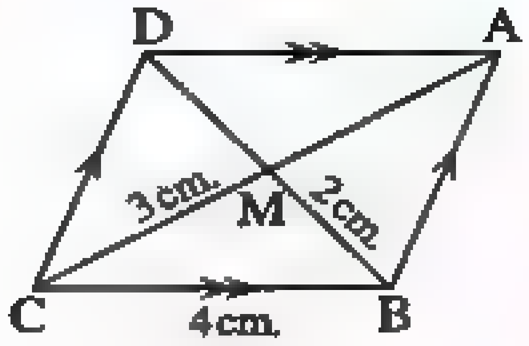
Example

In the opposite figure :

ABCD is a parallelogram whose diagonals intersect at M

If BC = 4 cm. , BM = 2 cm. and MC = 3 cm. ,

then find the perimeter of Δ AMD



Solution

Given ABCD is a parallelogram whose diagonals intersect at M , BC = 4 cm. , BM = 2 cm. and MC = 3 cm.

R.T.F. The perimeter of Δ AMD

Proof $\because$ ABCD is a parallelogram

$\therefore$ BC = AD = 4 cm. (Two opposite sides in a parallelogram)

$\because$ The two diagonals bisect each other

$\therefore$ MD = MB = 2 cm. and AM = MC = 3 cm.

$\therefore$ The perimeter of Δ AMD = AD + MD + AM = 4 + 2 + 3 = 9 cm.

(The req.)

Try by yourself

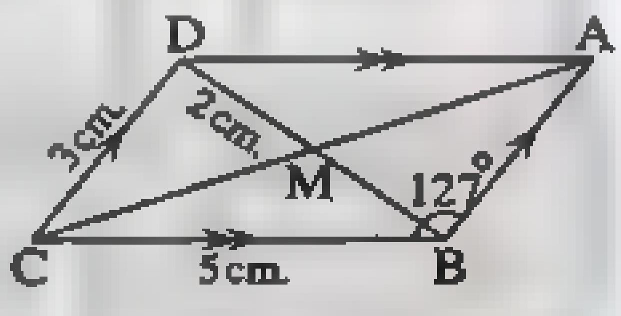
In the opposite figure :

ABCD is a parallelogram whose diagonals intersect at M

If BC = 5 cm. , DC = 3 cm. , DM = 2 cm.

and m ($\angle$ ABC) = 127° ,

complete the following :



1. AB = ... cm. and AD = ... cm.
2. BD = ... cm.
3. m ($\angle$ ADC) = ...° , m ($\angle$ BAD) = ...° and m ($\angle$ BCD) = ...°
4. The perimeter of ▱ ABCD = ... cm.

When does a quadrilateral represent a parallelogram ?

A quadrilateral represents a parallelogram if one of the following conditions satisfies

- Each two opposite sides are parallel.
- Each two opposite sides are equal in length.
- Two opposite sides are parallel and equal in length.
- Each two opposite angles are equal in measure
- The two diagonals bisect each other.

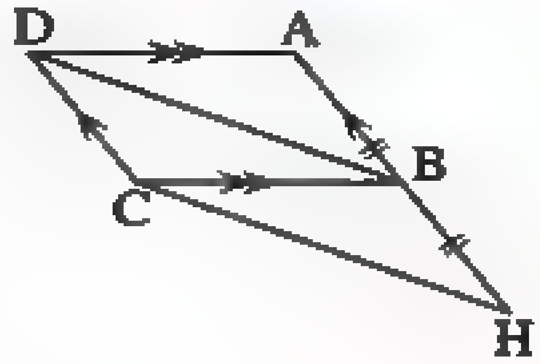
Example (3)

In the opposite figure :

ABCD is a parallelogram , $H \in \overrightarrow{AB}$

where AB = BH

Prove that : BHCD is a parallelogram



Solution

Given	ABCD is a parallelogram and AB = BH
R.T.P.	BHCD is a parallelogram.
Proof	$\because$ ABCD is a parallelogram $\quad\therefore$ AB = CD
	$\because$ AB = BH (Given) $\quad\therefore$ DC = BH (1)
	$\because \overline{AB} \parallel \overline{DC}$, $H \in \overrightarrow{AB}$ $\quad\therefore \overline{BH} \parallel \overline{DC}$ (2)
	From (1) and (2) : $\quad\therefore$ DC = BH and $\overline{DC} \parallel \overline{BH}$
	$\therefore$ BHCD is a parallelogram (Q.E.D.)

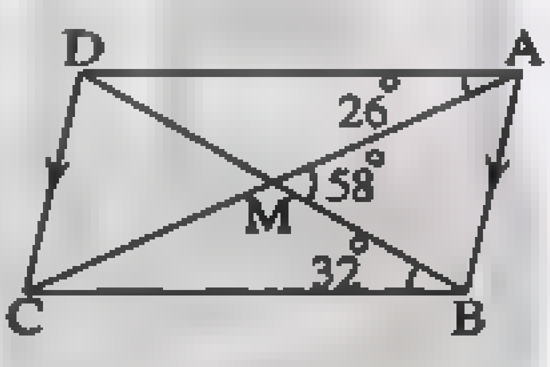
Try by yourself

In the opposite figure :

ABCD is a quadrilateral , its diagonals intersect at M ,

$\overline{AB} \parallel \overline{CD}$, m (∠ AMB) = 58° , m (∠ MBC) = 32°

and m (∠ MAD) = 26°



Complete the following proof to prove that ABCD is a parallelogram.

Given	....
R.T.P.	....
Proof	$\because M \in \overline{AC}$
	$\therefore$ m (∠ BMC) = 180° − ....° = ...°
	$\therefore$ In Δ BMC : m (∠ MCB) = 180° − (..........° + ...°) = ...°
	$\therefore$ m (∠ MCB) = m (∠ ....) and they are .... angles
	$\therefore \overline{......} \parallel \overline{......}$ $\quad\because \overline{......} \parallel \overline{......}$ (Given)
	$\therefore$ ABCD is a parallelogram. (Q.E.D.)

Unit 3

Lesson 2 The special cases of the parallelogram

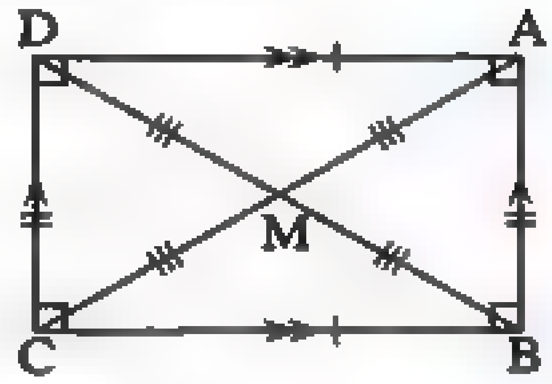
1 The rectangle :

A rectangle is a parallelogram with a right angle.

- **Properties of the rectangle :**
  - **The rectangle has the same properties as the parallelogram in addition :**

1 The four angles of a rectangle are all equal in measure and the measure of each is 90°

i.e. m (∠ A) = m (∠ B) = m (∠ C) = m (∠ D) = 90°



2 The two diagonals of a rectangle are equal in length.

i.e. AC = BD and as the two diagonals bisect each other , then AM = BM = CM = DM , where M is the point of intersection of the two diagonals.

- **The perimeter of the rectangle = (length + width) × 2**

2 The rhombus :

A rhombus is a parallelogram in which two adjacent sides are equal in length.

- **Properties of the rhombus :**
  - **The rhombus has the same properties as the parallelogram in addition :**

1 The four sides of a rhombus are all equal in length.

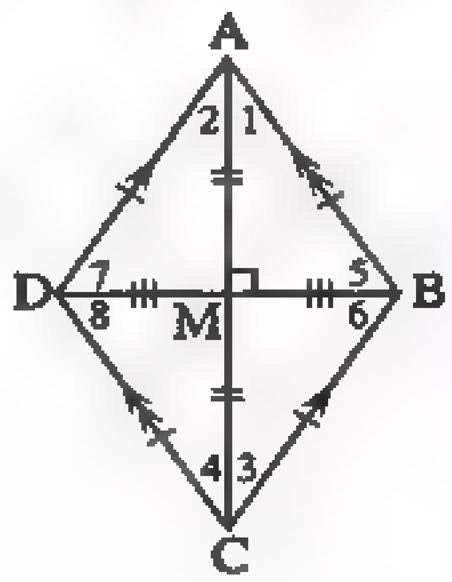
i.e. AB = BC = CD = DA

2 The two diagonals of the rhombus are perpendicular and bisect each of its interior angles.

i.e. • $\overline{AC} \perp \overline{BD}$

• m (∠ 1) = m (∠ 2) = m (∠ 3) = m (∠ 4)

• m (∠ 5) = m (∠ 6) = m (∠ 7) = m (∠ 8)



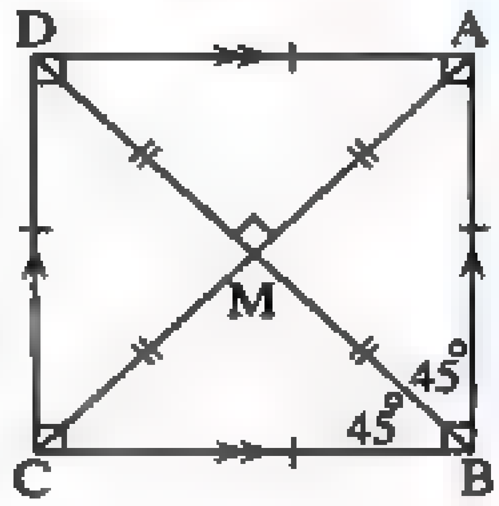
- **The perimeter of the rhombus = the length of one side × 4**

3

A square is a parallelogram with a right angle and two adjacent sides equal in length.

- **Properties of the square :**

  - **The square has the same properties as the parallelogram in addition :**



1 Its four sides are all equal in length

i.e. AB = BC = CD = DA

2 Its four angles are all equal in measure

and each of them is of measure 90°

i.e. m (∠ A) = m (∠ B) = m (∠ C) = m (∠ D) = 90°

3 Its two diagonals are equal in length , perpendicular and each diagonal bisects the two vertices angles which this diagonal joins.

i.e. • AC = BD and hence AM = BM = CM = DM , where $\overline{AC} \cap \overline{BD} = \{M\}$

• $\overleftrightarrow{AC} \perp \overline{BD}$

• m (∠ ABD) = m (∠ CBD) = 45°

- **The perimeter of the square = the length of one side × 4**

Notice that :

We can also define the square as follows :

1 A square is a rectangle with two adjacent sides equal in length.

2 A square is a rectangle with two perpendicular diagonals.

3 A square is a rhombus with a right angle.

4 A square is a rhombus with two diagnals equal in length.

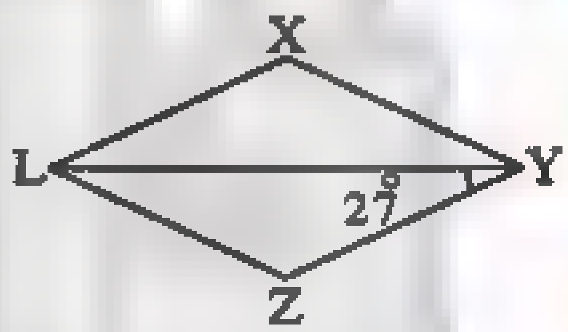
Notice that :

To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram , as we see in the previous lesson , then :

Unit 3

The parallelogram is

a rectangle	a rhombus	a square
if : One of its angles is a right angle **or** Its two diagonals are equal in length.	**if :** Two adjacent sides are equal in length. **or** Its two diagonals are perpendicular.	**if :** One of its angles is right and two adjacent sides are equal in length. **or** One of its angles is right and its two diagonals are perpendicular. **or** Its two diagonals are perpendicular and equal in length. **or** Two adjacent sides are equal in length and its two diagonals are equal in length.



In the opposite figure :

XYZL is a rhombus in which m (∠ LYZ) = 27°

Calculate the measures of the angles of the rhombus XYZL

Solution

Given XYZL is a rhombus in which m (∠ LYZ) = 27°

R.T.F. m (∠ XYZ) , m (∠ XLZ) , m (∠ X) and m (∠ Z)

Proof ∵ $\overline{YL}$ is a diagonal in the rhombus XYZL

∴ $\overrightarrow{YL}$ bisects ∠ XYZ ∴ m (∠ XYZ) = 2 × 27° = 54°

, ∵ Each two opposite angles in the rhombus are equal in measure.

∴ m (∠ XLZ) = 54°

∵ The rhombus is a special case of the parallelogram

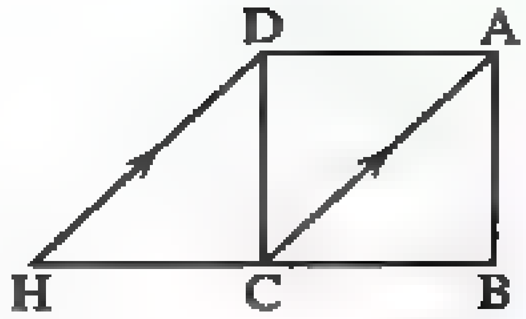
∴ m (∠ X) + m (∠ XYZ) = 180°

∴ m (∠ X) + 54° = 180° ∴ m (∠ X) = 126°

∴ m (∠ Z) = 126° (The req.)

Try to solve this example by another method using the properties of the rhombus

In the opposite figure :



ABCD is a square. Draw $\overrightarrow{DH} \parallel \overline{AC}$ to intersect $\overrightarrow{BC}$ at H

1 **Prove that :** CH = BC

2 **Find :** m (∠ ADH)

Solution

Given ABCD is a square and $\overrightarrow{DH} \parallel \overline{AC}$

R.T.P. CH = BC

R.T.F. m (∠ ADH)

Proof

$\because \overline{AD} \parallel \overline{BC}$ (Two opposite sides in the square) and $H \in \overrightarrow{BC}$

$\therefore \overline{AD} \parallel \overline{CH}$

$\because \overrightarrow{DH} \parallel \overline{AC}$ (Given)

$\therefore$ ACHD is a parallelogram $\therefore$ CH = AD

But AD = BC (Two opposite sides in the square)

$\therefore$ CH = BC (First req.)

$\because \overline{AC}$ is a diagonal in the square $\therefore \overrightarrow{CA}$ bisects ∠ BCD

$\because$ m (∠ BCD) = 90° $\therefore$ m (∠ ACD) = 45°

$\because \overrightarrow{DH} \parallel \overleftrightarrow{AC}$ and $\overleftrightarrow{CD}$ is their transversal

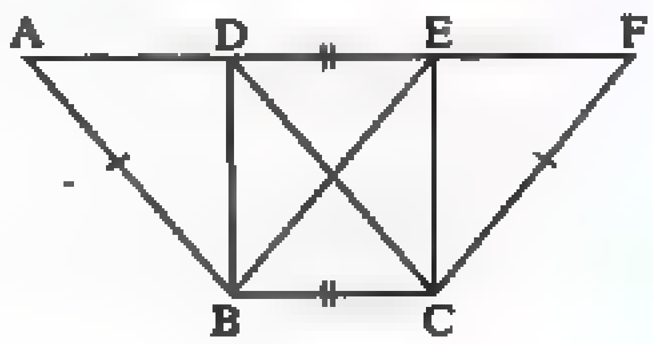
$\therefore$ m (∠ CDH) = m (∠ ACD) = 45° (Two alternate angles)

, $\because$ m (∠ ADC) = 90° (Property of the square)

$\therefore$ m (∠ ADH) = m (∠ ADC) + m (∠ CDH)

= 90° + 45° = 135° (Second req.)

In the opposite figure :



ABCD , EBCF are two parallelograms ,

D and E belong to $\overleftrightarrow{AF}$, AB = FC , BC = DE

Prove that : The figure DBCE is a rectangle.

Solution

Given | ABCD and EBCF are two parallelograms , AB = FC , BC = DE

R.T.P. | The figure DBCE is a rectangle.

Proof

∵ ABCD is a parallelogram. ∴ $\overline{AD} \parallel \overline{BC}$

, ∵ D and E belong to $\overleftrightarrow{AF}$ ∴ $\overline{DE} \parallel \overline{BC}$

, ∵ DE = BC ∴ DBCE is a parallelogram.

, ∵ ABCD is a parallelogram. ∴ AB = DC

, ∵ EBCF is a parallelogram. ∴ FC = EB but : AB = FC

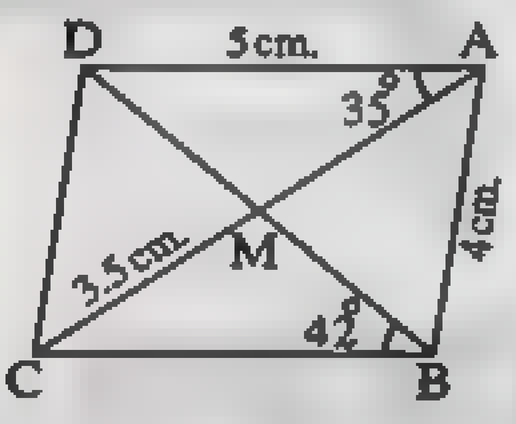
∴ DC = EB

∴ DBCE is a parallelogram and its diagonals are equal in length.

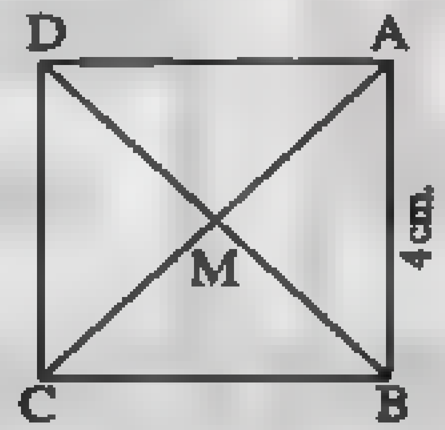
∴ DBCE is a rectangle. (Q.E.D.)

Try by yourself

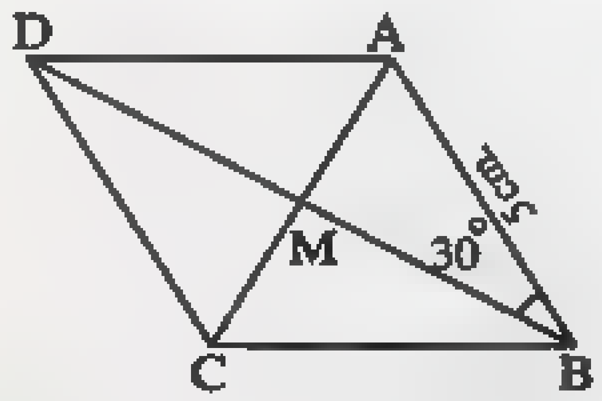
Using the given in each figure , complete where M is the intersection point of the diagonals :



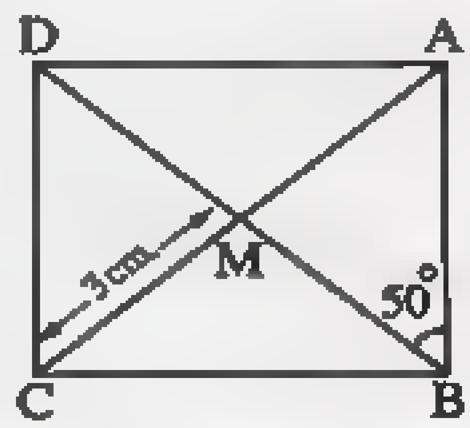
- ABCD is a parallelogram :
  - The perimeter of Δ ABC = ... cm.
  - m (∠ AMB) = ...°



- ABCD is a square :
  - The perimeter of the square = ... cm.
  - m (∠ BAC) = ...°



- ABCD is a rhombus :
  - AD = ... cm.
  - m (∠ BAM) = ...°

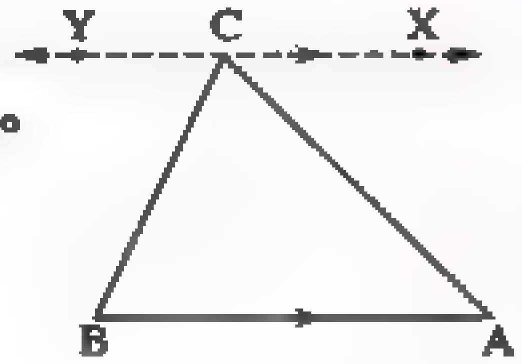


- ABCD is a rectangle :
  - BD = ... cm.
  - m (∠ MCD) = ...°

Lesson The triangle

Theorem (1) :

The sum of the measures of the interior angles of a triangle is 180°



Given ABC is a triangle

R.T.P. $m(\angle A) + m(\angle B) + m(\angle ACB) = 180^\circ$

Construction Draw $\overleftrightarrow{CX} \parallel \overline{AB}$

Proof

$\because \angle XCY$ is a straight angle

$\therefore m(\angle XCA) + m(\angle ACB) + m(\angle BCY) = 180^\circ$

$\because \overleftrightarrow{XY} \parallel \overline{AB}$

$\therefore m(\angle XCA) = m(\angle A)$ (alternate angles)

, $m(\angle YCB) = m(\angle B)$ (alternate angles)

$\therefore m(\angle A) + m(\angle ACB) + m(\angle B) = 180^\circ$ (Q.E.D.)

Example (1)

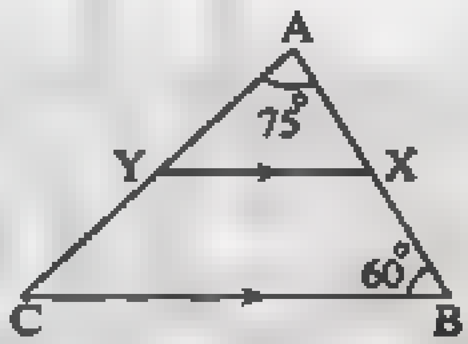
In the opposite figure :

ABC is a triangle in which $m(\angle A) = 75^\circ$,

$m(\angle B) = 60^\circ$, $X \in \overline{AB}$ and $Y \in \overline{AC}$

such that $\overline{XY} \parallel \overline{BC}$

Find : $m(\angle AYX)$



Solution

Given $\overline{XY} \parallel \overline{BC}$, $m(\angle A) = 75^\circ$ and $m(\angle B) = 60^\circ$

R.T.F. $m(\angle AYX)$

Proof

$\because m(\angle A) = 75^\circ$ and $m(\angle B) = 60^\circ$ (given)

, The sum of measures of the interior angles of $\Delta ABC = 180^\circ$

$\therefore m(\angle C) = 180^\circ - (75^\circ + 60^\circ) = 180^\circ - 135^\circ = 45^\circ$

$\because \overline{XY} \parallel \overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle C) = 45^\circ$ (corresponding angles) (The req.)

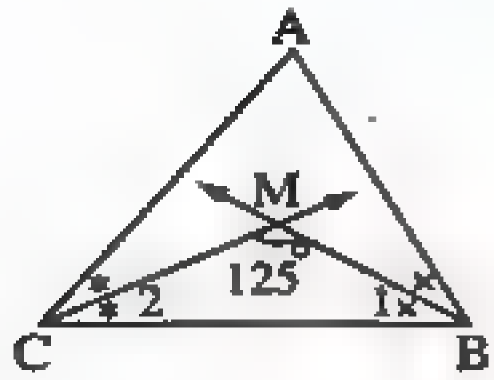
(Try to solve the example by another method.)

In the opposite figure :

$\overrightarrow{BM}$ bisects $\angle ABC$, $\overrightarrow{CM}$ bisects $\angle ACB$

and $m(\angle BMC) = 125°$

Find : $m(\angle A)$



Solution

Given | $\overrightarrow{BM}$ bisects $\angle ABC$, $\overrightarrow{CM}$ bisects $\angle ACB$ and $m(\angle BMC) = 125°$

R.T.F. | $m(\angle A)$

Proof

$\because$ The sum of measures of the interior angles of $\Delta MBC = 180°$

and $m(\angle BMC) = 125°$

$\therefore m(\angle 1) + m(\angle 2) = 180° - 125° = 55°$

But $m(\angle ABC) = 2\, m(\angle 1)$ and $m(\angle ACB) = 2\, m(\angle 2)$

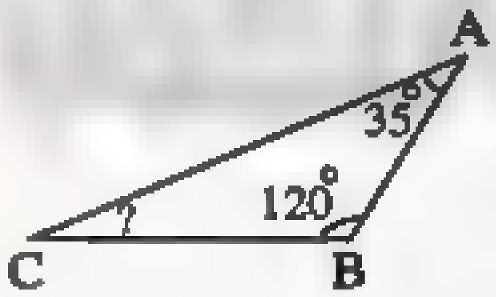
$\therefore m(\angle ABC) + m(\angle ACB) = 2 \times 55° = 110°$

$\because$ The sum of measures of the interior angles of $\Delta ABC = 180°$

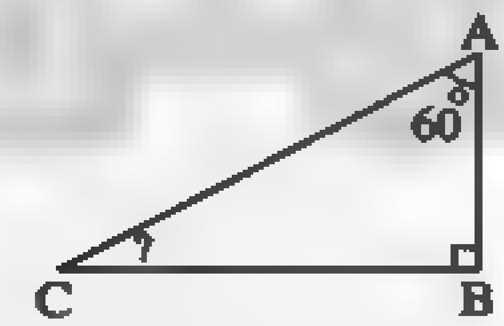
$\therefore m(\angle A) = 180° - 110° = 70°$ (The req.)

Try by yourself

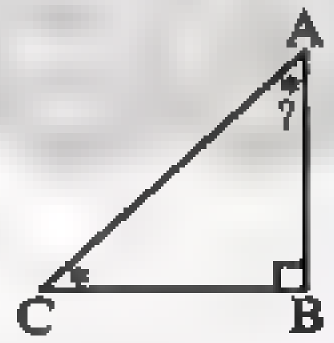
In each of the following figures , find the measure of the angle marked by (?) :



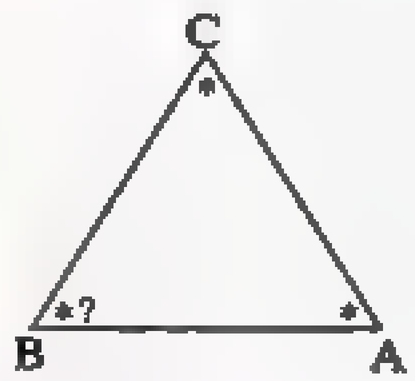
$m(\angle C) = ...°$



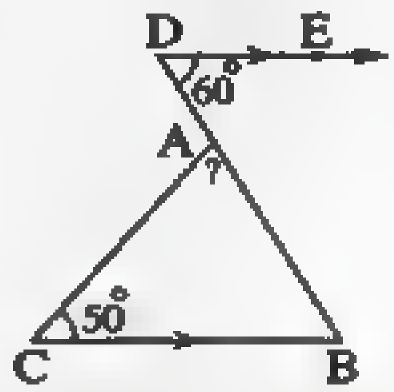
$m(\angle C) = ...°$



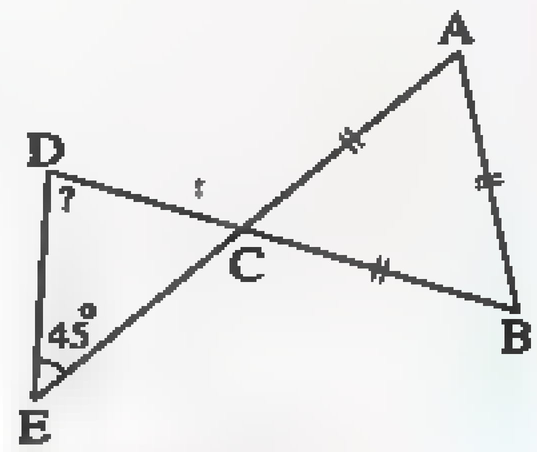
$m(\angle A) = ...°$



$m(\angle B) = ...°$



$m(\angle BAC) = ...°$

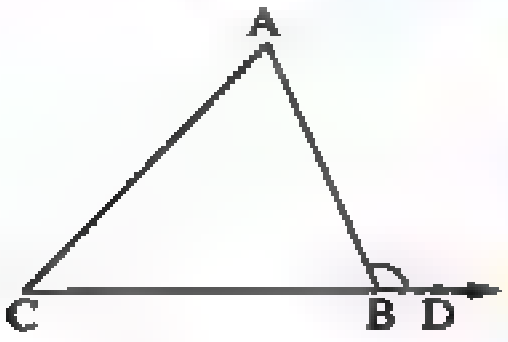


$m(\angle D) = ...°$

The exterior angle of the triangle

In the opposite figure :

If ABC is a triangle , $D \in \overrightarrow{CB}$ and $D \notin \overline{CB}$, then $\angle ABD$ is called an exterior angle of ΔABC

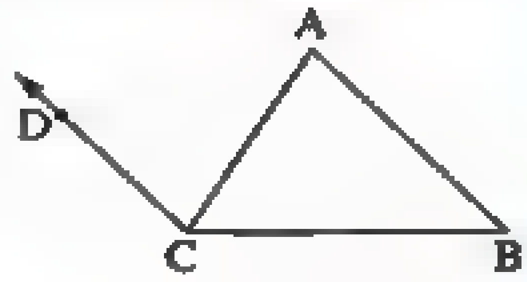


Notice that :

In the opposite figure :

$\angle ACD$ is not an exterior angle of ΔABC

because $D \notin \overrightarrow{BC}$



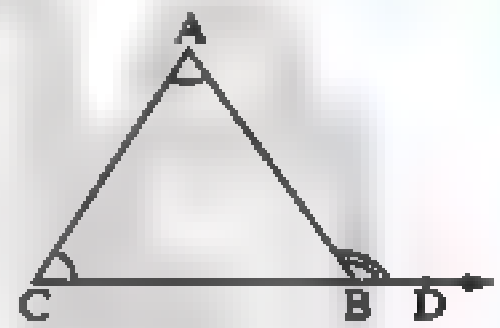
The measure of the exterior angle of a triangle :

The measure of the exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.

In the opposite figure :

If ABC is a triangle , $D \in \overrightarrow{CB}$ and $D \notin \overline{CB}$,

then $m(\angle ABD) = m(\angle A) + m(\angle C)$



We can prove that as follows :

$\because m(\angle A) + m(\angle C) + m(\angle ABC) = 180°$

$, m(\angle ABD) + m(\angle ABC) = 180°$

$\therefore m(\angle ABD) + m(\angle ABC) = m(\angle A) + m(\angle C) + m(\angle ABC)$

$\therefore m(\angle ABD) = m(\angle A) + m(\angle C)$ (Q.E.D.)

Notice that :

The measure of the exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

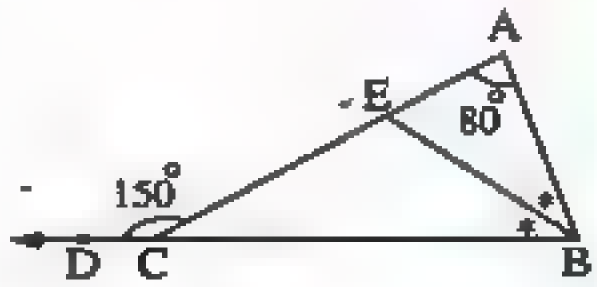
i.e. In the previous figure : $m(\angle ABD) > m(\angle A)$ and $m(\angle ABD) > m(\angle C)$

In the opposite figure :

ABC is a triangle , $D \in \overrightarrow{BC}$ and $E \in \overline{AC}$

where $\overrightarrow{BE}$ bisects $\angle ABC$, $m(\angle A) = 80°$ and

$m(\angle ACD) = 150°$



Find : **1** $m(\angle ABC)$ **2** $m(\angle BEC)$

Solution

Given $\overrightarrow{BE}$ bisects $\angle ABC$, $m(\angle A) = 80^\circ$ and $m(\angle ACD) = 150^\circ$

R.T.F. **1** $m(\angle ABC)$ **2** $m(\angle BEC)$

Proof $\because \angle ACD$ is an exterior angle of ΔABC

$\therefore m(\angle ACD) = m(\angle A) + m(\angle ABC)$

$\therefore 150^\circ = 80^\circ + m(\angle ABC)$

$\therefore m(\angle ABC) = 150^\circ - 80^\circ = 70^\circ$ (First req.)

$\because \overrightarrow{BE}$ bisects $\angle ABC$ (given)

$\therefore m(\angle ABE) = \frac{1}{2} m(\angle ABC) = \frac{70^\circ}{2} = 35^\circ$

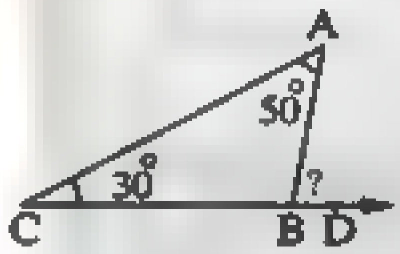
$\because \angle BEC$ is an exterior angle of ΔABE

$\therefore m(\angle BEC) = m(\angle A) + m(\angle ABE) = 80^\circ + 35^\circ = 115^\circ$ (Second req.)

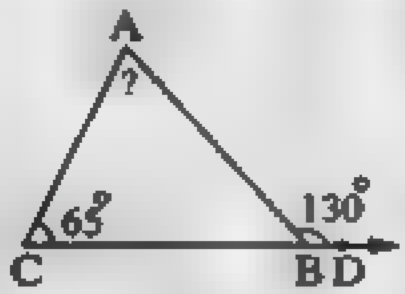
(Try to solve this example by another method.)

Try by yourself

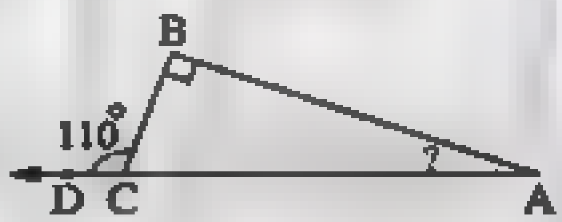
In each of the following figures , find the measure of each angle marked by (?)



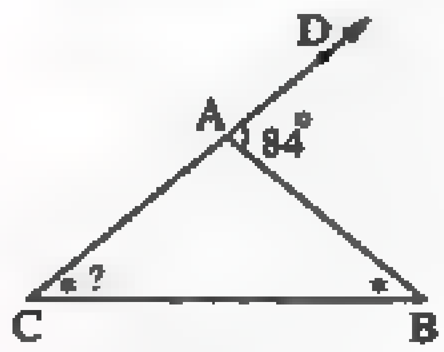
$m(\angle ABD) =$ ....°



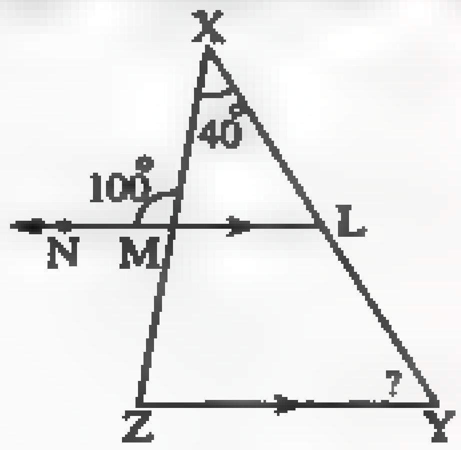
$m(\angle A) =$ ....°



$m(\angle A) =$ ....°



$m(\angle C) =$ ....°

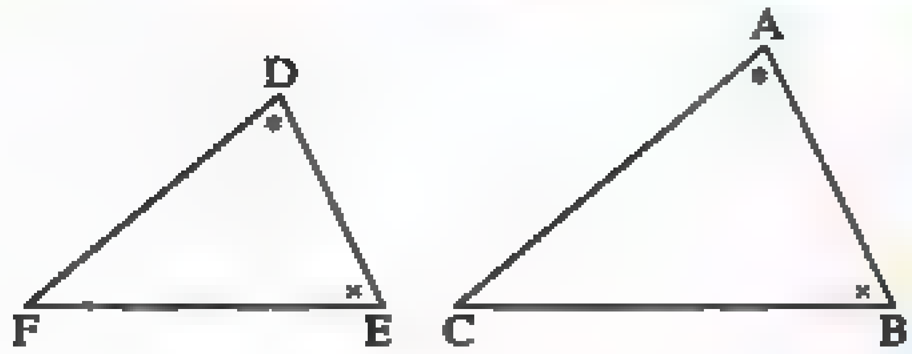


$m(\angle Y) =$ ....°

Remark (1)

If two angles of one triangle equal two angles of another triangle in measure , then the third angle of the first triangle is equal in measure to the third angle of the other triangle.

In ΔΔ ABC and DEF ,
if m (∠ A) = m (∠ D) and m (∠ B) = m (∠ E) ,
then m (∠ C) = m (∠ F)
"You can check the truth of the previous by measuring"

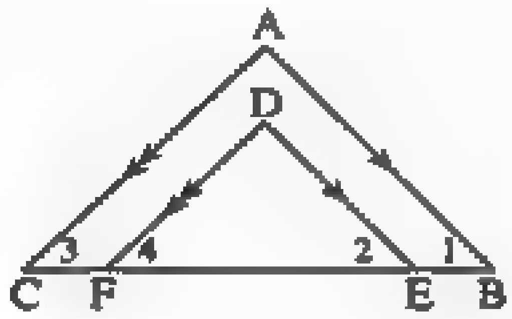


Example (1)

In the opposite figure :

ABC and DEF are two triangles ,
$E \in \overline{BC}$, $F \in \overline{BC}$, $\overline{DE} // \overline{AB}$ and $\overline{DF} // \overline{AC}$

Prove that : m (∠ A) = m (∠ D)



Solution

Given | $\overline{DE} // \overline{AB}$ and $\overline{DF} // \overline{AC}$

R.T.P. | m (∠ A) = m (∠ D)

Proof | ∵ $\overline{DE} // \overline{AB}$ and $\overleftrightarrow{BC}$ is a transversal to them.

∴ m (∠ 1) = m (∠ 2) (corresponding angles)

∵ $\overline{DF} // \overline{AC}$ and $\overleftrightarrow{BC}$ is a transversal to them.

∴ m (∠ 3) = m (∠ 4) (corresponding angles)

In Δ Δ ABC and DEF :

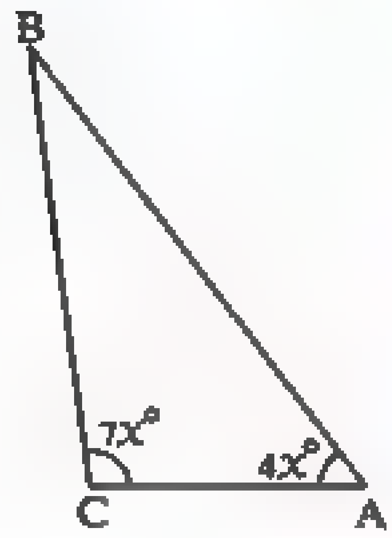
∵ m (∠ 1) = m (∠ 2) and m (∠ 3) = m (∠ 4)

∴ m (∠ A) = m (∠ D) (Q.E.D.)

Remark (2)

- If the sum of measures of two angles in a triangle equals 90° , then the third angle is right.
- If the sum of measures of two angles in a triangle is less than 90° , then the third angle is obtuse.
- If the sum of measures of two angles in a triangle is more than 90° , then the third angle is acute.

Example (5)



In the opposite figure :

ABC is a triangle in which m (∠ A) = 2 m (∠ B) = 4 $x°$ and m (∠ C) = 7 $x°$

Prove that : ∠ C is an obtuse angle.

Solution

Given m (∠ A) = 2 m (∠ B) = 4 $x°$ and m (∠ C) = 7 $x°$

R.T.P. ∠ C is an obtuse angle.

Proof ∵ 2 m (∠ B) = 4 $x°$ ∴ m (∠ B) = 2 $x°$

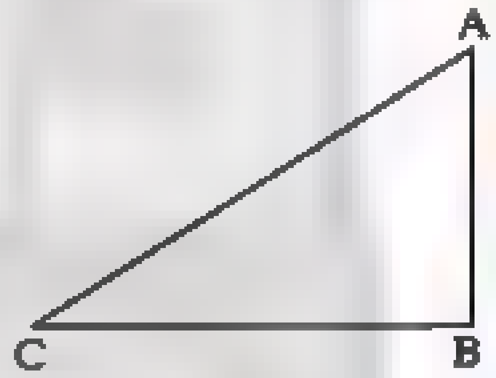
∴ m (∠ A) + m (∠ B) = 4 $x°$ + 2 $x°$ = 6 $x°$

∵ m (∠ C) = 7 $x°$ ∴ m (∠ A) + m (∠ B) < m (∠ C)

∴ ∠ C is an obtuse angle. (Q.E.D.)

Remark (3)

If the measure of an angle in a triangle equals the sum of measures of the other two angles , then the triangle is right-angled.



In the opposite figure :

If ABC is a triangle in which : m (∠ A) + m (∠ C) = m (∠ B)

, then m (∠ B) = $\frac{180°}{2}$ = 90°

i.e. Δ ABC is right-angled at B

Example (6)

ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 2 : 3 : 5

Prove without finding the measures of the angles of the triangle that the triangle is right-angled , then mention the right angle.

Solution

Given ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 2 : 3 : 5

R.T.P. Δ ABC is right-angled and mention the right angle.

Proof ∵ m (∠ A) + m (∠ B) is equivalent to 5 parts and m (∠ C) is equivalent to 5 parts

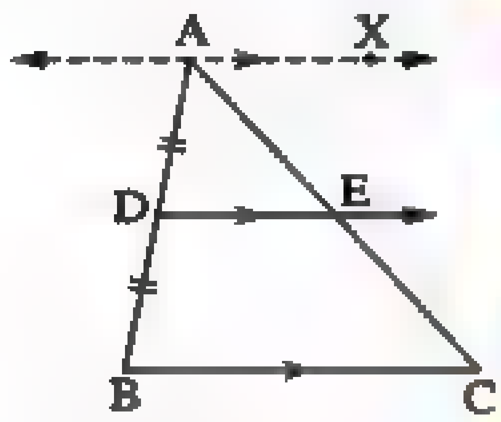
∴ m (∠ A) + m (∠ B) = m (∠ C)

∴ Δ ABC is right-angled at C (Q.E.D.)

Lesson Follow : The triangle

Theorem (2) :

The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.



Given D is the midpoint of $\overline{AB}$, $\overrightarrow{DE} \mathbin{/\!/} \overline{BC}$

R.T.P. E is the midpoint of $\overline{AC}$

Construction Draw $\overleftrightarrow{AX} \mathbin{/\!/} \overline{BC}$

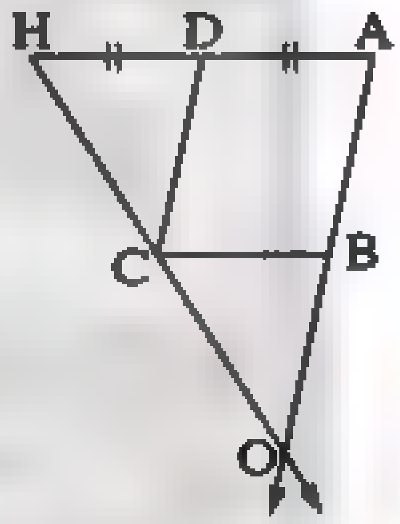
Proof $\because \overleftrightarrow{AX} \mathbin{/\!/} \overrightarrow{DE} \mathbin{/\!/} \overline{BC}$

, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two transversals to them at D and E respectively.

, $\because$ AD = DB $\therefore$ AE = EC

$\therefore$ E is the midpoint of $\overline{AC}$ (Q.E.D.)

Example (1)



In the opposite figure :

ABCD is a parallelogram , H $\in \overrightarrow{AD}$ such that AD = DH ,

$\overrightarrow{HC} \cap \overrightarrow{AB} = \{O\}$

Prove that : **1** HC = CO **2** AB = BO

Solution

Given ABCD is a parallelogram , AD = DH and $\overrightarrow{HC} \cap \overrightarrow{AB} = \{O\}$

R.T.P. **1** HC = CO **2** AB = BO

Proof In Δ HAO :

$\because$ D is the midpoint of $\overline{HA}$ (Given) ,

$\overline{DC} \mathbin{/\!/} \overline{AO}$ (Definition of the parallelogram)

$\therefore$ C is the midpoint of $\overline{HO}$

i.e. HC = CO (Theorem) (Q.E.D. 1)

$\because$ C is the midpoint of $\overline{OH}$ (Proved)

, $\overline{CB} \mathbin{/\!/} \overline{HA}$ (Definition of the parallelogram)

$\therefore$ B is the midpoint of $\overline{AO}$

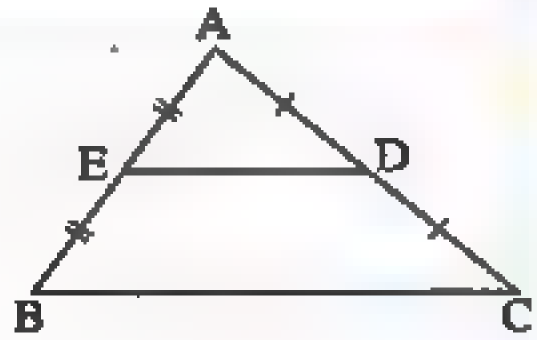
i.e. AB = BO (Theorem) (Q.E.D. 2)

Corollary

The line segment joining the midpoints of two sides of a triangle is parallel to the third side.

In the opposite figure :

If ABC is a triangle in which D is the midpoint of $\overline{AC}$,
E is the midpoint of $\overline{AB}$, then : $\overline{ED} \parallel \overline{BC}$

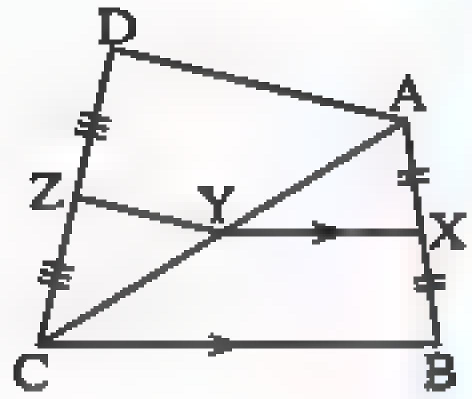


Example (2)

In the opposite figure :

X is the midpoint of $\overline{AB}$,
$\overline{XY} \parallel \overline{BC}$ and Z is the midpoint of $\overline{DC}$

Prove that : $\overline{YZ} \parallel \overline{AD}$



Solution

Given X is the midpoint of $\overline{AB}$, Z is the midpoint of $\overline{CD}$ and $\overline{XY} \parallel \overline{BC}$

R.T.P. $\overline{YZ} \parallel \overline{AD}$

Proof In Δ ABC :

$\because$ AX = XB , $\overline{XY} \parallel \overline{BC}$

$\therefore$ AY = YC (Theorem)

In Δ ACD : $\because$ AY = YC (Proved) , DZ = ZC (Given)

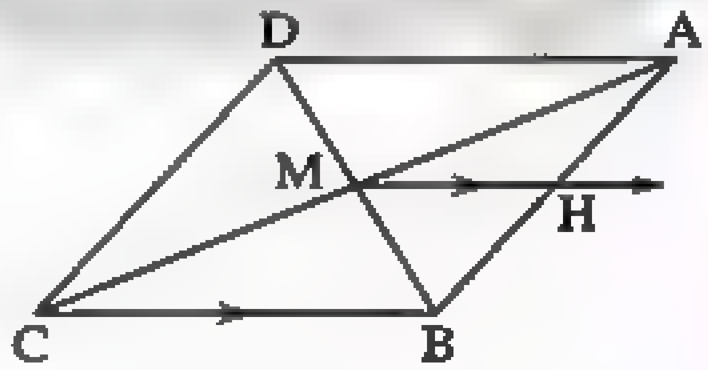
$\therefore$ $\overline{YZ} \parallel \overline{AD}$ (Corollary) (Q.E.D.)

Try by yourself

In the opposite figure :

ABCD is a parallelogram and M is the point of intersection of its two diagonals.
Draw $\overrightarrow{MH} \parallel \overline{BC}$ to cut $\overline{AB}$ at H

Complete the solution to prove that AH = HB



Given ....

R.T.P. ....

Proof $\because$ ABCD is a parallelogram

$\therefore$ M is the midpoint of ....

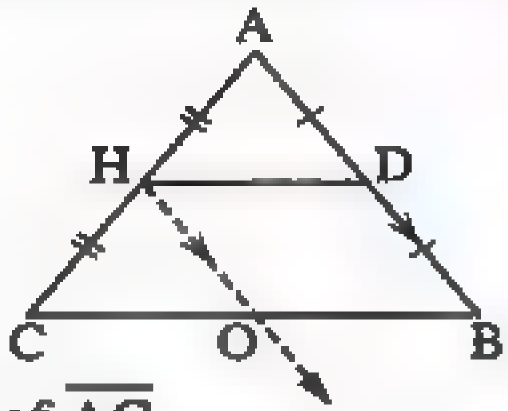
In Δ ABC :

$\because$ M is the midpoint of .... and .... // ....

$\therefore$ .... (Theorem) (Q.E.D.)

Theorem (3)

The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.



Given ABC is a triangle , D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$

R.T.P. $DH = \frac{1}{2} BC$

Construction Draw $\overrightarrow{HO} \,//\, \overline{AB}$ to cut $\overline{BC}$ at O

Proof $\because$ D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$

$\therefore \overline{DH} \,//\, \overline{BC}$ (Corollary)

, $\because \overline{HO} \,//\, \overline{AB}$ (Construction) , H is the midpoint of $\overline{AC}$

$\therefore$ O is the midpoint of $\overline{BC}$ $\therefore BO = \frac{1}{2} BC$

, $\because$ The figure DHOB is a parallelogram.

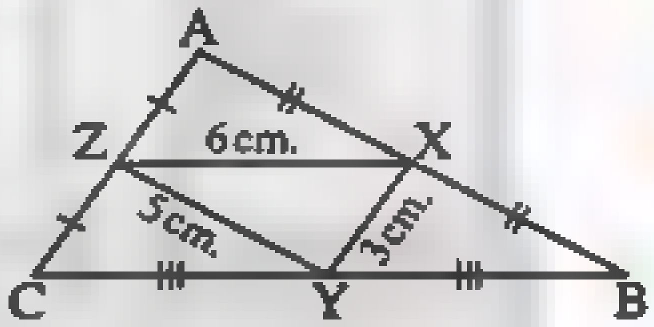
$\therefore DH = BO = \frac{1}{2} BC$ (Q.E.D.)

Example

In the opposite figure :

ABC is a triangle in which X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

If XY = 3 cm. , YZ = 5 cm. and ZX = 6 cm. , then find the perimeter of Δ ABC



Solution

Given ABC is a triangle in which X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively , XY = 3 cm. , YZ = 5 cm. and ZX = 6 cm.

R.T.F. The perimeter of Δ ABC

Proof In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$ and Z is the midpoint of $\overline{AC}$

$\therefore XZ = \frac{1}{2} BC$ (Theorem) $\therefore BC = 6 \times 2 = 12$ cm.

Similarly $\because$ X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC$ $\therefore AC = 3 \times 2 = 6$ cm.

Similarly $\because$ Y is the midpoint of $\overline{BC}$ and Z is the midpoint of $\overline{AC}$

$\therefore YZ = \frac{1}{2} AB$ $\therefore AB = 5 \times 2 = 10$ cm.

$\therefore$ The perimeter of $\Delta ABC = AB + BC + CA = 10 + 12 + 6 = 28$ cm.

(The req.)

Training 1

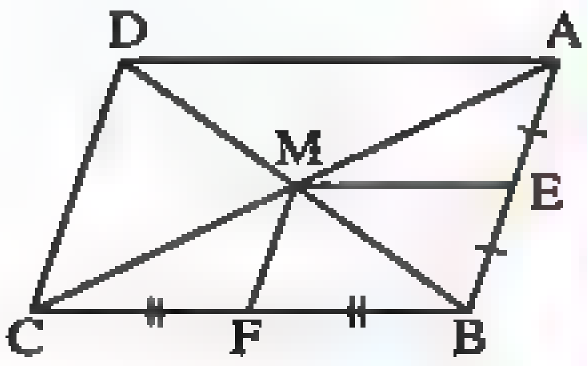
In the opposite figure :

ABCD is a parallelogram in which :

$\overline{AC} \cap \overline{BD} = \{M\}$, E is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{BC}$

Prove that : The figure EBFM is a parallelogram.



Solution

Given ABCD is a parallelogram , E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$

R.T.P. The figure EBFM is a parallelogram.

Proof

$\because$ ABCD is a parallelogram whose diagonals intersect at M

$\therefore$ M is the midpoint of each of $\overline{AC}$ and $\overline{BD}$

$\therefore$ In Δ ABC :

$\because$ E is the midpoint of $\overline{AB}$, M is the midpoint of $\overline{AC}$

$\therefore \overline{EM} \parallel \overline{BC}$ $\therefore \overline{EM} \parallel \overline{BF}$

, $EM = \frac{1}{2}$ BC (Theorem) $\therefore EM = BF$

$\therefore$ The figure EBFM is a parallelogram. (Q.E.D.)

Try by yourself

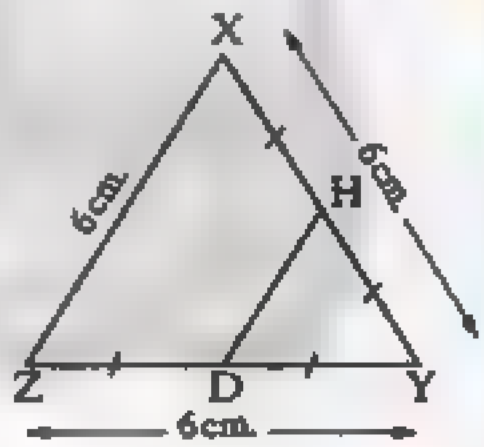
In the opposite figure :

XYZ is an equilateral triangle whose side is of length 6 cm. ,

D is the midpoint of $\overline{YZ}$ and H is the midpoint of $\overline{XY}$

Complete the following solution to prove that :

Δ HYD is an equilateral triangle and find its perimeter.



Given ....

R.T.P. ....

Proof

$\because$ D is the midpoint of .... $\therefore$ YD = ... cm. (1)

$\because$ H is the midpoint of .... $\therefore$ YH = ... cm. (2)

In Δ XYZ :

$\because$ D is the midpoint of .... and H is the midpoint of ....

$\therefore DH = \frac{1}{2}$ .... = ... cm. (3)

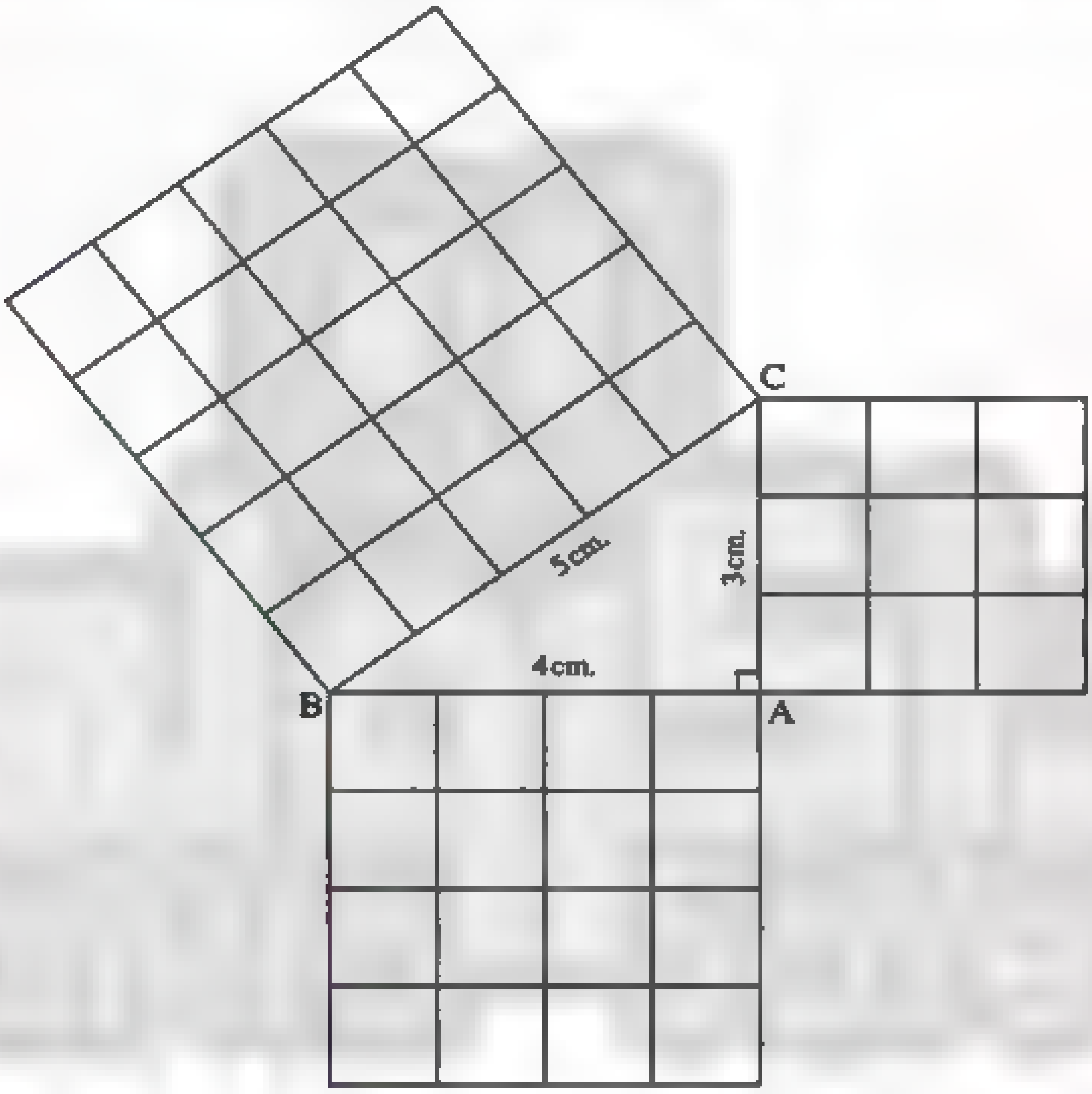
From (1) , (2) and (3) :

$\therefore \Delta$ HYD is .... and its perimeter = ... cm. (The req.)

Lesson Pythagoras' theorem

Prelude drill :

- Draw the right-angled triangle ABC at A in which AB = 4 cm., AC = 3 cm.
  If your drawing is accurate, you will find that : The length of the hypotenuse $\overline{BC}$ is 5 cm.



- Draw a square on each side of the triangle as in the opposite figure
  From the opposite figure, we find that :
  The area of the square drawn on $\overline{AB} = (AB)^2 = 16$ cm^2
  The area of the square drawn on $\overline{AC} = (AC)^2 = 9$ cm^2
  The area of the square drawn on $\overline{BC} = (BC)^2 = 25$ cm^2

i.e.: The area of the square drawn on $\overline{BC}$ is equal to the sum of the areas of the two squares drawn on $\overline{AB}$ and $\overline{AC}$

In other words : $(BC)^2 = (AB)^2 + (AC)^2$

The verbal formula of this relation is defined by Pythagoras' theorem.

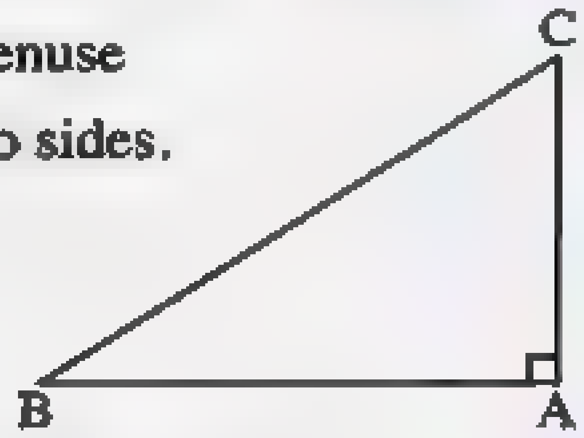

Pythagoras
(582-501 B.C.)

Pythagoras' theorem :

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.

We can also write the previous theorem as follows :

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



i.e.

If ABC is a right-angled triangle at A, then :

$(BC)^2 = (AB)^2 + (AC)^2$

- **From the previous relation, we can deduce the following two relations :**

$\boxed{(AB)^2 = (BC)^2 - (AC)^2}$ $\boxed{(AC)^2 = (BC)^2 - (AB)^2}$

Example 1

In each of the following figures, find the side length which is denoted by sign (?) :

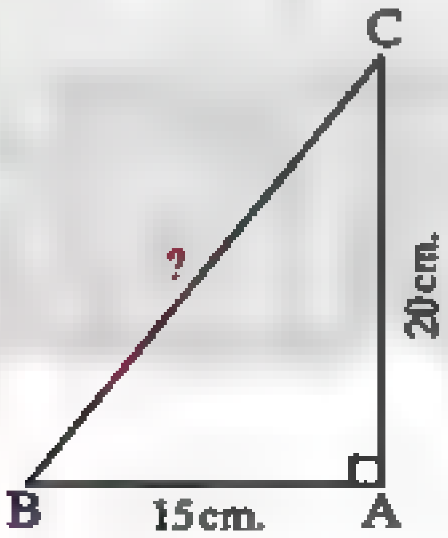


Fig. (1)

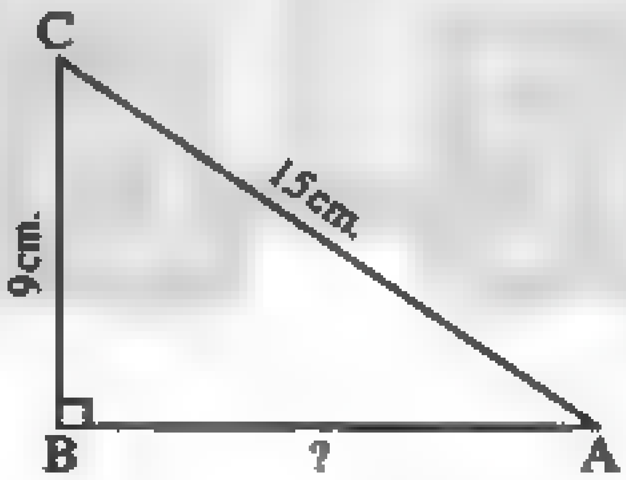


Fig. (2)

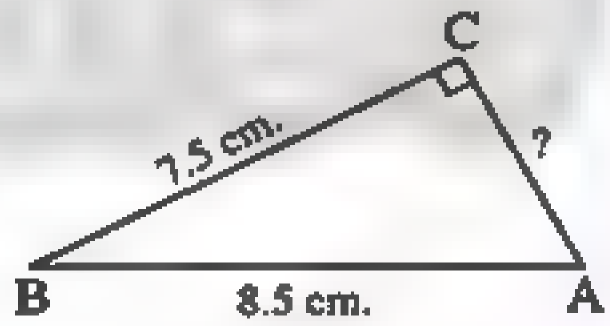


Fig. (3)

Solution

In fig. (1) :

$\because \Delta ABC$ is right-angled at A

$\therefore (BC)^2 = (AB)^2 + (AC)^2 = (15)^2 + (20)^2 = 225 + 400 = 625$

$\therefore BC = \sqrt{625} = 25$ cm.

In fig. (2) :

$\because$ Δ ABC is right-angled at B

$\because$ $(AB)^2 = (AC)^2 - (BC)^2 = (15)^2 - (9)^2 = 225 - 81 = 144$

$\therefore$ $AB = \sqrt{144} = 12$ cm.

In fig. (3) :

$\because$ Δ ABC is right-angled at C

$\therefore$ $(AC)^2 = (AB)^2 - (BC)^2 = (8.5)^2 - (7.5)^2 = 72.25 - 56.25 = 16$

$\therefore$ $AC = \sqrt{16} = 4$ cm.

Try by yourself

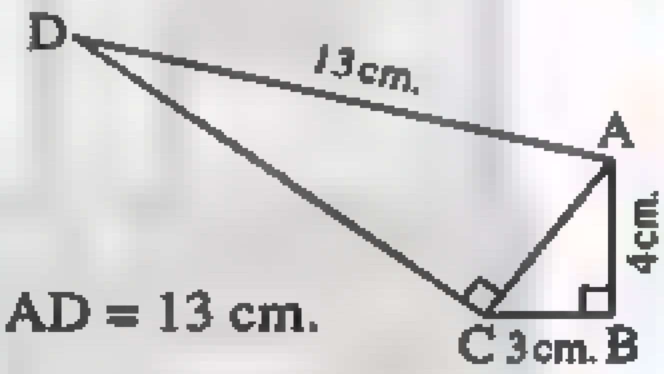
In the opposite figure :

Δ ABC is right-angled at A

Complete the following table :

AB	8 cm.	12 cm.	12 cm.	20 cm.	....	....
AC	6 cm.	9 cm.	....	....	12 cm.	4.5 cm.
BC	....	....	13 cm.	25 cm.	20 cm.	7.5 cm.

Example (2)



In the opposite figure :

ABCD is a quadrilateral in which :

m ($\angle$ B) = m ($\angle$ ACD) = 90° , AB = 4 cm. , BC = 3 cm. and AD = 13 cm.

Find : The length of each of $\overline{AC}$ and $\overline{CD}$

Solution

Given m ($\angle$ B) = m ($\angle$ ACD) = 90° , AB = 4 cm. , BC = 3 cm. and AD = 13 cm.

R.T.F. The length of each of $\overline{AC}$ and $\overline{CD}$

Proof

$\because$ Δ ABC is a right-angled triangle at B

$\therefore$ $(AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore$ $(AC)^2 = (4)^2 + (3)^2 = 16 + 9 = 25$

$\therefore$ $AC = \sqrt{25} = 5$ cm. (First req.)

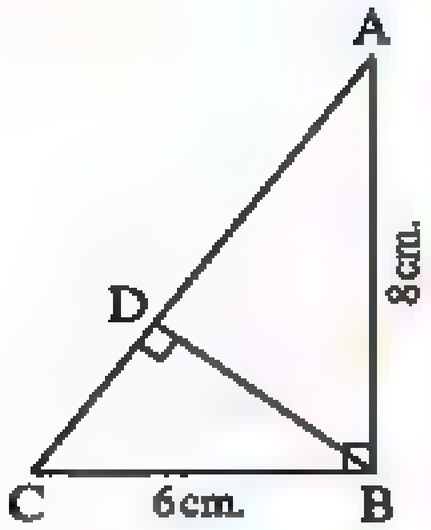
$\because$ Δ ACD is a right-angled triangle at C

$\therefore$ $(CD)^2 = (AD)^2 - (AC)^2$

$\therefore$ $(CD)^2 = (13)^2 - (5)^2 = 169 - 25 = 144$ (Pythagoras' theorem)

$\therefore$ $CD = \sqrt{144} = 12$ cm. (Second req.)

Example 3



In the opposite figure :

ABC is a right-angled triangle at B , $D \in \overline{AC}$

such that $\overline{BD} \perp \overline{AC}$, AB = 8 cm. and CB = 6 cm.

Find : The length of $\overline{BD}$

Solution

Given Δ ABC is right-angled at B , $\overline{BD} \perp \overline{AC}$, AB = 8 cm. and CB = 6 cm.

R.T.F. The length of $\overline{BD}$

Proof $\because \Delta$ ABC is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = 64 + 36 = 100$ $\therefore AC = \sqrt{100} = 10$ cm.

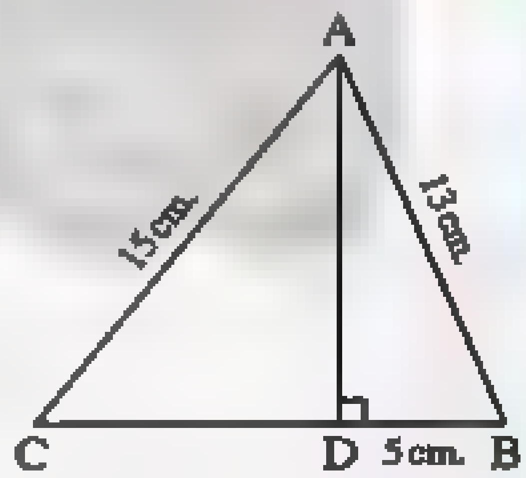
, $\because$ The area of $\Delta ABC = \frac{1}{2} BC \times AB = \frac{1}{2} \times 6 \times 8 = 24$ cm^2.

, $\because$ The area of $\Delta ABC = \frac{1}{2} AC \times BD$

$\therefore 24 = \frac{1}{2} \times 10 \times BD$ $\therefore 24 = 5\,BD$

$\therefore BD = \frac{24}{5} = 4.8$ cm. (The req.)

Try by yourself



In the opposite figure :

ABC is a triangle in which AB = 13 cm.

, AC = 15 cm. , $D \in \overline{BC}$ such that

$\overline{AD} \perp \overline{BC}$ and BD = 5 cm.

Complete the following proof to find the length of $\overline{DC}$

$\because \Delta$ ABD is right-angled at D

$\therefore (AD)^2 = (AB)^2 - (\cdots\cdots)^2 = (\cdots\cdots)^2 - (\cdots\cdots)^2 = \cdots\cdots$

$\therefore AD = \sqrt{\cdots\cdots} = \cdots\cdots$ cm.

, $\because \Delta$ ADC is right-angled at D

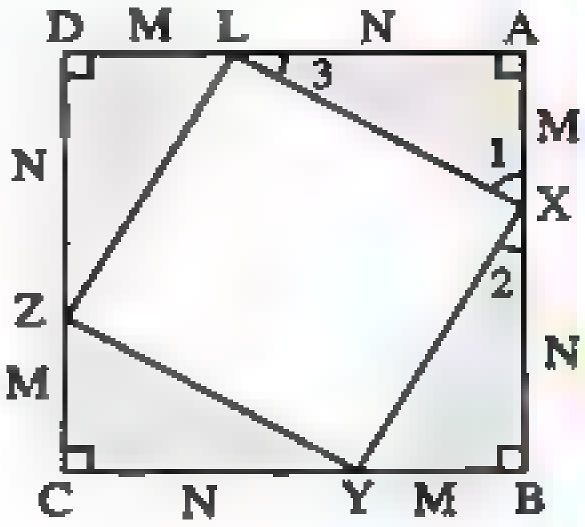
$\therefore (DC)^2 = (\cdots\cdots)^2 - (AD)^2 = (\cdots\cdots)^2 - (\cdots\cdots)^2 = \cdots\cdots$

$\therefore DC = \sqrt{\cdots\cdots} = \cdots\cdots$ cm. (The req.)

Can you prove Pethagoras' theorem theoretically ?

- We can prove Pethagoras' theorem which has been explained theoretically as follows :

In the opposite figure :



If ABCD is a square and the points X , Y , Z and L divide its sides $\overline{AB}$, $\overline{BC}$ $\overline{CD}$ and $\overline{DA}$ respectively into two parts , one part is M length unit and the other is N length unit

In $\Delta\Delta$ AXL and BYX : $\begin{cases} AL = BX = N \text{ length unit} \\ AX = BY = M \text{ length unit} \\ m(\angle A) = m(\angle B) = 90^\circ \text{ (properties of the square)} \end{cases}$

i.e. $\Delta AXL \equiv \Delta BYX$ (two sides and the included angle)

Similarly we can prove that :

$\Delta BYX \equiv \Delta CZY$, $\Delta CZY \equiv \Delta DLZ$, $\Delta DLZ \equiv \Delta AXL$

i.e. $\Delta AXL \equiv \Delta BYX \equiv \Delta CZY \equiv \Delta DLZ$

, then we deduce that : $XY = YZ = ZL = LX$

i.e. The figure XYZL is a rhombous

, $\because m(\angle 1) + m(\angle 3) = 90^\circ$

, $m(\angle 2) = m(\angle 3)$ (from congruence)

$\therefore m(\angle 1) + m(\angle 2) = 90^\circ$ $\therefore m(\angle LXY) = 90^\circ$

$\therefore$ The figure XYZL is a square

, then the area of the sqaure XYZL

= The area of the square ABCD – 4 the area of ΔAXL

$\therefore (XY)^2 = (M+N)^2 - 4 \times \frac{1}{2} \times MN$

$\therefore (XY)^2 = M^2 + 2\,MN + N^2 - 2\,MN$

$\therefore (XY)^2 = M^2 + N^2$ $\therefore (XY)^2 = (BY)^2 + (BX)^2$

i.e. In ΔXBY which is right-angled at B :

The square of the length of the hypotenuse = The sum of the squares of the lengths of the other two sides.

Enrichment information (for reading only)

You can get three numbers representing the lengths of sides of a right-angled triangle as follows :

1. If M is an even number bigger than 2, then the numbers M, $\left(\frac{M}{2}\right)^2 - 1$, $\left(\frac{M}{2}\right)^2 + 1$ represent three lengths of sides of a right-angled triangle as shown in the following table :

M	$\left(\frac{M}{2}\right)^2 - 1$	$\left(\frac{M}{2}\right)^2 + 1$
4	$\frac{16}{4} - 1 = 3$	$\frac{16}{4} + 1 = 5$
6	$\frac{36}{4} - 1 = 8$	$\frac{36}{4} + 1 = 10$
8	$\frac{64}{4} - 1 = 15$	$\frac{64}{4} + 1 = 17$

2. If M is an odd number bigger than 2, then the numbers M, $\frac{M^2 - 1}{2}$, $\frac{M^2 + 1}{2}$ represent three lengths of sides of a right-angled triangle as shown in the following table :

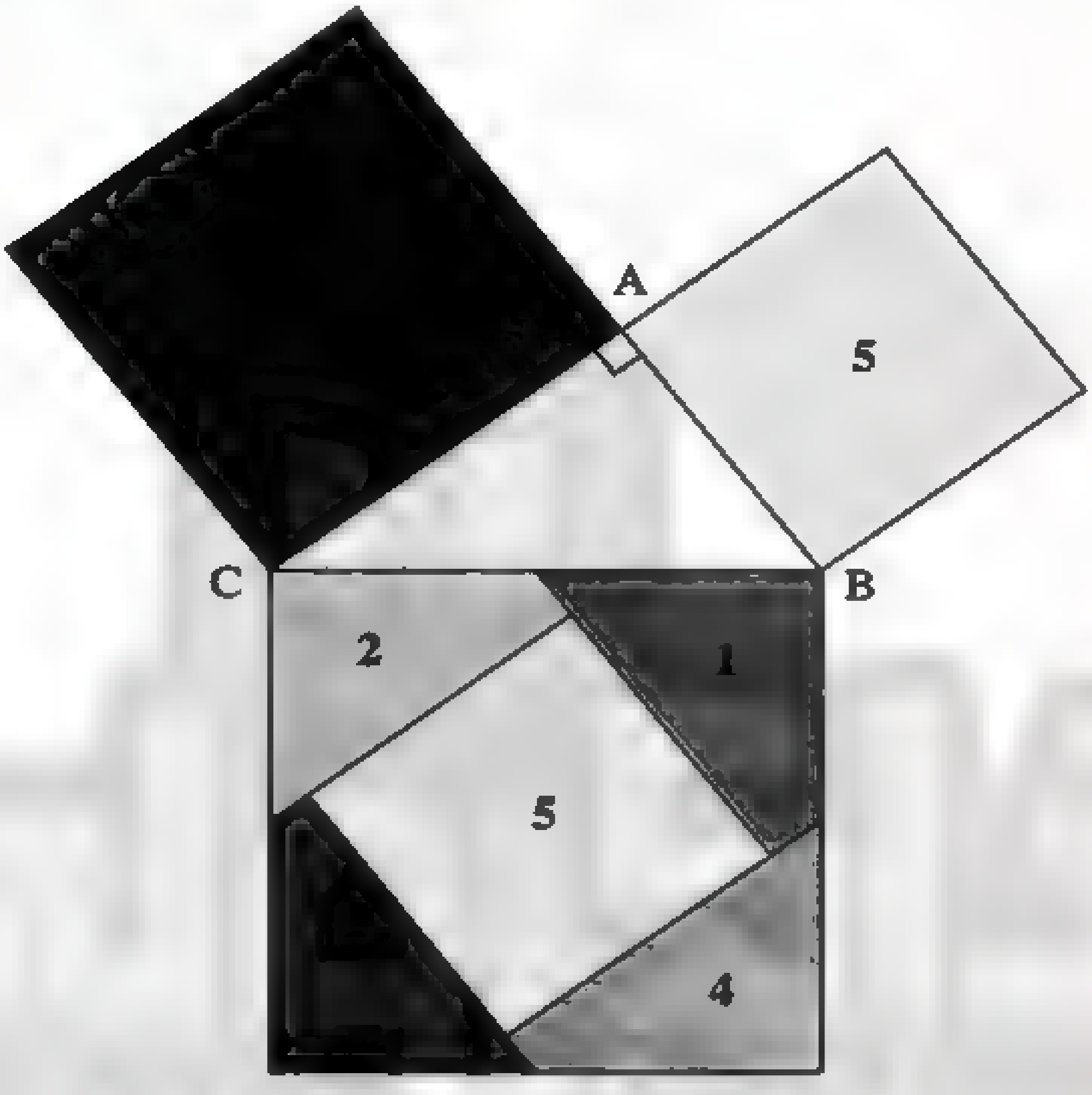
M	$\frac{M^2 - 1}{2}$	$\frac{M^2 + 1}{2}$
5	$\frac{25 - 1}{2} = 12$	$\frac{25 + 1}{2} = 13$
7	$\frac{49 - 1}{2} = 24$	$\frac{49 + 1}{2} = 25$
9	$\frac{81 - 1}{2} = 40$	$\frac{81 + 1}{2} = 41$

Enrichment Activity 1

You can prove Pethagoras' theorem by using computer through the Paint programme as shown in the computer activities at the end of that book.

Enrichment Activity 2

A practical activity to prove Pethagoras' theorem



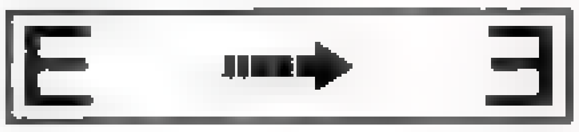
- Cut the square drawn on $\overline{AB}$ and the square drawn on $\overline{AC}$
- Cut the numbered areas by numbers ❶ , ❷ , ❸ and ❹
- Try to stick the areas you cut on the areas with their corresponding numbers on the square drawn on the chord $\overline{BC}$
- If you are accurate in your work, you will find that they are completely congruent, so we prove Pethagoras' theorem practically which says :

> The area of the square drawn on the hypotenuse $\overline{BC}$ is equal to the sum of areas of the two squares drawn on the sides of the right angle $\overline{AB}$ and $\overline{AC}$

Lesson 1 Geometric transformations

Bassim and Ayman play with a set of plastic letters. In each time , they notice the change which occurs to a certain letter as (E) if they :

1 Reflect its position.



Reflection

2 Move it straight a certain distance.

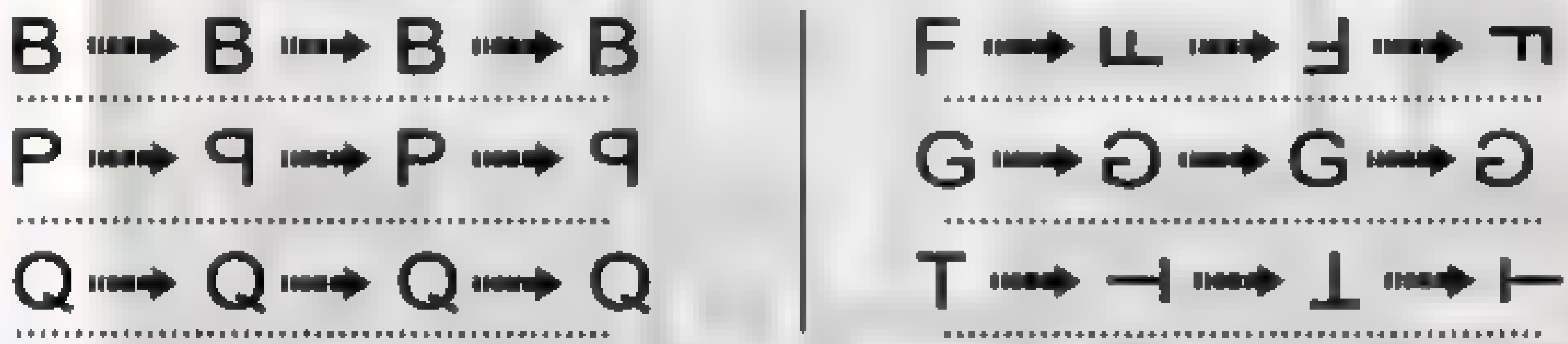
Translation

3 Rotate it with a certain angle.

Rotation anticlockwise   Rotation clockwise

Try by yourself

By noticing the change that occurs to every letter compared with the figure preceding it , write down each figure the suitable word (reflection , translation , rotation) :



In each of the following figures , notice the image of Δ ABC and deduce what happened to it :

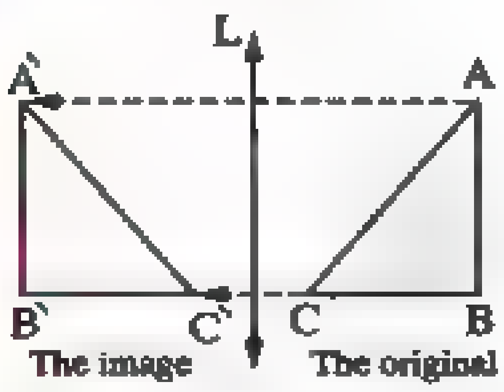


Fig. (1)

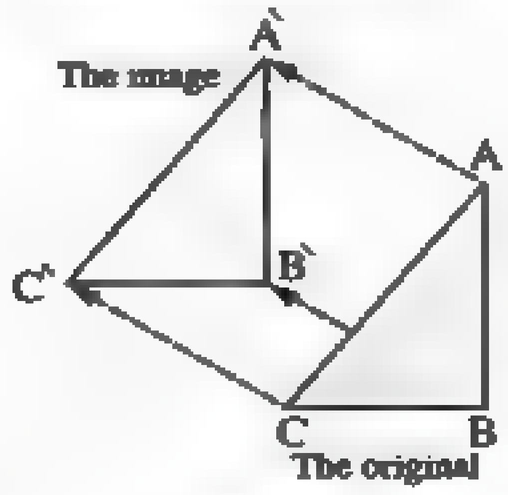


Fig. (2)

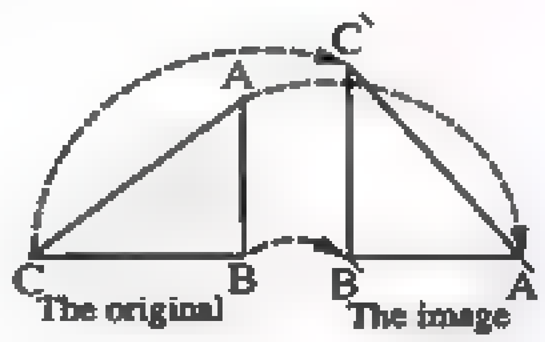


Fig. (3)

- Fig. (1) : Δ ABC has been reflected.
- Fig. (2) : Δ ABC has been translated.
- Fig. (3) : Δ ABC has been rotated.

Unit 3

In each of the previous figures , notice that :

- The point A has been transferred to $\grave{A}$ *i.e.* $A \longrightarrow \grave{A}$
- The point B has been transferred to $\grave{B}$ *i.e.* $B \longrightarrow \grave{B}$
- The point C has been transferred to $\grave{C}$ *i.e.* $C \longrightarrow \grave{C}$

Thus , all the points of Δ ABC have been transferred to another position , then we say that Δ ABC has been transformed from position to another position.

From the previous, we deduce that :

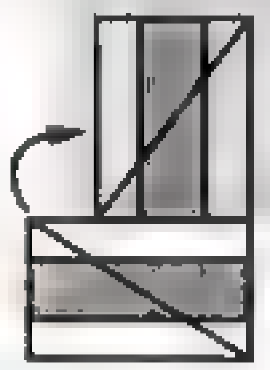
If all the points of a geometrical figure have moved according to a certain system , then we will obtain an image in a new position to the same figure , then we say that this figure has been under the effect of a geometric transformation.

i.e. The geometric transformation maps each point P in the plane onto an image point $\grave{P}$ in the same plane.

Geometric transformations are many types as reflection , translation and rotation which we will study each of them in detail in the following lessons.

Try by yourself

Describe the type of the geometric transformation (reflection , translation or rotation) in each of the following figures :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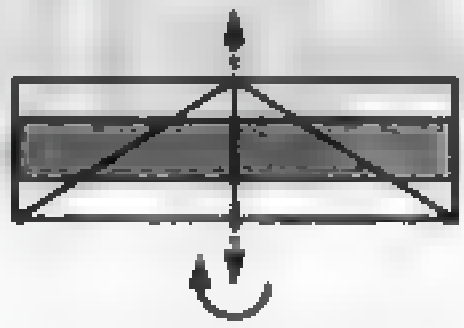
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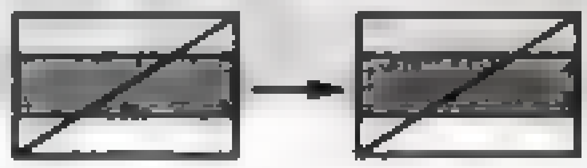
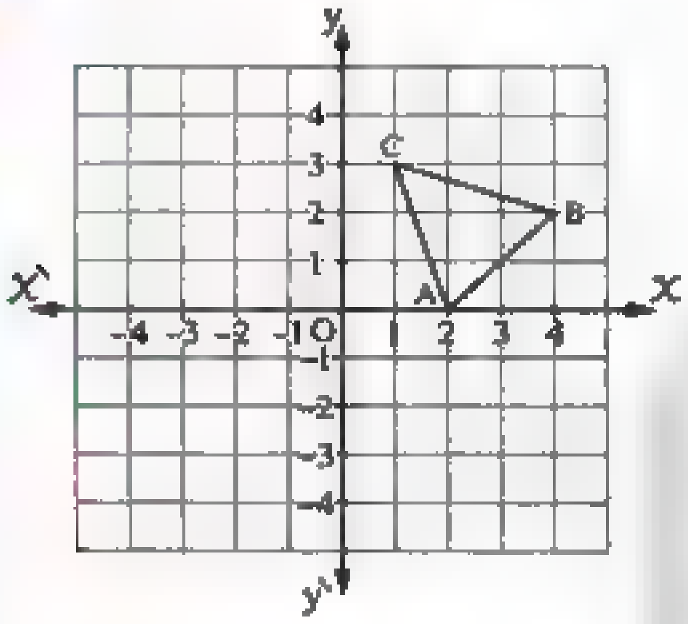

....

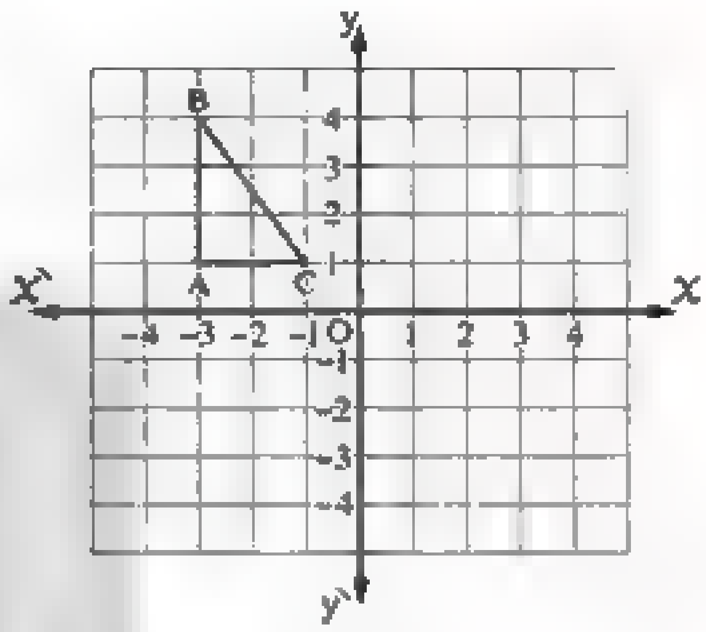
Example (1)

Draw the image of each of the following figures according to the illustrated geometric transformation , then describe its type :

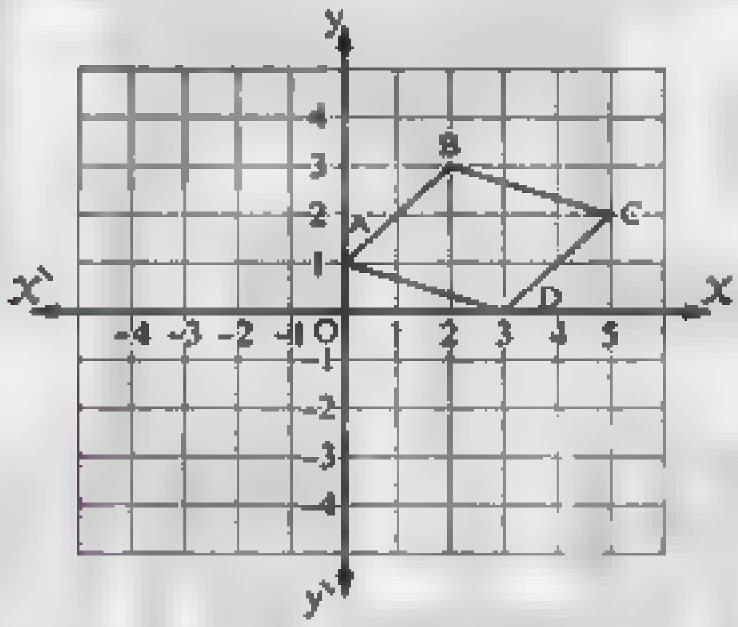
1 $(x , y) \longrightarrow (x - 4 , y + 1)$



2 $(x , y) \longrightarrow (-x , -y)$



3 $(x , y) \longrightarrow (x , -y)$



Solution

1 $(x , y) \longrightarrow (x - 4 , y + 1)$

i.e. x transferred to $x - 4$ and

y transferred to y + 1 , therefore we get :

$A (2 , 0) \longrightarrow A' (2 - 4 , 0 + 1)$

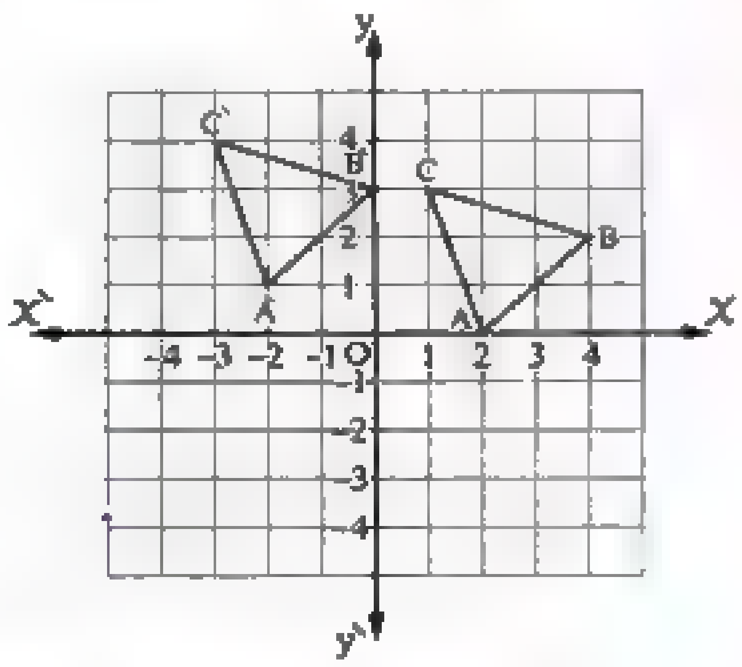
i.e. $A' (-2 , 1)$

$B (4 , 2) \longrightarrow B' (4 - 4 , 2 + 1)$

i.e. $B' (0 , 3)$

$C (1 , 3) \longrightarrow C' (1 - 4 , 3 + 1)$

i.e. $C' (-3 , 4)$



From the graph , it is shown that ΔABC has been translated to become $\Delta A'B'C'$

2 $(x , y) \longrightarrow (-x , -y)$

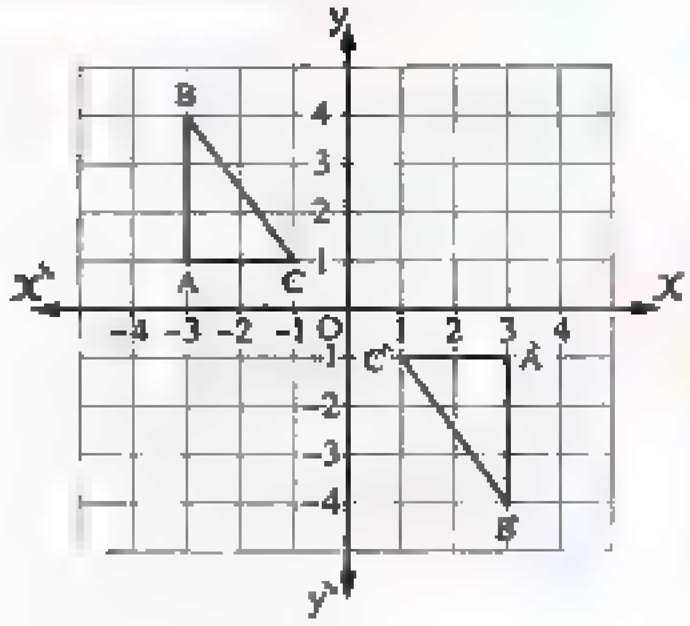
i.e.

A $(-3 , 1) \longrightarrow$ À $(3 , -1)$ [Note : $-(-3) = 3$]

B $(-3 , 4) \longrightarrow$ B̀ $(3 , -4)$

C $(-1 , 1) \longrightarrow$ C̀ $(1 , -1)$

From the graph , it is shown that Δ ABC has been rotated to become Δ ÀB̀C̀



3 $(x , y) \longrightarrow (x , -y)$

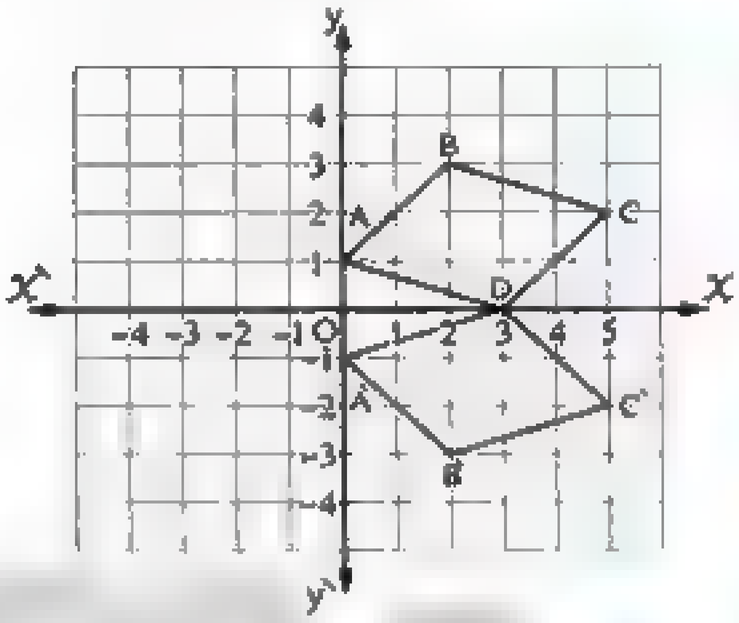
A $(0 , 1) \longrightarrow$ À $(0 , -1)$

B $(2 , 3) \longrightarrow$ B̀ $(2 , -3)$

C $(5 , 2) \longrightarrow$ C̀ $(5 , -2)$

D $(3 , 0) \longrightarrow$ D $(3 , 0)$

From the graph , it is shown that the shape ABCD has been reflected to become ÀB̀C̀D



Example (2)

Draw the image of the quadrilateral ABCD where A (1 , 1) , B (4 , 1) , C (4 , 3) , D (1 , 5) according to each of the following transformations , then describe its type :

1 $(x , y) \longrightarrow (-x , y)$ **2** $(x , y) \longrightarrow (-y , x)$ **3** $(x , y) \longrightarrow (x , y - 5)$

Solution

1 $(x , y) \longrightarrow (-x , y)$

i.e.

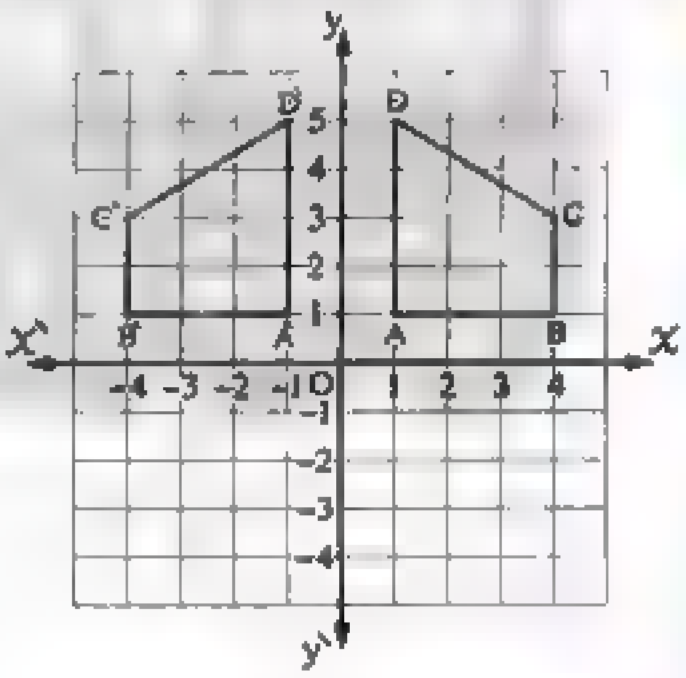
A $(1 , 1) \longrightarrow$ À $(-1 , 1)$

B $(4 , 1) \longrightarrow$ B̀ $(-4 , 1)$

C $(4 , 3) \longrightarrow$ C̀ $(-4 , 3)$

D $(1 , 5) \longrightarrow$ D̀ $(-1 , 5)$

The transformation is reflection



2 $(x , y) \longrightarrow (-y , x)$

i.e.

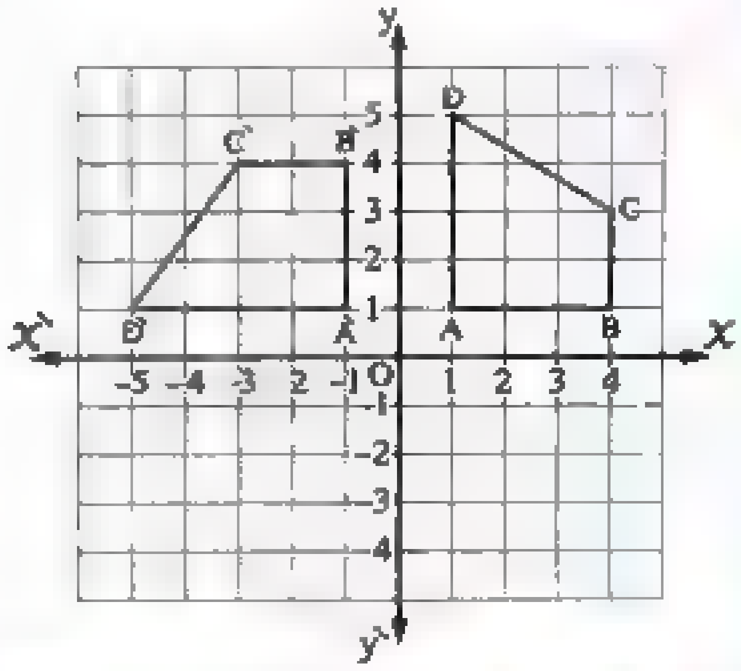
A $(1 , 1) \longrightarrow$ À $(-1 , 1)$

B $(4 , 1) \longrightarrow$ B̀ $(-1 , 4)$

C $(4 , 3) \longrightarrow$ C̀ $(-3 , 4)$

D $(1 , 5) \longrightarrow$ D̀ $(-5 , 1)$

The transformation is rotation.



3 $(x , y) \longrightarrow (x , y - 5)$

i.e.

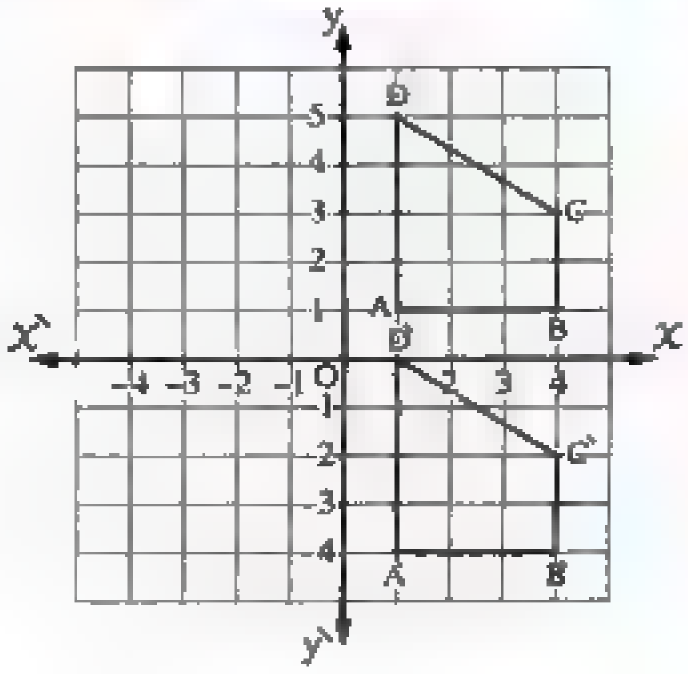
$A(1 , 1) \longrightarrow \grave{A}(1 , -4)$

$B(4 , 1) \longrightarrow \grave{B}(4 , -4)$

$C(4 , 3) \longrightarrow \grave{C}(4 , -2)$

$D(1 , 5) \longrightarrow \grave{D}(1 , 0)$

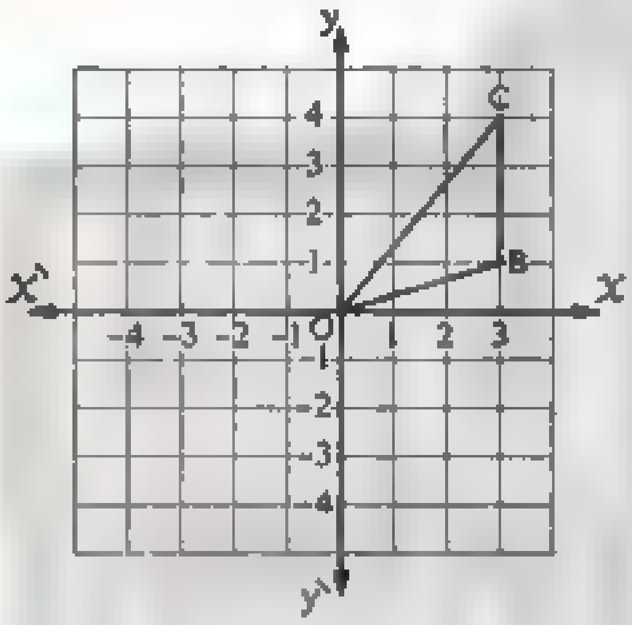
The transformation is translation.



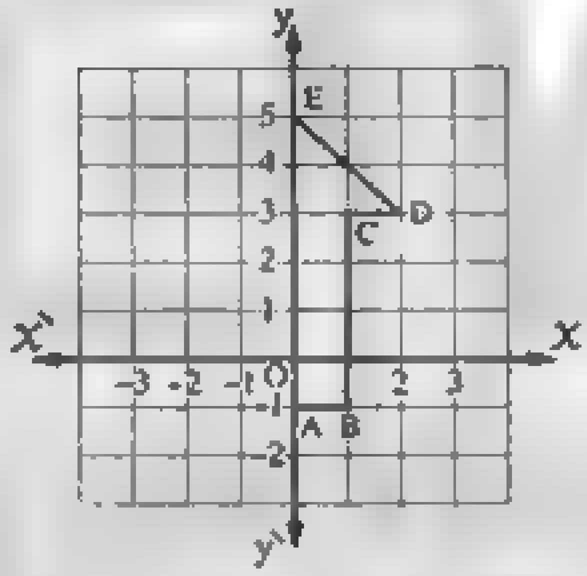
Try by yourself

Map each of the following shapes due to the geometric transformation above it , then describe its type :

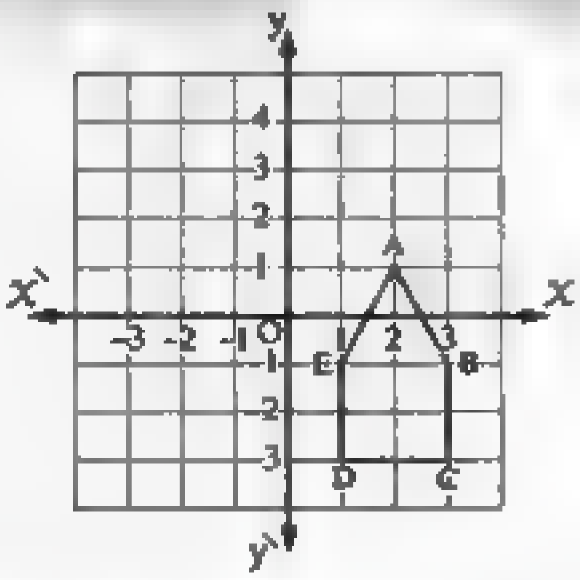
1 $(x , y) \longrightarrow (-x , -y)$



2 $(x , y) \longrightarrow (-x , y)$



3 $(x , y) \longrightarrow (x , y + 2)$



Unit 3

Lesson 1 Reflection (Reflection in a straight line)

Prelude

If you stand in front of a plane mirror , then you will see your picture (image) reflected in the mirror in the same size and details and you will notice also that the distance between the image and the mirror equals the real distance between you and the mirror. If you approach the mirror , then you will find that your image approaches also the mirror.

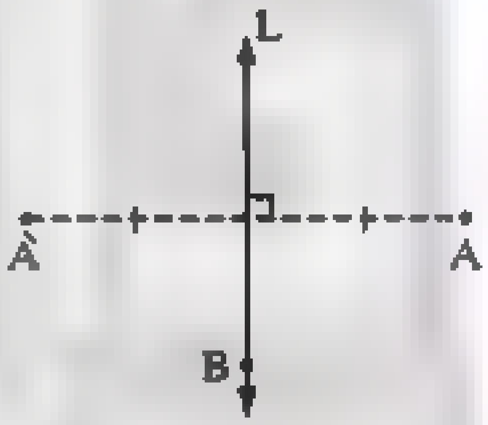
Reflection in a straight line

Reflection in the straight line L maps each point A to the point $\grave{A}$ in the same plane such that :

1 If $A \notin L$, then the straight line L is the perpendicular bisector to the line segment $\overline{A\grave{A}}$

2 If $B \in L$, then B is reflected onto itself

i.e. $\grave{B}$ coincides B

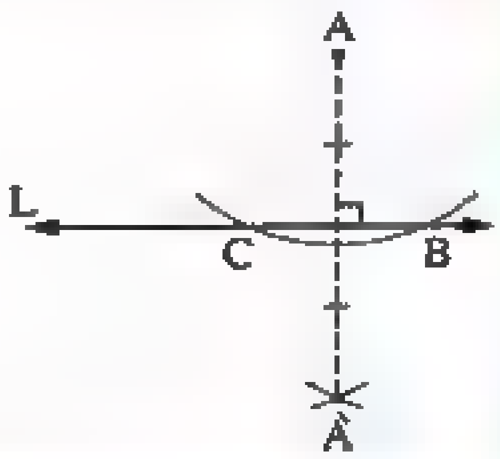


Finding the image of a point by reflection in a given straight line

- **To find $\grave{A}$ which is the image of A by reflection in the straight line L , we do as follows :**

1 Draw an arc of a circle with centre A to cut L at B and C

2 With the same radius length taking B and C as centres , draw two arcs in the other side of the straight line L to intersect at $\grave{A}$, then $\grave{A}$ is the image of A by reflection in L

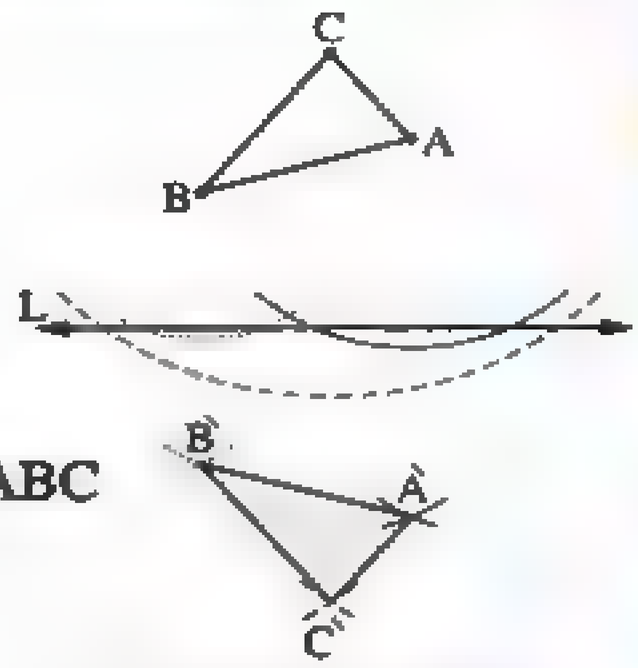


Check by measuring that :

$\overline{A\grave{A}} \perp L$ and L bisects $\overline{A\grave{A}}$

Finding the image of a polygon by reflection in a given straight line

- **To find the image of a polygon as ΔABC by reflection in the straight line L , we do as follows :**



1 Find the image of each vertex of ΔABC by reflection in the straight line L as we did before ($\grave{A}$ is the image of A , $\grave{B}$ is the image of B and $\grave{C}$ is the image of C)

2 Draw $\overline{\grave{A}\grave{B}}$, $\overline{\grave{B}\grave{C}}$ and $\overline{\grave{C}\grave{A}}$, then $\Delta \grave{A}\grave{B}\grave{C}$ is the image of ΔABC by reflection in the straight line L

$\boxed{\Delta ABC \equiv \Delta \grave{A}\grave{B}\grave{C}}$

Check by measuring that :

- $AB = \grave{A}\grave{B}$, $BC = \grave{B}\grave{C}$ and $CA = \grave{C}\grave{A}$
- $m(\angle A) = m(\angle \grave{A})$, $m(\angle B) = m(\angle \grave{B})$ and $m(\angle C) = m(\angle \grave{C})$

From the previous, we deduce that :

Reflection is a geometrical transformation which transforms the geometrical shape into another one congruent to it (equal to it in its side lengths and angle measures), but the direction of reading the shape is the opposite direction of reading the image.

Notice that :

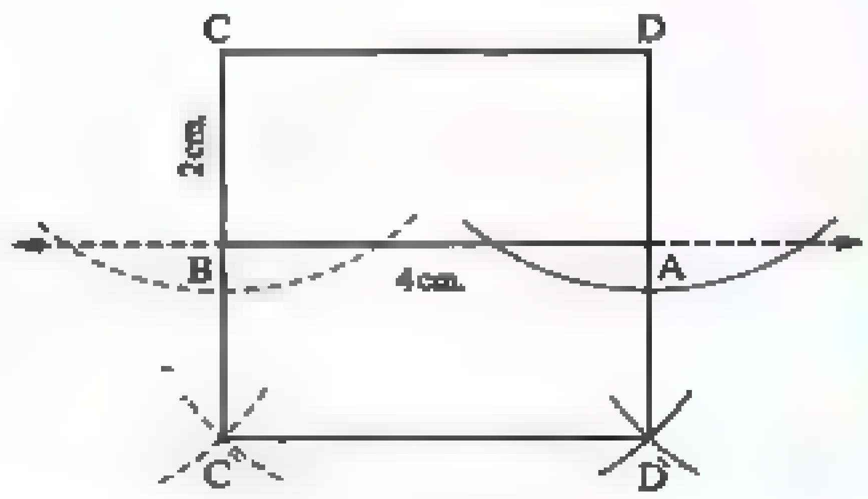
The reading of ΔABC is clockwise while the reading of $\Delta \grave{A}\grave{B}\grave{C}$ is anticlockwise.

Properties of reflection in a straight line

Illustrated example

Draw the image of the rectangle ABCD in which AB = 4 cm. , BC = 2 cm. by reflection in $\overleftrightarrow{AB}$

Solution



First : We draw the rectangle ABCD in which AB = 4 cm. and BC = 2 cm.

Second : To find the image of the rectangle ABCD by reflection in $\overleftrightarrow{AB}$ we do as follows :

1 The images of A and B by reflection in $\overleftrightarrow{AB}$ are the same because they belong to $\overleftrightarrow{AB}$

2 We find the image of D by reflection in $\overleftrightarrow{AB}$, let it be $\grave{D}$, the image of C by reflection in $\overleftrightarrow{AB}$ let it be $\grave{C}$, then we get the rectangle $AB\grave{C}\grave{D}$ to be the image of the rectangle ABCD by reflection in $\overleftrightarrow{AB}$

Notice that :

1 $AD = A\grave{D}$, $DC = \grave{D}\grave{C}$, $CB = \grave{C}B$ and $\overline{AB}$ is a common side.

i.e. Reflection in a straight line reserves the lengths of the line segments.

2 $m(\angle BAD) = m(\angle BA\grave{D})$, $m(\angle ABC) = m(\angle AB\grave{C})$, $m(\angle C) = m(\angle \grave{C})$ and $m(\angle D) = m(\angle \grave{D})$

i.e. Reflection in a straight line reserves the measures of angles.

3 From the rectangle ABCD : $\overline{AD} \parallel \overline{BC}$, from the rectangle $AB\grave{C}\grave{D}$: $\overline{A\grave{D}} \parallel \overline{B\grave{C}}$

$\therefore$ The images of the two parallel line segments are also two parallel line segments.

i.e. Reflection in a straight line reserves parallelism.

4 The reading of the rectangle ABCD is in the clockwise direction while the reading of the rectangle $AB\grave{C}\grave{D}$ is in anticlockwise direction.

i.e. Reflection in a straight line doesn't reserve the orientation of the vertices of the figure.

5 If a point lies on $\overline{DC}$ and we find its image by reflection in $\overleftrightarrow{AB}$, we find its image lie on $\overline{\grave{D}\grave{C}}$

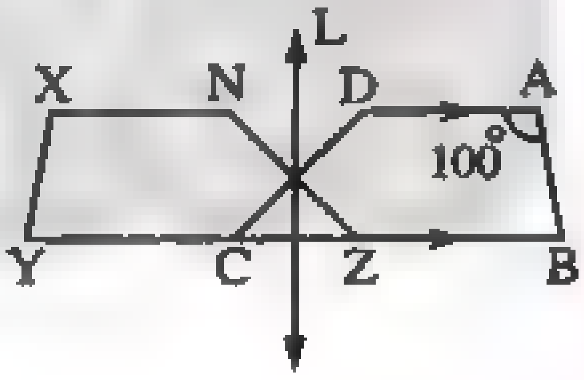
i.e. Reflection in a straight line reserves the betweenness.

Example 1

In the opposite figure :

ABCD is a quadrilateral in which $m(\angle A) = 100°$ and $\overline{AD} \parallel \overline{BC}$ If the figure XYZN is the image of the figure ABCD by reflection in the straight line L

Find : $m(\angle Y)$



Solution

$\because$ The figure XYZN is the image of the figure ABCD by reflection in L

$\therefore$ X is the image of A by reflection in L $\qquad \therefore m(\angle X) = m(\angle A) = 100°$

(Because the reflection in the straight line reserves the measures of angles)

$\because$ $\overline{XN}$ is the image of $\overline{AD}$ and $\overline{YZ}$ is the image of $\overline{BC}$ by reflection in L and $\because$ $\overline{AD} \parallel \overline{BC}$

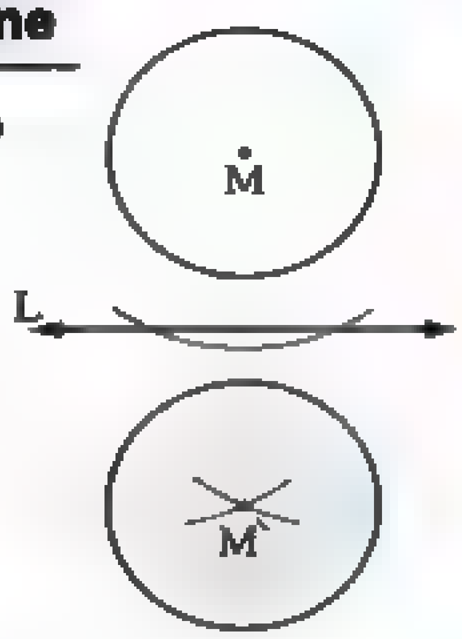
$\therefore$ $\overline{XN} \parallel \overline{YZ}$ (Because the reflection in a straight line reserves parallelism)

$\therefore m(\angle X) + m(\angle Y) = 180°$ (Two interior angles in the same side of the transversal)

$\therefore m(\angle Y) = 180° - 100° = 80°$ (The req.)

Finding the image of a circle by reflection in a given straight line

- **To find the image of a circle M by reflection in the straight line L , we do as follows :**

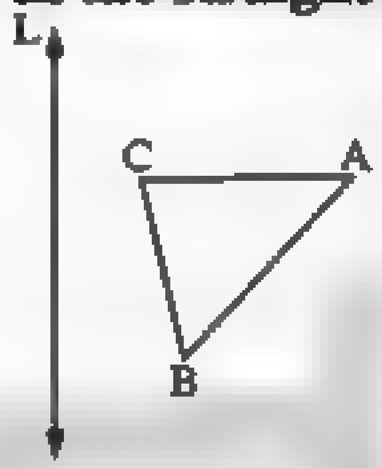


1 Find the image of the centre M by reflection in L as we did before , say M̀

2 Use the compasses with radius length equal to the radius length of the circle M to draw a circle with centre M̀ that will be the image of the circle M by reflection in L

Try by yourself

Draw the image of the triangle by reflection in the straight line L :



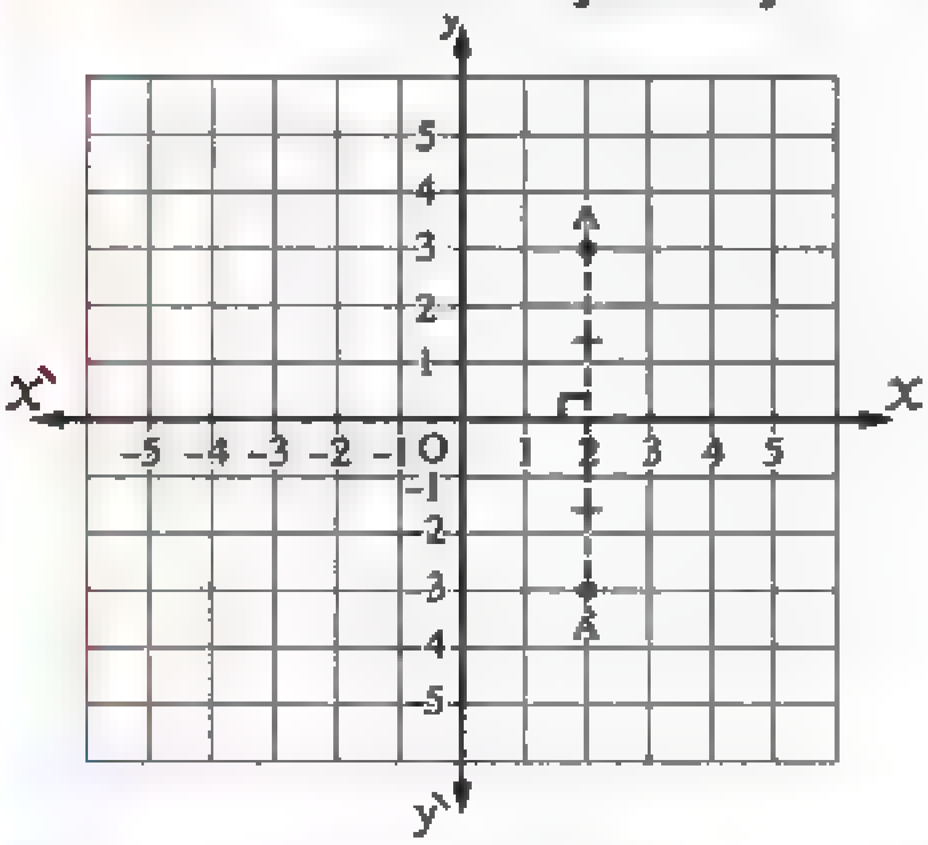
Optical illusion

Look at the picture. What do you see ?

Reflection in the Cartesian coordinates plane

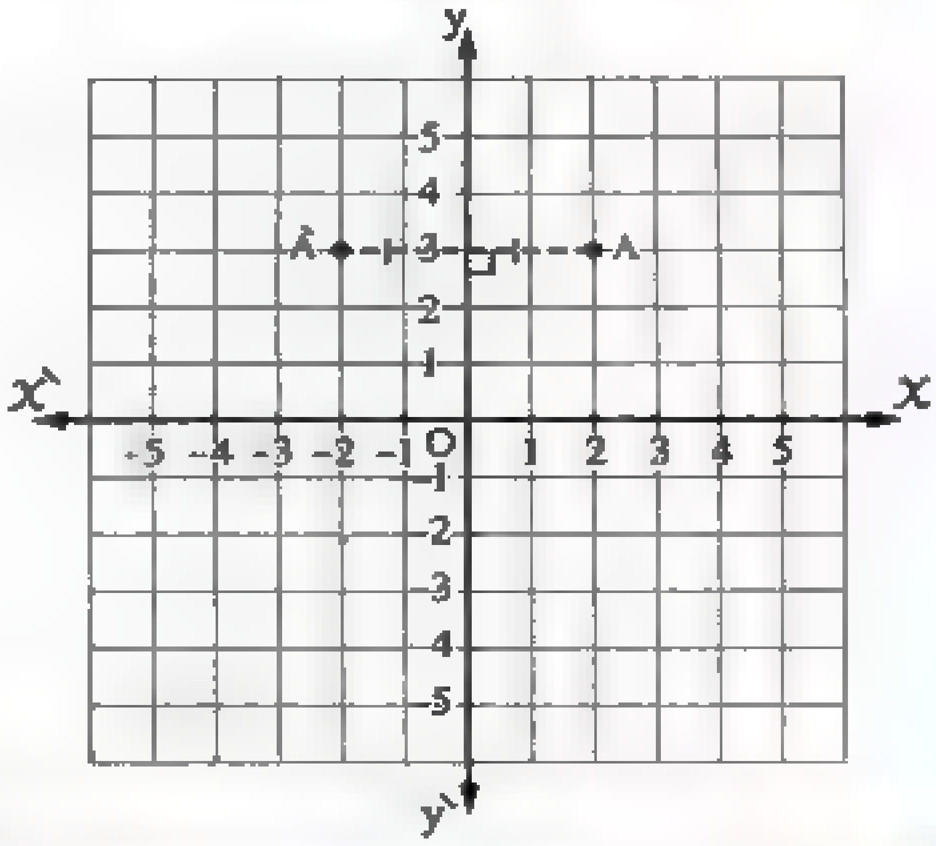
Reflection in the x-axis

- If A (2 , 3) is a point in the Cartesian coordinate plane and the required is finding its image by reflection in $\overleftrightarrow{x\acute{x}}$ (the x-axis) , then draw $\overline{A\acute{A}}$ such that $\overleftrightarrow{x\acute{x}}$ is the line of symmetry of it :



Reflection in the y-axis

- If A (2 , 3) is a point in the Cartesian coordinate plane and the required is finding its image by reflection in $\overleftrightarrow{y\acute{y}}$ (the y-axis) , then draw $\overline{A\acute{A}}$ such that $\overleftrightarrow{y\acute{y}}$ is the line of symmetry of it :



Then we find that :

$A(2, 3) \longrightarrow \grave{A}(2, -3)$

i.e. The reflection in the x-axis changes the sign of the 2nd projection (y-coordinates)

$A(x, y) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} \grave{A}(x, -y)$

For example :

- $(2, 4) \longrightarrow (2, -4)$
- $(-3, 1) \longrightarrow (-3, -1)$
- $(5, -6) \longrightarrow (5, 6)$
- $(-2, -1) \longrightarrow (-2, 1)$

Then we find that :

$A(2, 3) \longrightarrow \grave{A}(-2, 3)$

i.e. The reflection in the y-axis changes the sign of the 1st projection (x-coordinates)

$A(x, y) \xrightarrow[\text{the y-axis}]{\text{by reflection in}} \grave{A}(-x, y)$

For example :

- $(2, 4) \longrightarrow (-2, 4)$
- $(-3, 1) \longrightarrow (3, 1)$
- $(5, -6) \longrightarrow (-5, -6)$
- $(-2, -1) \longrightarrow (2, -1)$

Remarks

1. The image of the point $(x, 0)$ by reflection in the x-axis is itself because it lies on the x-axis

   For example : $(5, 0) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (5, 0)$

2. The image of the point $(0, y)$ by reflection in the y-axis is itself because it lies on the y-axis

   For example : $(0, -3) \xrightarrow[\text{the y-axis}]{\text{by reflection in}} (0, -3)$

3. The image of the point $(0, 0)$ by reflection in the x-axis or the y-axis is itself because it lies on both two axes.

Try by yourself

Complete the following table :

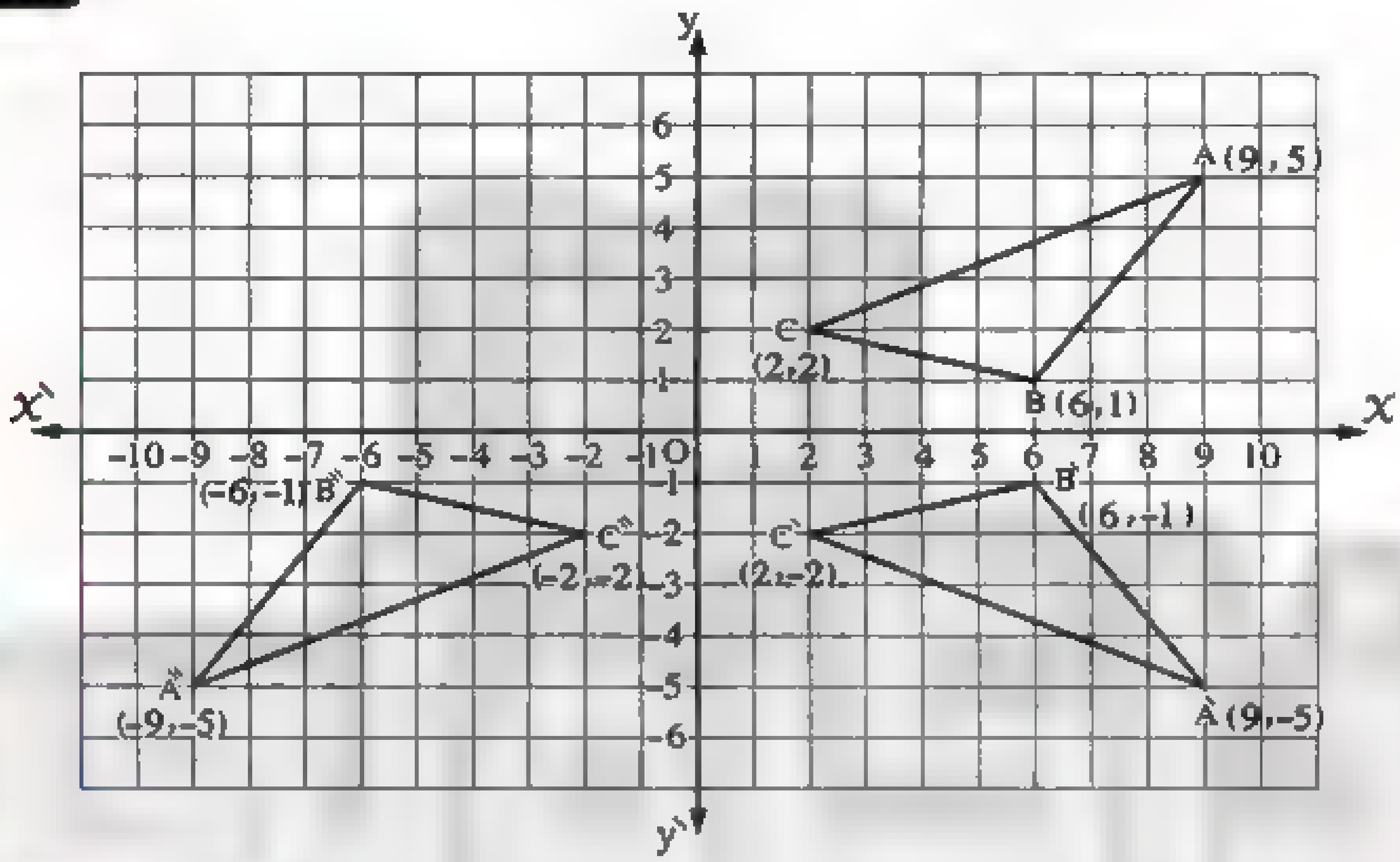
The point	(5, 1)	(2, −3)	(−1, 4)	(−2, −6)	(0, −1)	(3, 0)	(0, 0)
Its image by reflection in the x-axis							
Its image by reflection in the y-axis							

Example 2

Draw on a square lattice the triangle ABC where A (9 , 5) , B (6 , 1) and C (2 , 2) :

1. Draw Δ ÀB̀C̀ which is the image of Δ ABC by reflection in the X-axis
2. Draw Δ À̀B̀̀C̀̀ which is the image of Δ ÀB̀C̀ by reflection in the y-axis

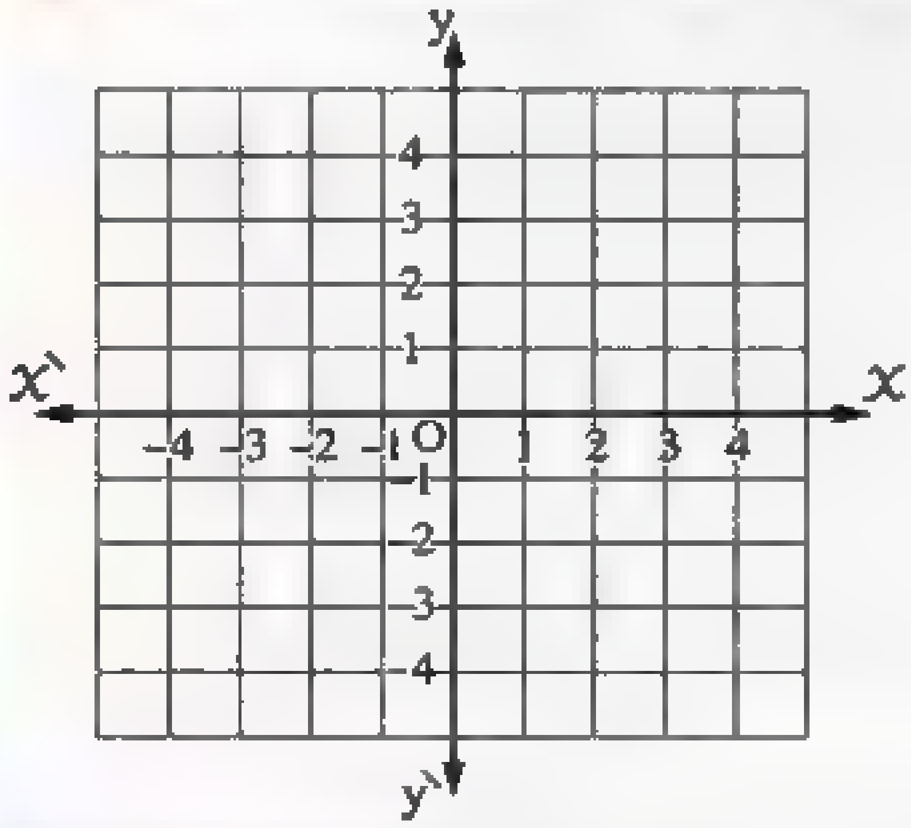
Solution

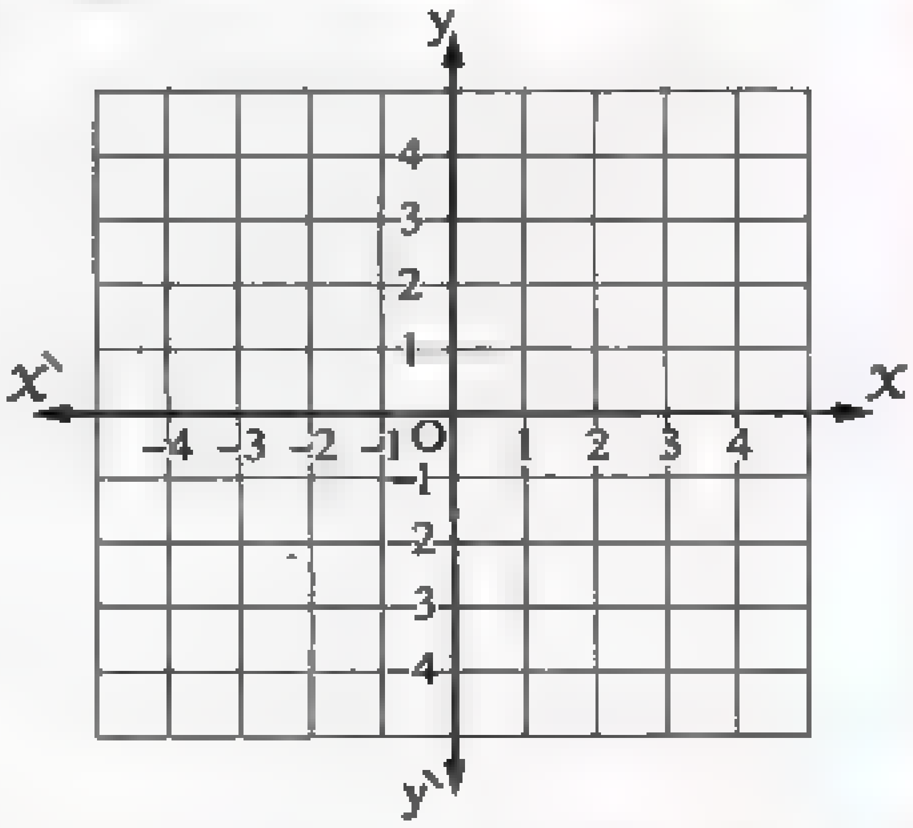


Try by yourself

Draw Δ ABC where A (1 , 1) , B (4 , 1) and C (3 , 3) , then draw its image by reflection in :

1 The X-axis **2 The y-axis**

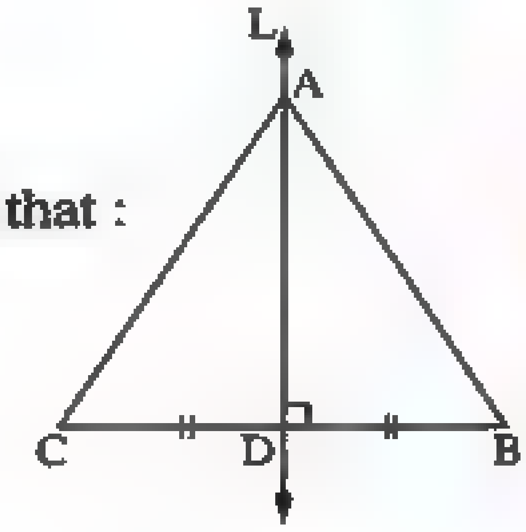




Unit 3

Symmetry

- **In the opposite figure :**



ABC is a triangle , $\overleftrightarrow{AD} \perp \overline{BC}$, D is the midpoint of $\overline{BC}$, we find that :

- The image of A by reflection in L is itself (A)
- The image of B by reflection in L is C
- The image of C by reflection in L is B

i.e. The image of Δ ABC by reflection in L is Δ ACB

We can say that Δ ABC is transformed to itself by reflection in the straight line L ,

Therefore the straight line L is called the axis of symmetry of Δ ABC

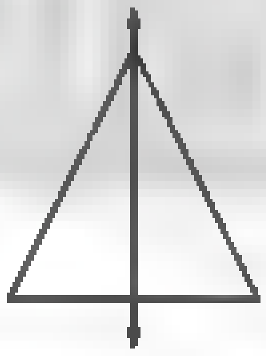
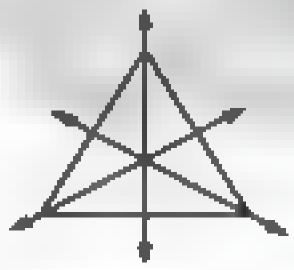
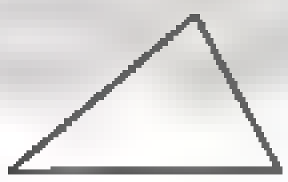
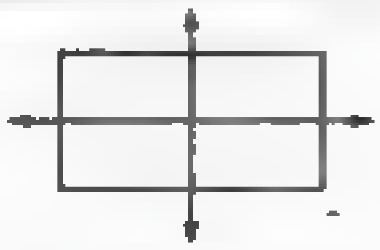
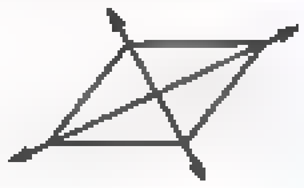
From the previous, we can deduce the definition of the axis of symmetry as follows :

If the reflection in a line transforms the figure to itself , then this line is called an axis of symmetry of the figure.

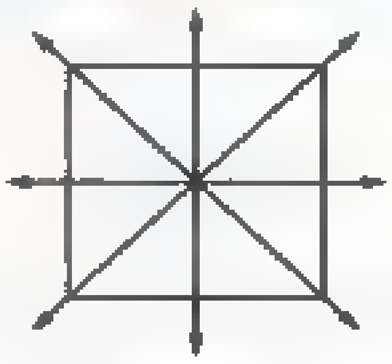
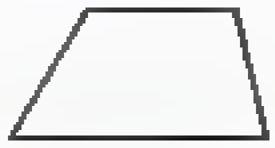
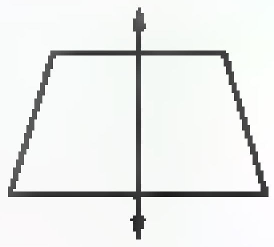
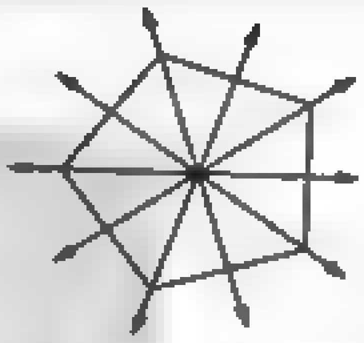
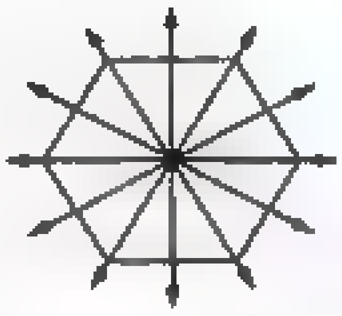
Remark

The axis of symmetry divides the figure into two congruent figures.

The axes of symmetry of some geometric figures

The figure	 An isosceles triangle	 An equilateral triangle	 Scalene
Number of axes of symmetry	1	3	Zero (does not exist)
The figure	 Parallelogram	 Rectangle	 Rhombus
Number of axes of symmetry	Zero (does not exist)	2	2

Lesson Nine

The figure	 Square	 Trapezium	 An isosceles trapezium
Number of axes of symmetry	4	Zero (does not exist)	1
The figure	 The circle	 The regular pentagon	 The regular hexagon
Number of axes of symmetry	An infinite number	5	6

Example

On a square lattice , determine the point A , B and C in each case , then find the image of Δ ABC by reflection in $\overleftrightarrow{XX'}$ (X-axis).

Mention if $\overleftrightarrow{XX'}$ is an axis of symmetry of Δ ABC or not :

1 A (1 , 3) , B (3 , 1) , C (0 , – 1)

2 A (1 , 2) , B (4 , 0) , C (1 , – 2)

Solution

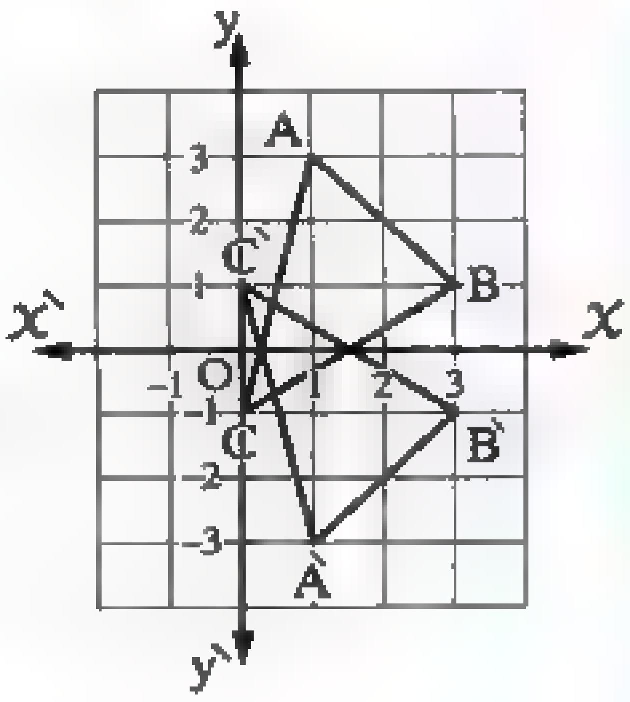
1 • A (1 , 3) $\longrightarrow$ A' (1 , – 3)

B (3 , 1) $\longrightarrow$ B' (3 , – 1)

C (0 , – 1) $\longrightarrow$ C' (0 , 1)

$\therefore$ $\Delta A'B'C'$ is the image of Δ ABC by reflection in X-axis

• $\overleftrightarrow{XX'}$ is not an axis of symmetry of Δ ABC because it does not transform the figure ABC to itself by reflection.



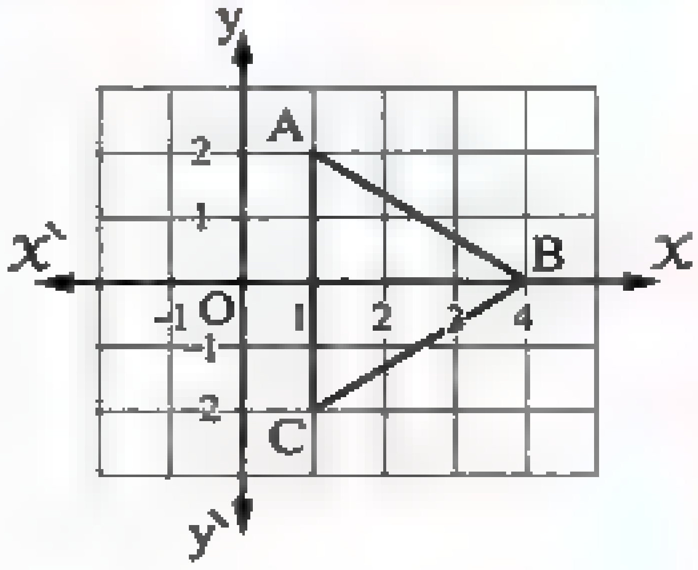
2 • A (1 , 2) ⟶ C (1 , −2)

B (4 , 0) ⟶ B (4 , 0) itself

C (1 , −2) ⟶ A (1 , 2)

∴ Δ CBA is the image of Δ ABC by reflection in $\overleftrightarrow{XX'}$ (X-axis)

• $\overleftrightarrow{XX'}$ is the axis of symmetry of Δ ABC because it transforms Δ ABC by reflection in it to itself.



Try by yourself

In the opposite figure :

ABC is an equilateral triangle. X , Y and Z are the midpoints of its sides.

Complete the following :

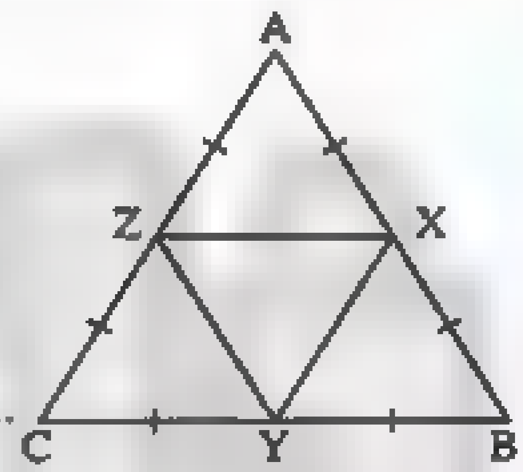
1 The image of Δ AXZ by reflection in $\overleftrightarrow{XZ}$ is ....

2 The image of the figure AZYX by reflection in $\overleftrightarrow{AY}$ is ....

3 Δ ABC is the image of Δ ACB by reflection in ....

4 The number of axes of symmetry of the figure ABYZ is ....

5 The number of axes of symmetry of Δ ABC is ....



Lesson Follow : Reflection (Reflection in a point)

Reflection in a point M maps each point A in the plane to the point A' in the same plane where M is the midpoint of the line segment $\overline{AA'}$, the point M is called the centre of reflection and the image of M by reflection in M is itself.

Finding the image of a point by reflection in a given point

- **To find the image of a point as A by reflection in M , we do as follows :**

1. Draw $\overrightarrow{AM}$
2. Using the compasses with open length equals MA , then use M as a centre and draw an arc to intersect $\overrightarrow{AM}$ at a point as A' , then A' is the image of the point A by reflection in the point M

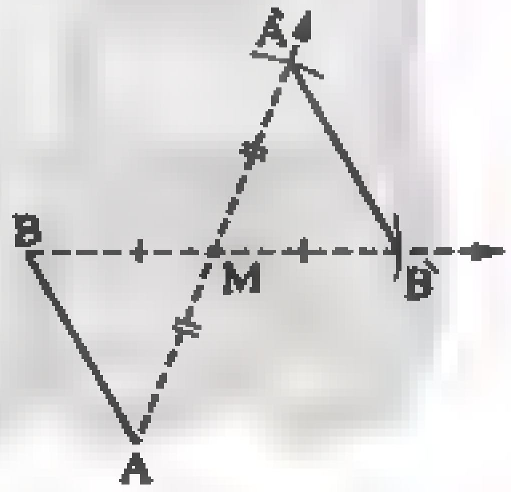
3. From the previous , we found that : $MA = MA'$

Finding the image of a line segment by reflection in a given point

- **To find the image of a line segment as $\overline{AB}$ by reflection in M , we do as follows :**

1. Find the image of A by reflection in M to be A' as we mentioned before.
2. Similarly find the image of B by reflection in M to be B'
3. Draw $\overline{A'B'}$ to be the image of $\overline{AB}$ by reflection in the point M

Notice that : $A'B' = AB$ and $\overline{A'B'} \,/\!/\, \overline{AB}$



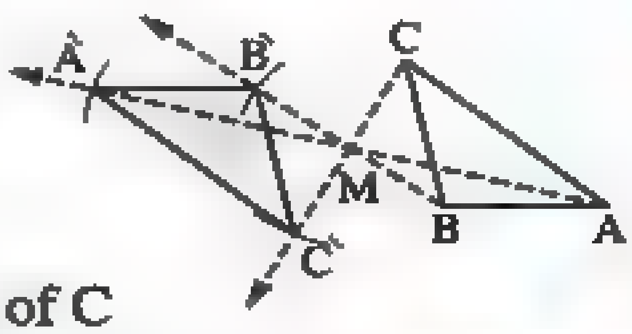
i.e.: The image of a line segment by reflection in a point is a line segment parallel to the original one and its length equals the length of the original line segment.

Finding the image of a polygon by reflection in a given point

- **To find the image of a polygon as the triangle ABC by reflection in M , we do as follows :**

1. Find the image of each vertex of the vertices of the triangle ABC by reflection in the point M as we mentioned before to be :

   A' is the image of A , B' is the image of B and C' is the image of C



2 Draw $\overline{\grave{A}\grave{B}}$, $\overline{\grave{B}\grave{C}}$ and $\overline{\grave{C}\grave{A}}$ to get $\Delta\, \grave{A}\grave{B}\grave{C}$ which is the image of Δ ABC by reflection in the point M

Notice that :

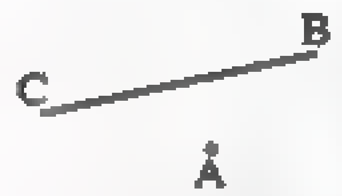
$\Delta ABC \equiv \Delta\, \grave{A}\grave{B}\grave{C}$, therefore it is said that the reflection in a point is **isometric**.

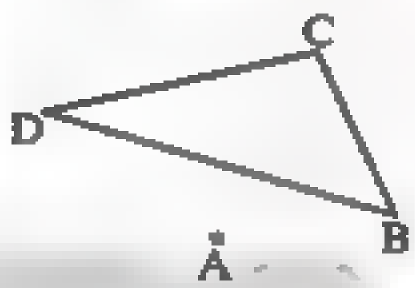
From the previous, we deduce that :

Reflection in a point is a geometric transformation that maps the geometric figure to another geometric figure congruent to it and has the same orientation of its vertices.

Try by yourself

In each of the following , map the image of each figure by reflection in the point A :



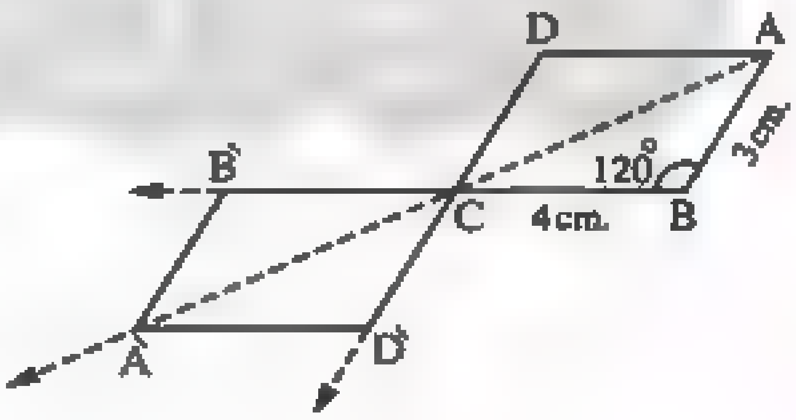


Properties of reflection in a point

Illustrated example

Draw the parallelogram ABCD in which AB = 3 cm. , BC = 4 cm. and m ($\angle$ B) = 120° , then draw its image by reflection in the point C and show what you observe.

Solution



Finding the image of each vertex of the vertices of ▱ ABCD by reflection in the point C , we find the image of ▱ ABCD by reflection in the point C is ▱ $\grave{A}\grave{B}C\grave{D}$

Notice that :

1 $\grave{A}\grave{B}$ = AB , $\grave{B}C$ = BC , $C\grave{D}$ = CD and $\grave{D}\grave{A}$ = DA

i.e. Reflection in a point reserves the lengths of the line segments.

2 m ($\angle \grave{A}$) = m ($\angle$ A) , m ($\angle \grave{B}$) = m ($\angle$ B),

m ($\angle \grave{B}C\grave{D}$) = m ($\angle$ BCD) and m ($\angle \grave{D}$) = m ($\angle$ D)

i.e. Reflection in a point reserves the measures of angles.

3 **From the parallelogram ABCD : $\overline{AB} \parallel \overline{DC}$,**

From the parallelogram $\grave{A}\grave{B}\grave{C}\grave{D}$: $\overline{\grave{A}\grave{B}} \parallel \overline{\grave{D}\grave{C}}$

$\therefore$ The images of the two parallel line segments are also two parallel line segments.

i.e. Reflection in a point reserves parallelism.

4 The reading of the parallelogram ABCD is in the clockwise direction and the reading of the parallelogram $\grave{A}\grave{B}\grave{C}\grave{D}$ is in the clockwise direction also.

i.e. Reflection in a point reserves the orientation of vertices of the figure.

5 Putting a point belongs to $\overline{AB}$, we find its image by reflection in C belongs to $\overline{\grave{A}\grave{B}}$

i.e. Reflection in a point reserves the betweenness.

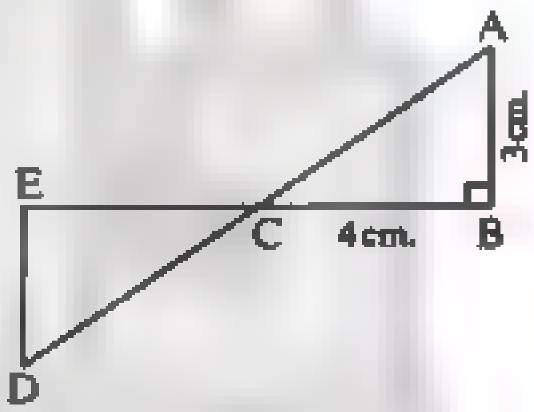
Example (1)

In the opposite figure :

ABC is a triangle in which m ($\angle$ B) = 90° ,

AB = 3 cm. , BC = 4 cm. and Δ DEC

is the image of Δ ABC by reflection in C



1 **Find :** the length of $\overline{DC}$

2 **Prove that :** $\overline{AB} \parallel \overline{DE}$

Solution

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore$ $(AC)^2 = (AB)^2 + (BC)^2 = (3)^2 + (4)^2 = 25$ (Pythagoras' theorem)

$\therefore$ AC = 5 cm.

, $\because$ Δ DEC is the image of Δ ABC by reflection in C

$\therefore$ DC = AC = 5 cm. (The reflection reserves the lengths of the line segments) (First req.)

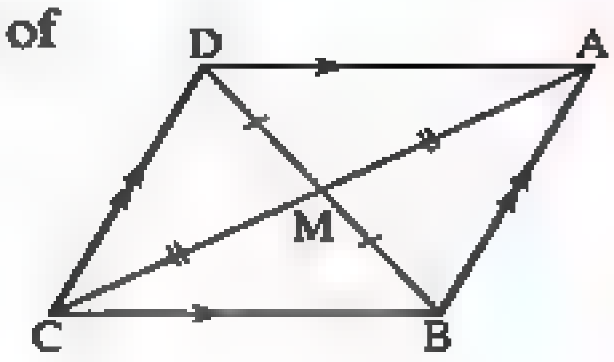
, $\because$ m ($\angle$ E) = m ($\angle$ B) = 90° (Properties of reflection in a point) and they are alternate angles

$\therefore$ $\overline{AB} \parallel \overline{DE}$ (Second req.)

Unit 3

Using reflection in a point to prove that a quadrilateral is a parallelogram

- We mentioned before that the image of a line segment by reflection in a point is a line segment parallel to it and has the same length of the main line segment.



If $\overline{CD}$ is the image of $\overline{AB}$ by reflection in the point M , then $\overline{AB} \,/\!/\, \overline{DC}$, AB = DC

We can prove that the quadrilateral is a parallelogram by several methods as follows :

1 $\because$ AB = DC and $\overline{AB} \,/\!/\, \overline{DC}$

$\therefore$ The quadrilateral ABCD is a parallelogram.

Remember that : The quadrilateral in which two opposite sides are parallel and equal in length is a parallelogram.

2 $\because$ $\overline{CD}$ is the image of $\overline{AB}$ by reflection in the point M

$\therefore$ MA = MC and MB = MD

$\therefore$ The quadrilateral ABCD is a parallelogram.

Remember that : The quadrilateral whose diagonals bisect each other is a parallelogram.

3 $\because$ MA = MC and MB = MD

$\therefore$ $\overline{AD}$ is the image of $\overline{CB}$ by reflection in the point M

, $\because$ $\overline{CD}$ is the image of $\overline{AB}$ by reflection in the point M

$\therefore$ $\overline{AB} \,/\!/\, \overline{DC}$ and $\overline{AD} \,/\!/\, \overline{BC}$

$\therefore$ The quadrilateral ABCD is a parallelogram.

Remember that : The quadrilateral in which each two opposite sides are parallel is a parallelogram.

4 $\because$ $\overline{AD}$ is the image of $\overline{CB}$ and $\overline{CD}$ is the image of $\overline{AB}$ by reflection in the point M

$\therefore$ AD = CB , CD = AB

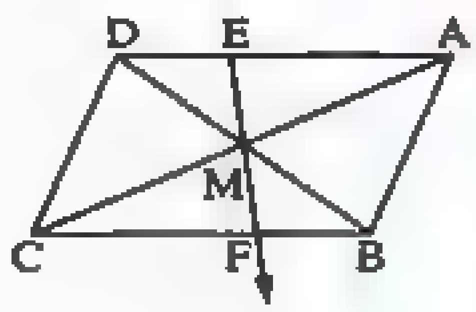
$\therefore$ The quadrilateral ABCD is a parallelogram.

Remember that : The quadrilateral in which each two opposite sides are equal in length is a parallelogram.

Example 2

In the opposite figure :

ABCD is a parallelogram , M is the point of intersection of its diagonals , E $\in \overline{AD}$ and $\overrightarrow{EM} \cap \overline{BC} = \{F\}$

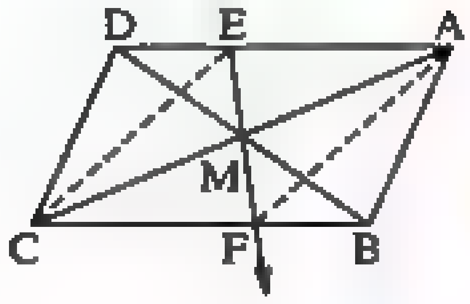


Prove that :

1 F is the image of E by reflection in M

2 The quadrilateral AFCE is a parallelogram.

Solution



$\because$ ABCD is a parallelogram. $\therefore \overline{AD} \parallel \overline{BC}$

$\therefore$ In $\Delta\Delta$ AME and CMF :

$\begin{cases} m(\angle DAC) = m(\angle BCA) \text{ (alternate angles)} \\ m(\angle AME) = m(\angle FMC) \text{ (V.O.A.)} \\ AM = MC \text{ (properties of parallelogram)} \end{cases}$

$\therefore \Delta AME \equiv \Delta CMF$, then we deduce that EM = FM

, $\because F \in \overrightarrow{EM}$

$\therefore$ F is the image of E by reflection in the point M (Q.E.D. 1)

, $\because$ AM = CM and $A \in \overrightarrow{CM}$

$\therefore$ A is the image of C by reflection in the point M

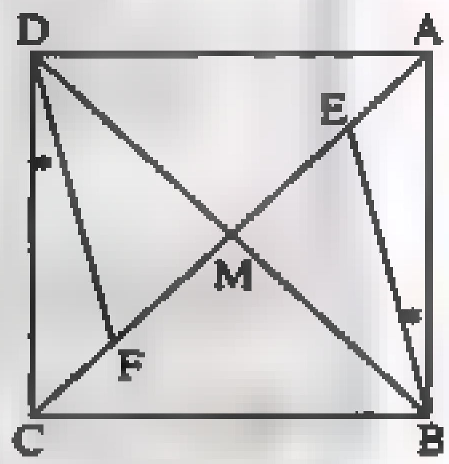
$\therefore \overline{AF}$ is the image of $\overline{CE}$ by reflection in the point M

$\therefore$ AF = CE and $\overline{AF} \parallel \overline{CE}$

$\therefore$ The quadrilateral AFCE is a parallelogram. (Q.E.D. 2)

Example

In the opposite figure :



ABCD is a square whose diagonals intersect at M ,

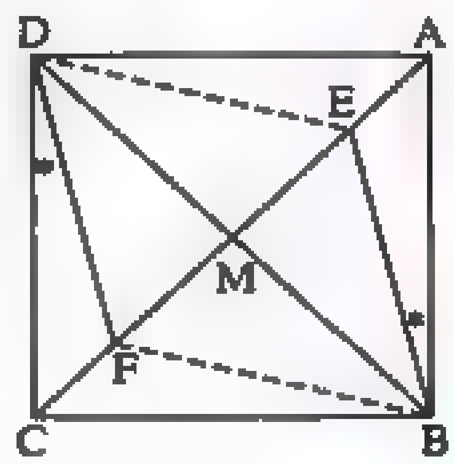
$E \in \overline{AC}$ and $F \in \overline{AC}$ where $m(\angle ABE) = m(\angle CDF)$

Prove that :

1 Δ ABE is the image of Δ CDF by reflection in M

2 The quadrilateral EBFD is a rhombus.

Solution



In $\Delta\Delta$ ABE and CDF :

$\begin{cases} AB = CD \text{ (properties of the square)} \\ m(\angle BAE) = m(\angle DCF) = 45^\circ \text{ (properties of the square)} \\ m(\angle ABE) = m(\angle CDF) \text{ (given)} \end{cases}$

$\therefore \Delta ABE \equiv \Delta CDF$, then we deduce that : AE = CF

, $\because$ AM = CM (properties of the square) $\therefore$ AM – AE = CM – CF

$\therefore$ EM = FM , $E \in \overrightarrow{FM}$ $\therefore$ E is the image of F by reflection in M (1)

, AM = CM , $A \in \overrightarrow{CM}$ $\therefore$ A is the image of C by reflection in M

, BM = DM , $B \in \overrightarrow{DM}$ $\therefore$ B is the image of D by reflection in M (2)

∴ Δ ABE is the image of Δ CDF by reflection in M (Q.E.D. 1)

From (1) and (2) :

∴ $\overline{EB}$ is the image of $\overline{FD}$ by reflection in M ∴ $\overline{EB}$ // $\overline{FD}$ and EB = FD

∴ The quadrilateral EBFD is a parallelogram.

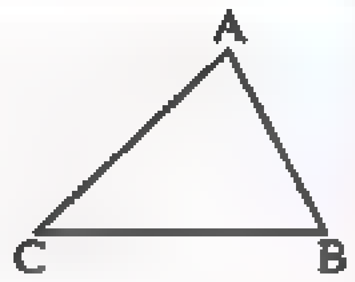
, ∵ $\overline{DB} \perp \overline{EF}$ (properties of the square ABCD)

∴ The quadrilateral EBFD is a rhombus. (Q.E.D. 2)

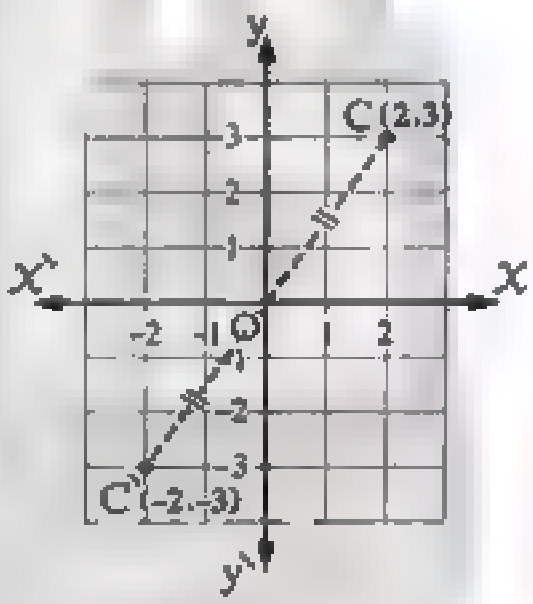
Try by yourself

In the following figure :

Draw Δ $\grave{A}\grave{B}C$ as the image of Δ ABC by reflection in C , then prove that the quadrilateral $AB\grave{A}\grave{B}$ is a parallelogram.



Reflection in the origin point



- If the point C is a point in the Cartesian coordinates plane where C (2 , 3)
- To find the image of the point C by reflection in the origin point O using the same previous method , we will find it $\grave{C}$ (– 2 , – 3)
- Notice that :

The signs of the two projections of the ordered pair (2 , 3) have been changed , hence we can define the reflection in the origin point as follows :

Definition :

If A (x , y) is a point in the Cartesian coordinates plane , then the image of the point A by reflection in the origin point O is $\grave{A}$ (– x , – y)

i.e. Reflection in the origin point converts the signs of each of the coordinates of the point.

∴ **The image of the point (x , y) $\xrightarrow[\text{the origin point}]{\text{by reflection in}}$ (– x , – y)**

For example :

- The image of the point $(2 , 3) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-2 , -3)$
- The image of the point $(-4 , 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (4 , -1)$
- The image of the point $(3 , -5) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-3 , 5)$
- The image of the point $(-1 , -3) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (1 , 3)$

Remark

The image of the point (0 , 0) by reflection in the origin point is itself.

Example (4)

Draw ΔABC where A (4 , 1) , B (2 , 4) and C (– 1 , 3) , then map its image by reflection in the origin point.

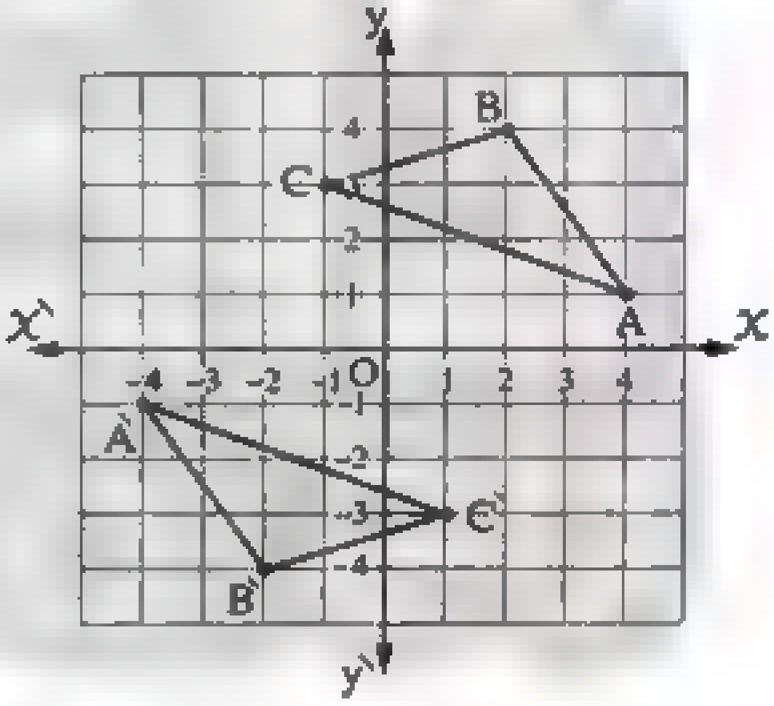
Solution

$\because (x , y) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-x , -y)$

$\therefore A (4 , 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{A} (-4 , -1)$

$, B (2 , 4) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{B} (-2 , -4)$

$, C (-1 , 3) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{C} (1 , -3)$



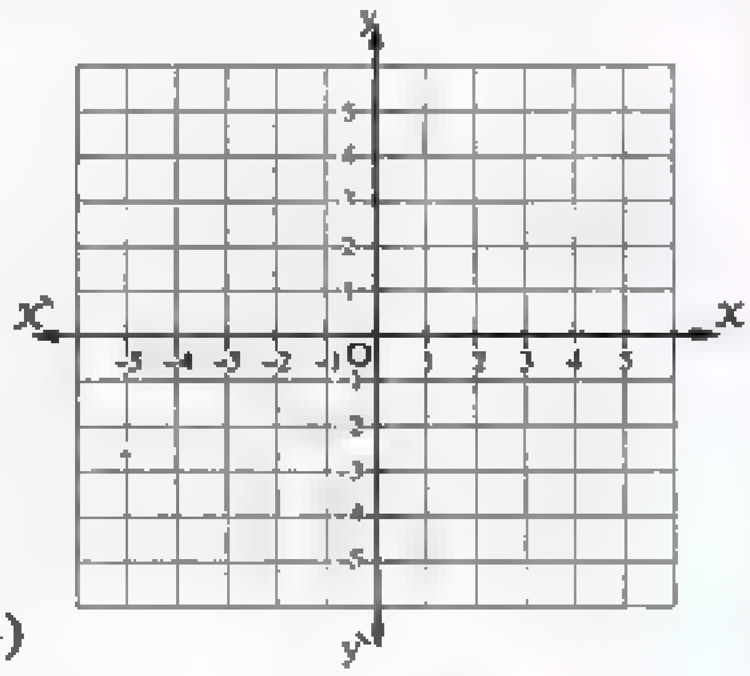
Try by yourself

Draw on a square lattice ΔABC , where A (– 2 , 1) , B (4 , – 2) and C (2 , 3) , then draw its image by reflection in the origin point

$A (-2 , 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{A}$ (.......... , ...)

$, B (4 , -2) \xrightarrow[\text{.....................}]{\text{.....................}} \grave{B}$ (.......... , ...)

, C (.......... , ...) $\xrightarrow[\text{.....................}]{\text{.....................}} \grave{C}$ (.......... , ...)



Unit 3

Lesson Translation

Prelude

If a car moved a distance 25 metres in a straight line forward , then we say that :

The car translated for a distance 25 metres forward.

***i.e.* To determine the new position of the car after its movement , we should know two important elements which are :**

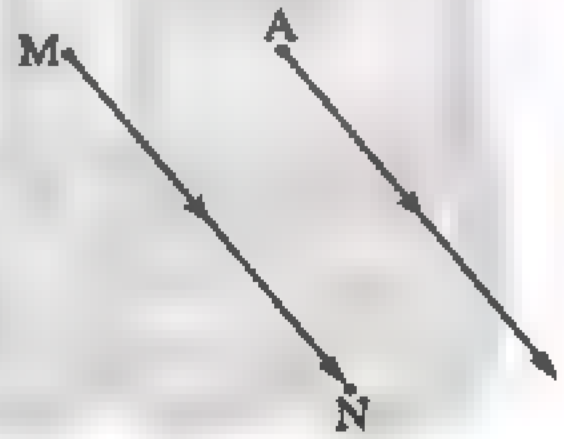
1. The magnitude of the translation (25 metres).
2. The direction of the translation (forward in a straight line).

According to this, we can say that :

Translation is a geometrical transformation which maps each point A in the plane to another point $\grave{A}$ in the same plane with a constant distance in a certain direction.

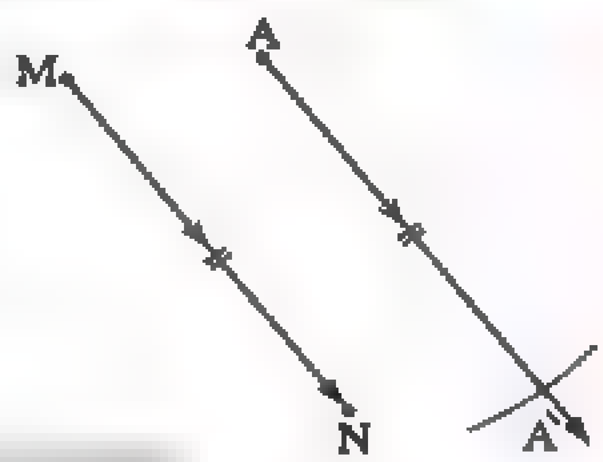
Translation in the plane

Finding the image of a point by a given translation



- **To find $\grave{A}$ which is the image of A by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :**

1. Draw from A a ray parallel to $\overrightarrow{MN}$ and in the same direction.
2. By the compasses in A as a centre with radius = MN , draw an arc to intersect the ray drawn from A at the point $\grave{A}$ ($A\grave{A}$ = MN and $\overrightarrow{A\grave{A}} \parallel \overrightarrow{MN}$)



- **Then $\grave{A}$ is the image of A by translation of magnitude MN in the direction of $\overrightarrow{MN}$**

Finding the image of a line segment by a given translation

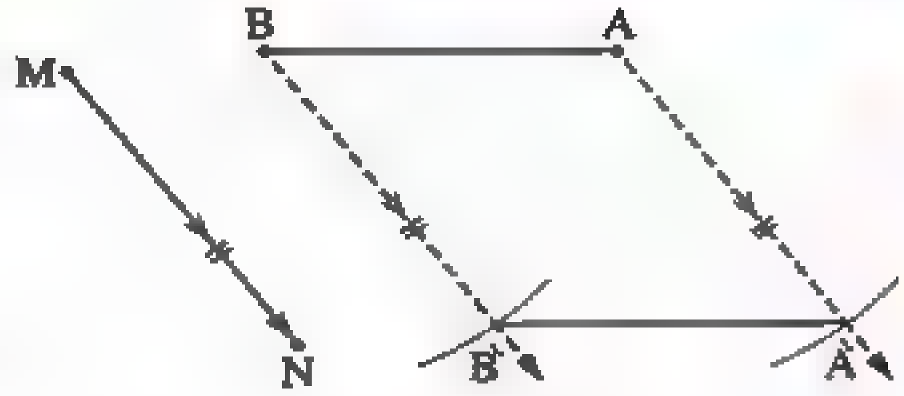
- **To find the image of $\overline{AB}$ by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :**

1. Find the image of the point A by translation MN in the direction of $\overrightarrow{MN}$ as we mentioned before , say $\grave{A}$

2 Similarly , we find the image of the point B by translation MN in the direction of $\overrightarrow{MN}$, say $\grave{B}$

3 Draw $\overline{\grave{A}\grave{B}}$ to be the image of $\overline{AB}$ by translation MN in the direction of $\overrightarrow{MN}$

Check that : $AB = \grave{A}\grave{B}$ and $\overline{AB} \parallel \overline{\grave{A}\grave{B}}$



Finding the image of a polygon by a given translation

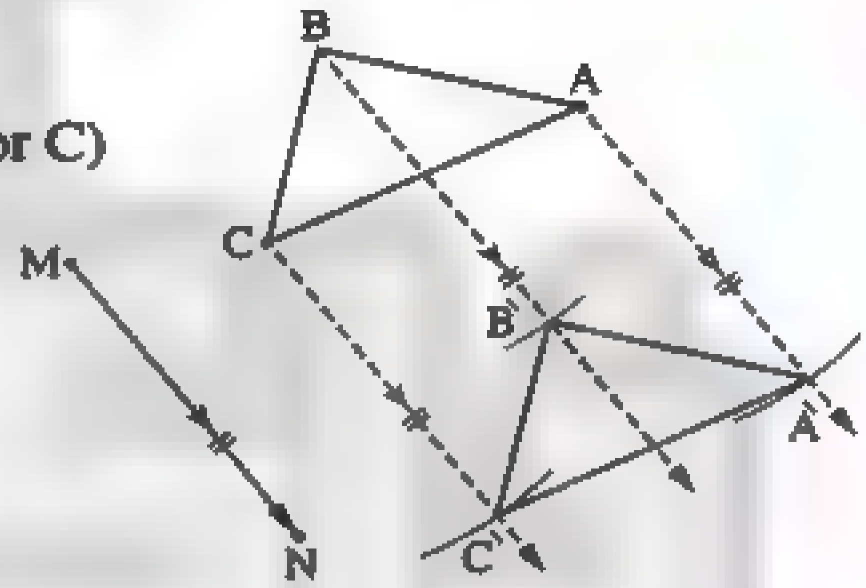
- **To find the image of a polygon as Δ ABC by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :**

1 Find the image of each vertex of the vertices of Δ ABC by translation MN in the direction of $\overrightarrow{MN}$ as we mentioned before (say $\grave{A}$ for A , $\grave{B}$ for B and $\grave{C}$ for C)

2 Draw $\overline{\grave{A}\grave{B}}$, $\overline{\grave{B}\grave{C}}$ and $\overline{\grave{C}\grave{A}}$ then Δ $\grave{A}\grave{B}\grave{C}$ is the image of Δ ABC by translation MN in the direction of $\overrightarrow{MN}$

Check that :

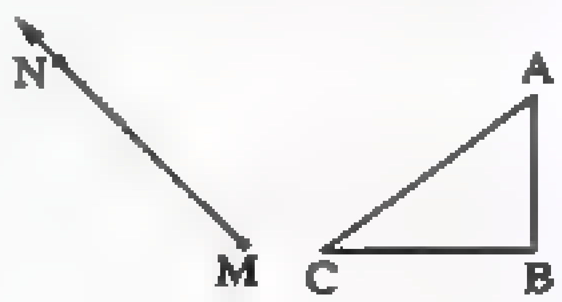
- $AB = \grave{A}\grave{B}$, $BC = \grave{B}\grave{C}$ and $CA = \grave{C}\grave{A}$
- $m(\angle A) = m(\angle \grave{A})$, $m(\angle B) = m(\angle \grave{B})$, $m(\angle C) = m(\angle \grave{C})$



From the previous , we deduce that translation is a geometrical transformation which maps the geometrical figure to another geometrical figure congruent to it.

Try by yourself

Using the geometrical tools , draw the image of Δ ABC by translation MN in the direction of $\overrightarrow{MN}$ as shown:



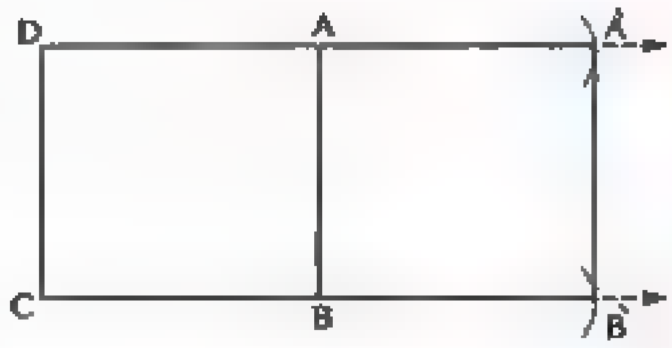
Unit 3

Properties of translation

Illustrated example

Draw the square ABCD whose side length is 4 cm., then draw its image by translation AB in the direction $\overrightarrow{DA}$

Solution



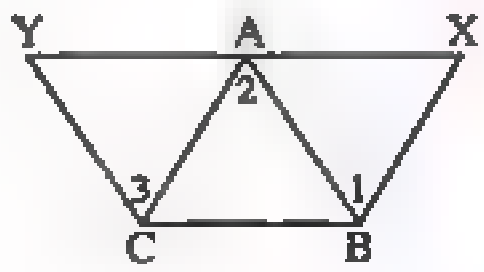
The square $\grave{A}\grave{B}BA$ is the image of the square ABCD by translation AB in the direction $\overrightarrow{DA}$

Notice that :

1. $\grave{A}\grave{B} = AB$, $AB = DC$
   i.e. Translation reserves the lengths of the line segments.
2. $m(\angle \grave{A}) = m(\angle BAD)$, $m(\angle \grave{B}) = m(\angle CBA)$
   i.e. Translation reserves the measures of angles.
3. From the square ABCD : $\overline{AB} \parallel \overline{DC}$, from the square $\grave{A}\grave{B}BA$: $\overline{\grave{A}\grave{B}} \parallel \overline{AB}$
   $\therefore$ The images of the two parallel line segments are also two parallel line segment.
   i.e. Translation reserves the parallelism.
4. The reading of the square ABCD is in the clockwise direction and the reading of the square $\grave{A}\grave{B}BA$ is in the clockwise direction also.
   i.e. Translation reserves the orientation of vertices of the figure.
5. If you take a point lies on $\overline{AB}$ and find its image by the previous translation , you will find its image lies on $\overline{\grave{A}\grave{B}}$
   i.e. Translation reserves the betweenness.

Example 1

In the opposite figure :



ABC is a triangle, X is the image of B by translation of a distance CA in the direction of $\overrightarrow{CA}$,
Y is the image of C by translation of a distance BA in the direction of $\overrightarrow{BA}$

1. **Prove that : $\Delta XAB \equiv \Delta AYC$**
2. Determine the translation which makes ΔAYC the image of ΔXAB

Solution

$\because$ X is the image of B by translation of a distance CA in the direction $\overrightarrow{CA}$

$\therefore$ BX = CA and $\overline{BX}$ // $\overline{CA}$

$\therefore$ m ($\angle$ 1) = m ($\angle$ 2) (alternate angles) (1)

, $\because$ Y is the image of C by translation of a distance BA in the direction $\overrightarrow{BA}$

$\therefore$ CY = BA , $\overline{CY}$ // $\overline{BA}$

$\therefore$ m ($\angle$ 2) = m ($\angle$ 3) (alternate angles) (2)

From (1) and (2) : We deduce that : m ($\angle$ 1) = m ($\angle$ 3)

$\therefore$ In $\Delta\Delta$ XAB , AYC :

$\left\{\begin{array}{l}\text{BX = CA (properties of translation)}\\ \text{BA = CY (properties of translation)}\\ \text{m}(\angle 1) = \text{m}(\angle 3)\ \text{(proved)}\end{array}\right.$

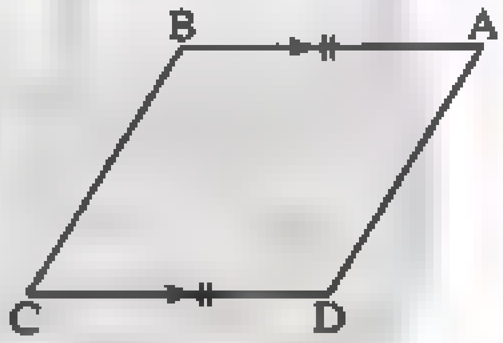
$\therefore$ Δ XAB $\equiv$ Δ AYC (First req.)

The translation which makes Δ AYC is the image of Δ XAB is the translation of a distance BC in the direction of $\overrightarrow{BC}$ (Second req.)

Using the properties of translation to prove that the figure is a parallelogram :

We noticed that :

The image of a line segment by a translation is another line segment parallel to it and has the same length of the original line segment.



For example :

In the opposite figure :

If $\overline{AB}$ is the image of $\overline{DC}$ by a translation,

then : $\overline{AB}$ // $\overline{DC}$ and AB = DC

i.e. The figure ABCD is a parallelogram.

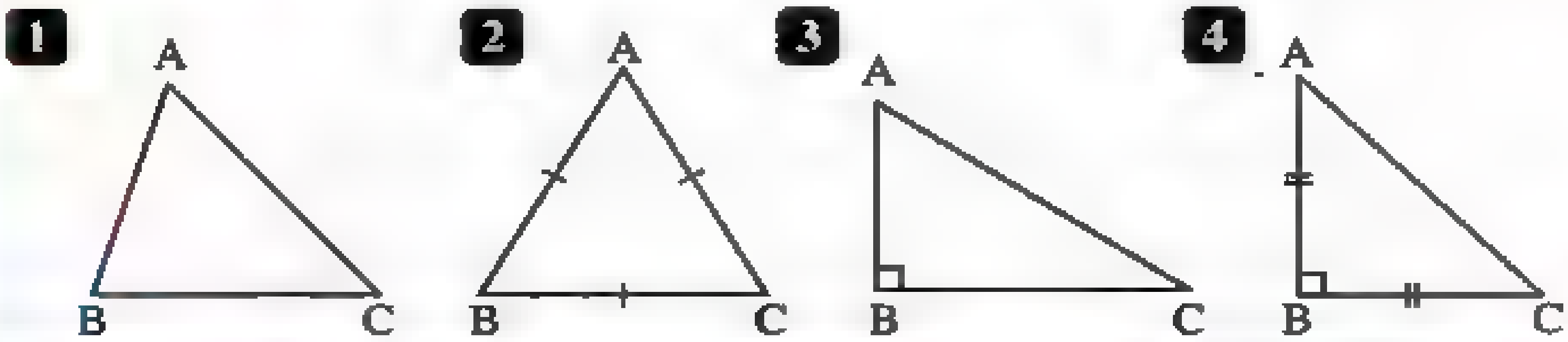
Remember that :

The parallelogram is a quadrilateral in which two opposite sides are parallel and equal in length.

Example 2

In each of the following figures :

Draw Δ À̀B̀B as the image of Δ ABC by translation of a distance CB in the direction of $\overrightarrow{CB}$ then show the kind of the figure ABB̀À in each case :



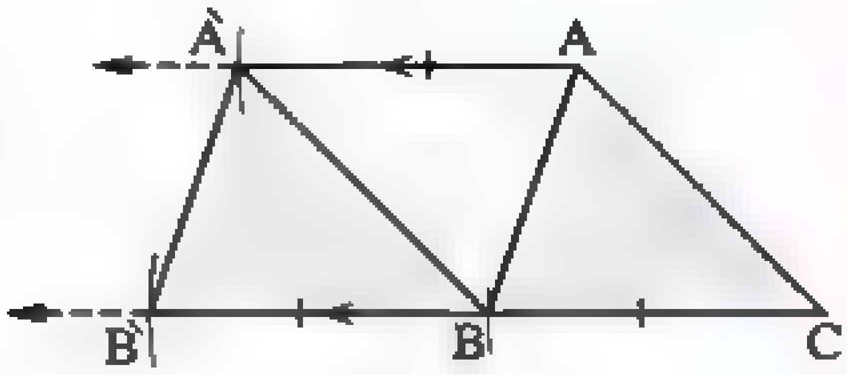
Solution

1 $\because \Delta \grave{A}\grave{B}B$ is the image of ΔABC

$\therefore \overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$

$\therefore \overline{\grave{A}\grave{B}} \parallel \overline{AB}$, $\grave{A}\grave{B} = AB$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a parallelogram.



2 $\because \Delta \grave{A}\grave{B}B$ is the image of ΔABC

$\therefore \overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$

$\therefore \overline{\grave{A}\grave{B}} \parallel \overline{AB}$, $\grave{A}\grave{B} = AB$

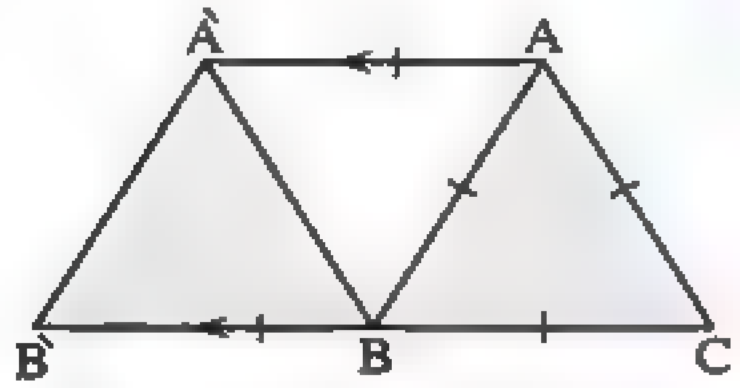
$\therefore$ The figure $AB\grave{B}\grave{A}$ is a parallelogram.

, $\because \Delta ABC \equiv \Delta \grave{A}\grave{B}B$

$\therefore \Delta \grave{A}\grave{B}B$ is an equilateral triangle

$\therefore \grave{A}\grave{B} = \grave{B}B$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a rhombus.



Remember that :

The rhombus is a parallelogram in which two adjacent sides are equal in length.

3 $\because \Delta \grave{A}\grave{B}B$ is the image of ΔABC

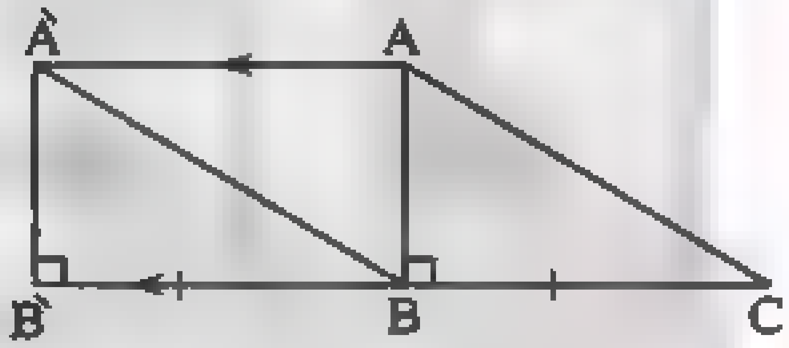
$\therefore \overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$

$\therefore \overline{\grave{A}\grave{B}} \parallel \overline{AB}$, $\grave{A}\grave{B} = AB$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a parallelogram.

, $\because m(\angle \grave{B}) = m(\angle B) = 90°$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a rectangle.



Remember that :

The rectangle is a parallelogram one of its angles is right.

4 $\because \Delta \grave{A}\grave{B}B$ is the image of ΔABC

$\therefore \overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$

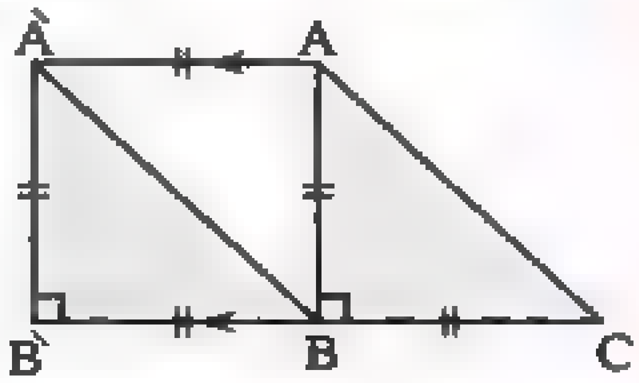
$\therefore \overline{\grave{A}\grave{B}} \parallel \overline{AB}$, $\grave{A}\grave{B} = AB$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a parallelogram.

$\because m(\angle \grave{B}) = m(\angle B) = 90°$

, $\grave{A}\grave{B} = \grave{B}B$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a square.



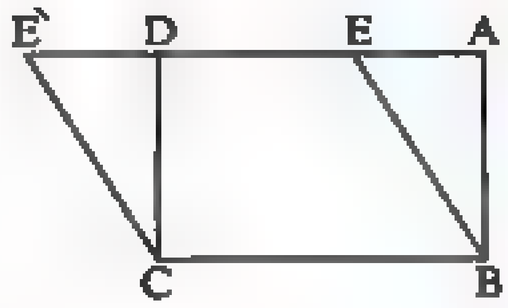
Remember that :

The square is a parallelogram in which one of its angles is right and two adjacent sides are equal in length.

Example (3)

Draw the rectangle ABCD. Take the point E $\in \overline{AD}$, then find the image of E by translation DA in the direction of $\overrightarrow{AD}$. Then prove that the figure EBCÈ is a parallelogram.

Solution



- We take È $\in \overrightarrow{AD}$ such that ÈE = AD , then È is the image of E by translation DA in the direction of $\overrightarrow{AD}$

$\because$ ABCD is a rectangle.

$\therefore$ $\overline{AD}$ // $\overline{BC}$ and AD = BC

$\therefore$ C is the image of B by translation AD in the direction of $\overrightarrow{AD}$

, $\because$ È is the image of E by the same translation.

$\therefore$ $\overline{ÈC}$ is the image of $\overline{EB}$ by translation AD in the direction of $\overrightarrow{AD}$

$\therefore$ EB = ÈC and $\overline{EB}$ // $\overline{ÈC}$

$\therefore$ The figure EBCÈ is a parallelogram.

Try by yourself

In the following figure :

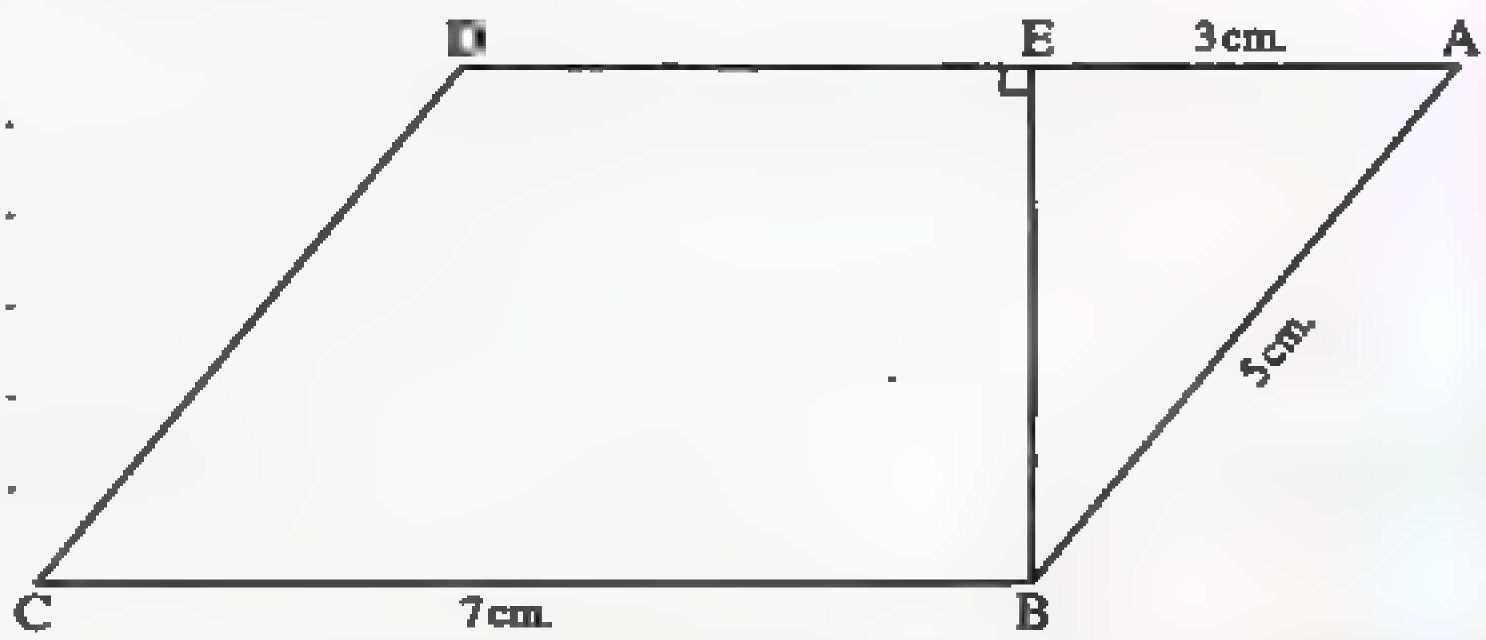
ABCD is a parallelogram in which AB = 5 cm. , BC = 7 cm. and E $\in \overline{AD}$ where AE = 3 cm.

If $\overline{BE} \perp \overline{AD}$, draw Δ DCÈ as the image of Δ ABE by translation of a distance 7 cm. in the direction of $\overrightarrow{AD}$:

1. Prove that the figure EBCÈ is a rectangle.
2. Determine the distance and the direction of the translation which transforms $\overline{BC}$ to $\overline{EÈ}$

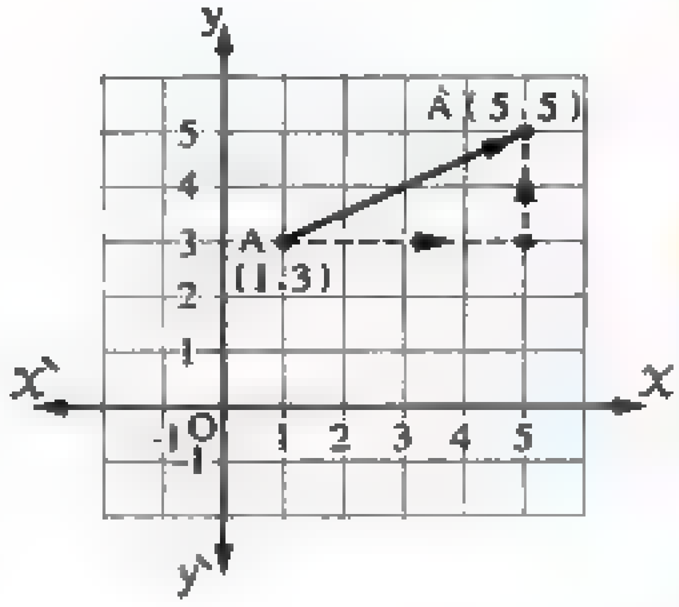
Solution

....
....
....
....
....



Unit 3

Translation in the Cartesian plane



If A (1 , 3) is a point in the orthogonal coordinates plane and to find its image À by translation with magnitude 4 length units in the direction of $\overrightarrow{OX}$ followed by a translation with magnitude 2 length units in the direction of $\overrightarrow{OY}$

From the graph , we get À to be the point (5 , 5)

i.e. À (1 + 4 , 3 + 2)

According to this :

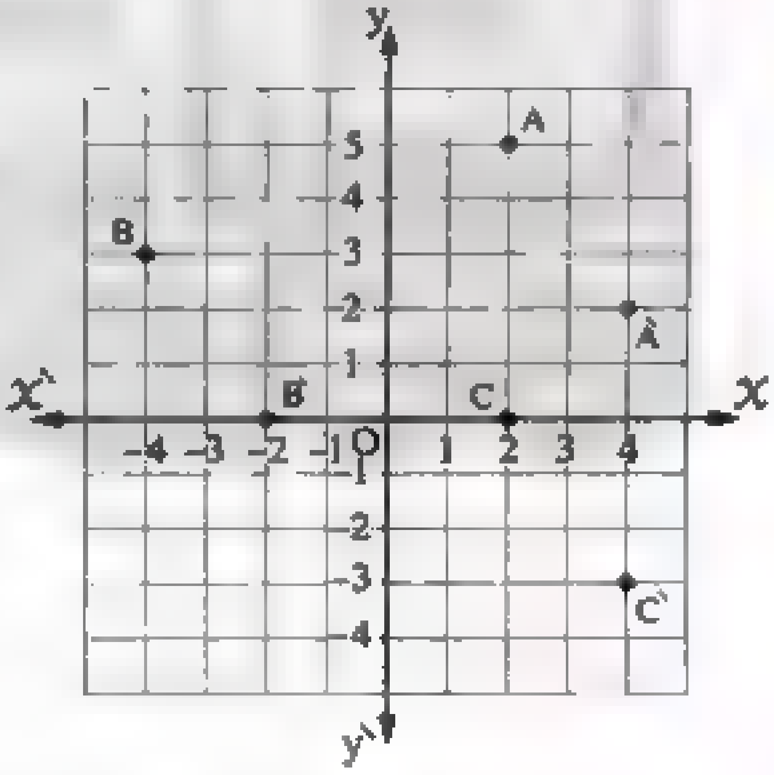
Translation in the orthogonal Cartesian coordinates plane transforms each point by a displacement a in the direction of the x-axis followed by a displacement b in the direction of the y-axis

i.e. The image of the point A (x , y) ⟶ the point À (x + a , y + b)

Example 4

Find the images of the points A (2 , 5) , B (– 4 , 3) and C (2 , 0) by translation (x , y) ⟶ (x + 2 , y – 3)

Solution



∵ (x , y) ⟶ (x + 2 , y – 3) , then :

- The image of A (2 , 5) is À (2 + 2 , 5 – 3)
  i.e. À (4 , 2)
- The image of B (– 4 , 3) is B̀ (– 4 + 2 , 3 – 3)
  i.e. B̀ (– 2 , 0)
- The image of C (2 , 0) is C̀ (2 + 2 , 0 – 3)
  i.e. C̀ (4 , – 3)

Notice that :

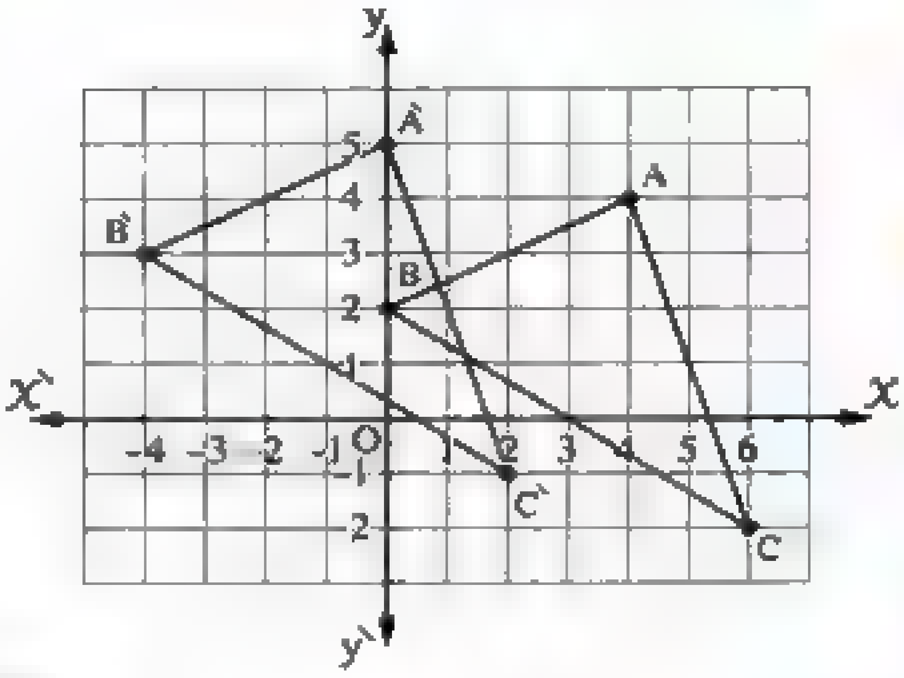
The translation (x , y) ⟶ (x + 2 , y – 3) transforms each point to another point by a right horizontal displacement of 2 units and a vertical displacement of 3 units downwards.

Example 5

Draw on a square lattice Δ ABC where A (4 , 4) , B (0 , 2) , C (6 , – 2) , then find its image by translation (x , y) ⟶ (x – 4 , y + 1)

Solution

The point	Its image by the translation
(x , y)	$(x - 4 , y + 1)$
A (4 , 4)	$\grave{A}$ (0 , 5)
B (0 , 2)	$\grave{B}$ (– 4 , 3)
C (6 , – 2)	$\grave{C}$ (2 , – 1)



$\therefore \Delta \grave{A}\grave{B}\grave{C}$ is the image of Δ ABC by translation

$(x , y) \longrightarrow (x - 4 , y + 1)$

Remark

The translation $(x , y) \longrightarrow (x + a , y + b)$ can be written as the translation (a , b) for example :
The translation $(x , y) \longrightarrow (x + 2 , y - 1)$ can be written as the translation (2 , – 1)

Try by yourself

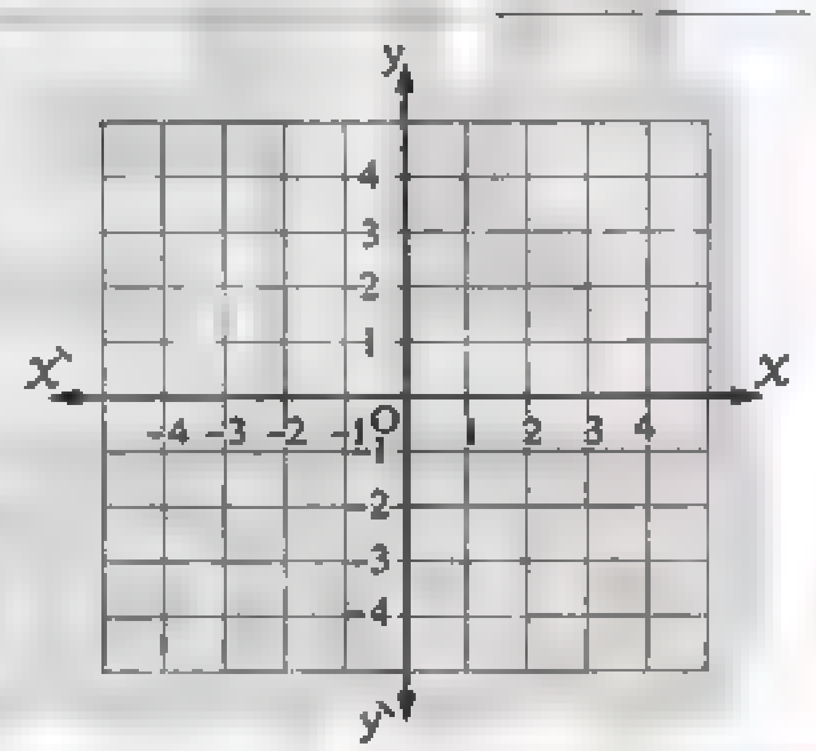
On a square lattice , draw Δ ABC where A (– 3 , 2) , B (– 1 , 1) , C (– 2 , 0) , then find its image by translation :

$(x , y) \longrightarrow (x + 2 , y + 1)$

$\grave{A}$ = (……… , ………)

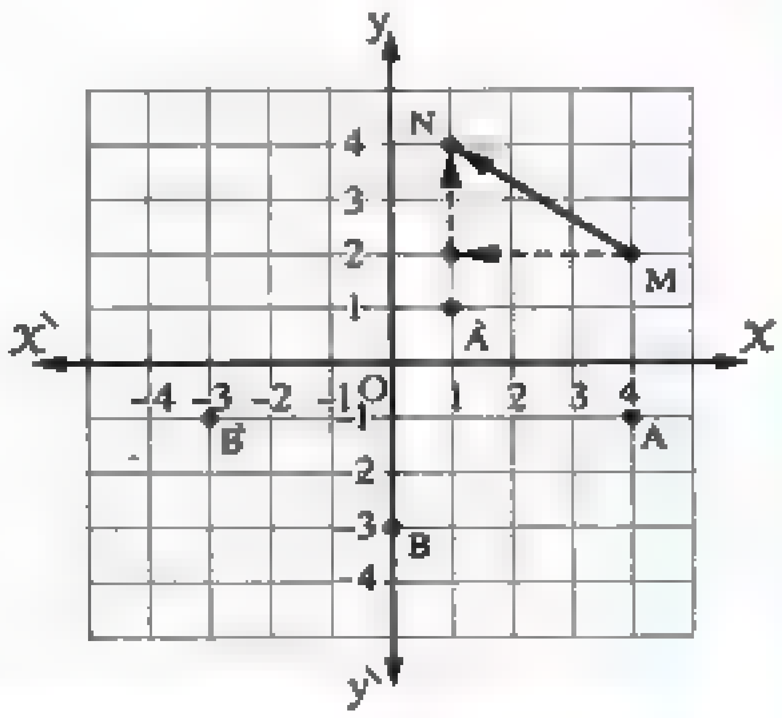
$\grave{B}$ = (……… , ………)

$\grave{C}$ = (……… , ………)



Example (6)

Find the image of each of the two points A (4 , – 1) and B (0 , – 3) by translation with magnitude MN in the direction of $\overrightarrow{MN}$ where M (4 , 2) and N (1 , 4)



Solution

By noticing the opposite graph , we find that the translation with magnitude MN in the direction of $\overrightarrow{MN}$ where M (4 , 2) and N (1 , 4) is equivalent to :

- Horizontal displacement from 4 to 1

  i.e. a displacement = 3 units to the left (– 3)

- Vertical displacement from 2 to 4

  i.e. a displacement = 2 units upwards (2)

  i.e. $(x , y) \longrightarrow (x - 3 , y + 2)$, thus we get :

  $A (4 , -1) \longrightarrow \grave{A} (4 - 3 , -1 + 2)$

  i.e. $\grave{A} (1 , 1)$

  $B (0 , -3) \longrightarrow \grave{B} (0 - 3 , -3 + 2)$

  i.e. $\grave{B} (-3 , -1)$

Notice that :

The translation with magnitude MN in the direction of $\overrightarrow{MN}$ where M (4 , 2) and N (1 , 4) is equivalent to :

- A horizontal displacement (in the x-axis direction) from 4 to 1 = 1 – 4 = – 3
- A vertical displacement (in the y-axis direction) from 2 to 4 = 4 – 2 = 2

  i.e. The rule of translation is $(x , y) \longrightarrow (x - 3 , y + 2)$

Example (7)

Draw the image of Δ ABC where A (5 , 2) , B (4 , 5) and C (2 , 2) by translation BC in the direction of $\overrightarrow{BC}$ and write the rule of the translation.

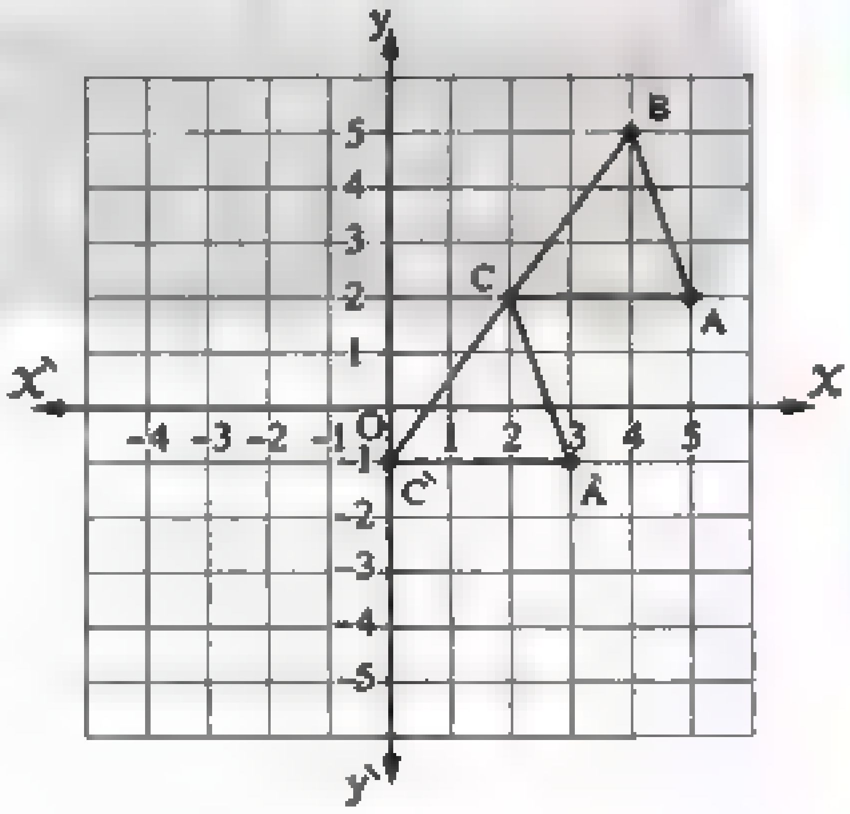
Solution

∵ B (4 , 5) , C (2 , 2)

∴ The translation BC in the direction of $\overrightarrow{BC}$ is equivalent to :

- Horizontal displacement = 2 – 4 = – 2
- Vertical displacement = 2 – 5 = – 3

Thus the rule of translation is $(x , y) \longrightarrow (x - 2 , y - 3)$



Thus :

$A (5 , 2) \longrightarrow \grave{A} (5 - 2 , 2 - 3)$

i.e. $\grave{A} (3 , -1)$

$B (4 , 5) \longrightarrow \grave{B} (4 - 2 , 5 - 3)$

i.e. $\grave{B} (2 , 2)$

$C (2 , 2) \longrightarrow \grave{C} (2 - 2 , 2 - 3)$

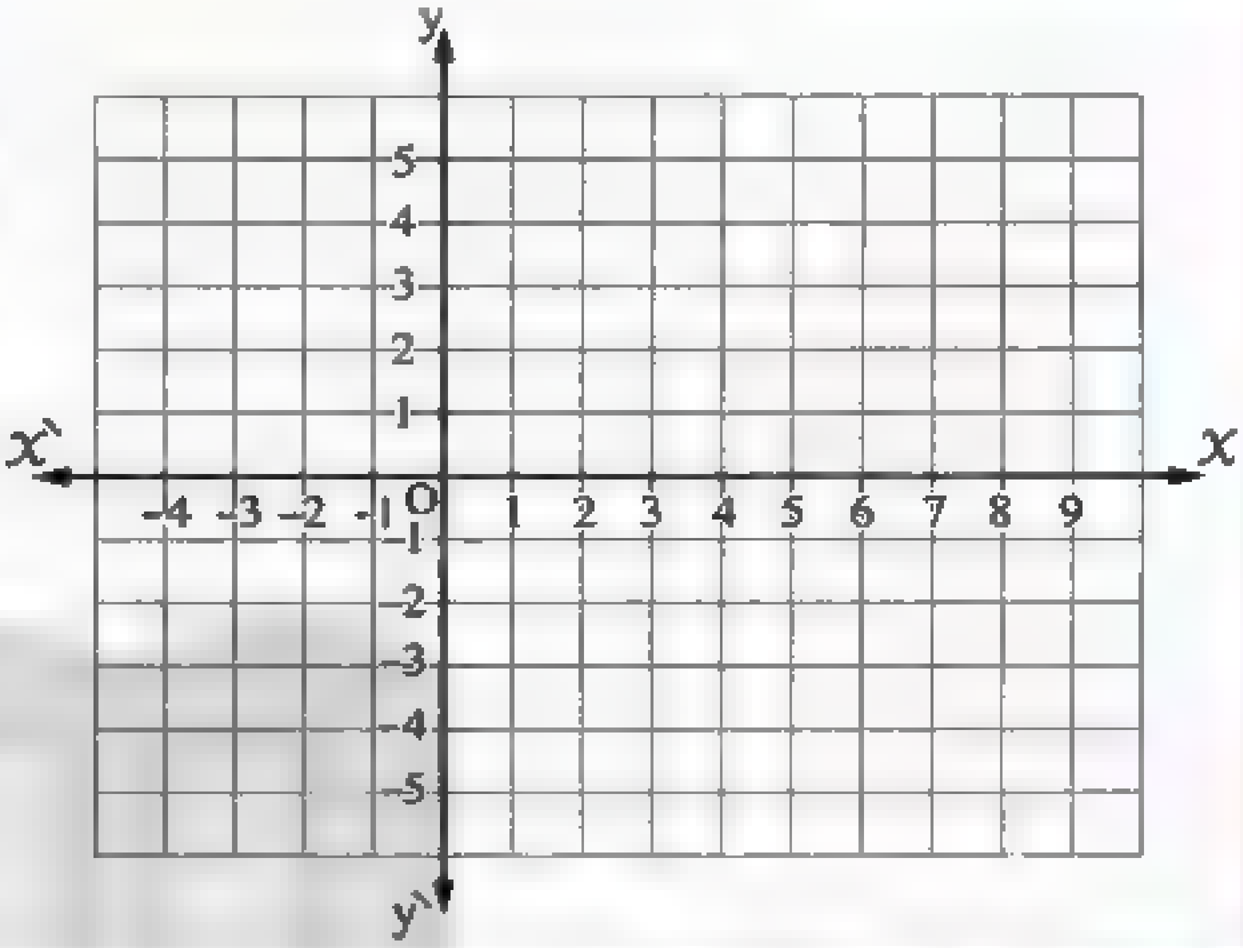
i.e. $\grave{C} (0 , -1)$

Notice that : $\grave{B}$ coincides C

i.e. Δ $\grave{A}C\grave{C}$ is the image of Δ ABC by translation BC in the direction of $\overrightarrow{BC}$

Try by yourself

Draw the square ABCD where A (4 , − 2) , B (4 , − 5) , C (1 , − 5) and D (1 , − 2) , then find its image by translation CA in the direction of $\overrightarrow{CA}$



Example 8

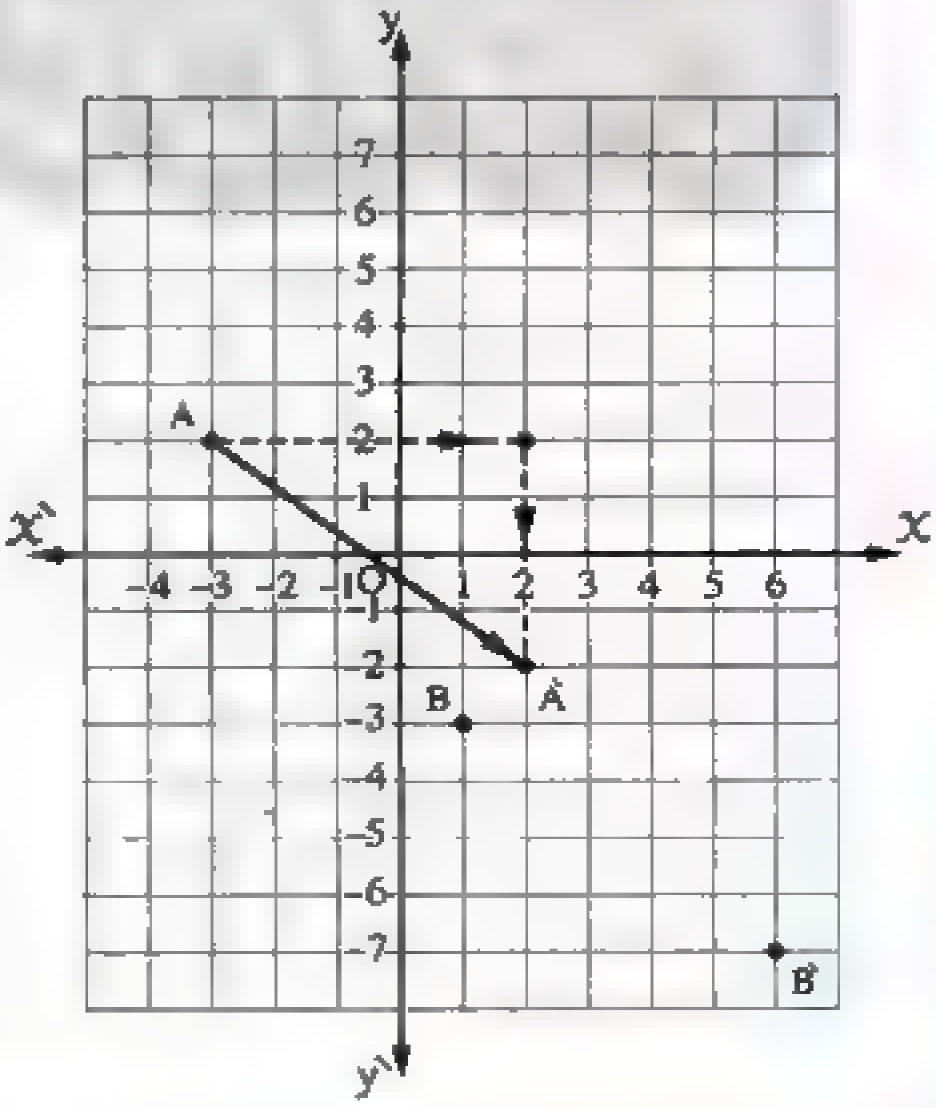
If the image of the point A (− 3 , 2) by translation in the Cartesian coordinates plane is Á (2 , − 2) :

1. Find the rule of translation.
2. Find the image of B (1 , − 3) by the same translation.

Solution

1. By noticing the opposite graph , we find that the translation which makes Á (2 , − 2) the image of A (− 3 , 2) is equivalent to :
   - Horizontal displacement of 5 units to the right side (5)
   - Vertcial displacement of 4 units downwards (− 4)

   ∴ The rule of translation is
   $(x , y) \longrightarrow (x + 5 , y - 4)$



2. B (1 , − 3) ⟶ B́ (1 + 5 , − 3 − 4)
   i.e. B́ (6 , − 7)

Example 9

If $\grave{A}$ (7 , − 2) is the image of A by the translation whose rule is $(x , y) \longrightarrow (x - 3 , y + 1)$, find A

Solution

Let A be (x , y)

$\therefore A(x , y) \longrightarrow \grave{A}(x - 3 , y + 1)$

$, \because \grave{A}(7 , -2)$

$\therefore (x - 3 , y + 1) = (7 , -2)$

$\therefore x - 3 = 7$

$\therefore x = 10$

$, y + 1 = -2$ $\qquad \therefore y = -3$

$\therefore A(10 , -3)$

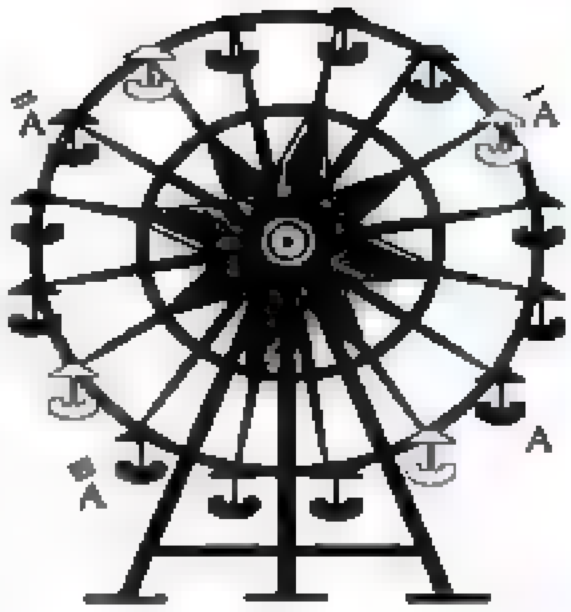
Notice that :

If $(x , y) = (a , b)$, then $x = a$, $y = b$

Lesson — Rotation

Prelude

If you stand in front of the spinning wheel play in the funfair , you will find that the carriage moves in a circular motion around a fixed point in a certain direction with a certain angle. It will take different positions as shown in the opposite picture. The carriage moves from position A to position A' , then to position A'' , then to position A''' , then it returns back to its original position to complete the perfect circulation and so on.



The concept of rotation

If M is a fixed point in the plane, then the rotation around M with an angle of measure θ is a geometric transformation transforming each point A in the plane to another point A' in the same plane such that $m(\angle AMA') = \theta$, $MA = MA'$

It is denoted by $R(M , \theta)$ where :

- M is the centre of rotation.
- θ is the measure of the angle of rotation.

According to this concept , the rotation is determined completely if we know the following elements :

1. The centre of rotation.
2. The measure of the angle of rotation (θ)
3. The direction of rotation.

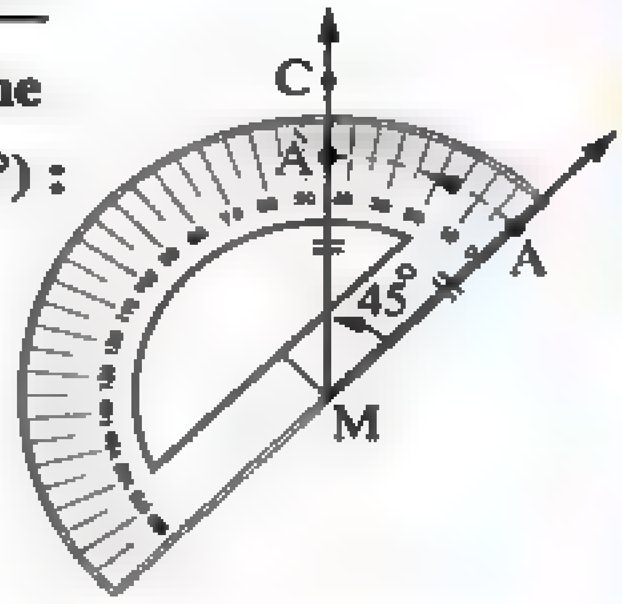
Remark

The measure of rotation angle is positive if the rotation is anticlockwise and it is negative if the rotation is clockwise.

Unit 3

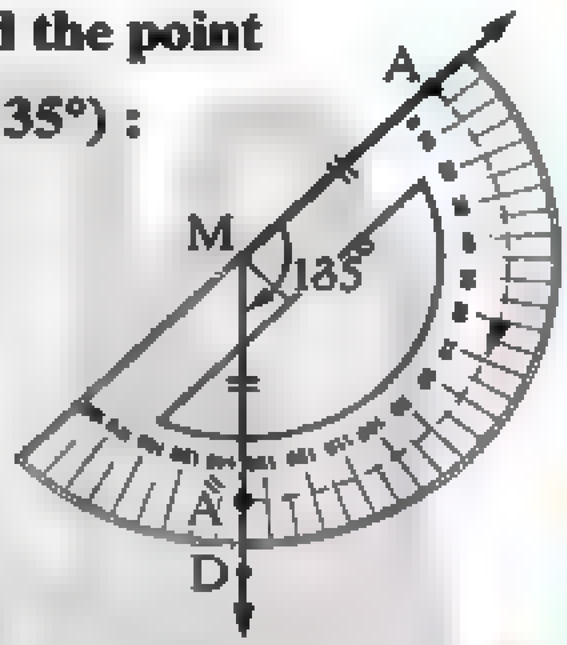
Rotation in the plane

Finding the image of a given point by a given rotation



Firstly : **Finding the image of the point A by rotation around the point M with an angle of measure 45°** ***i.e.*** **R (M , 45°) :**

- Draw the ray $\overrightarrow{MA}$
- Put the protractor with its straight edge on $\overrightarrow{MA}$ and in the anticlockwise direction , then draw $\overrightarrow{MC}$ such that m ($\angle$ AMC) = 45°
- Use the compasses at the point M as a centre with radius = MA , draw an arc to cut $\overrightarrow{MC}$ at A' then A' is the image of the point A by rotation around M with an angle of measure 45°



Secondly : **Finding the image of the point A by rotation around the point M with an angle of measure (– 135°)** ***i.e.*** **R (M , – 135°) :**

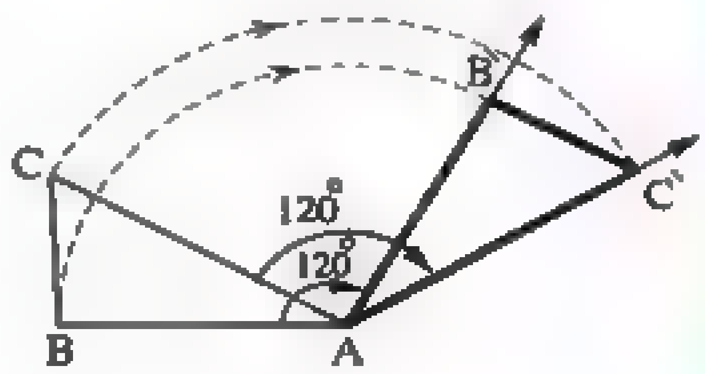
- Repeat the same previous steps , then draw $\overrightarrow{MD}$ in the clockwise direction such that m ($\angle$ AMD) = 135° , then determine on it the point A' such that $MA' = MA$, then A' is the image of A by rotation around M with an angle of measure (– 135°)

Remark

If A' is the image of A by rotation around M with an angle θ , then A is the image of A' by rotation around M with an angle of measure (– θ)

Finding the image of a polygon

The opposite figure shows how to find the image of Δ ABC by the rotation R (A , – 120°) by finding the image of each vertex of its vertices , then Δ $AB'C'$ is the image of Δ ABC by rotation R (A , – 120°)



Notice that : Δ $AB'C'$ ≡ Δ ABC

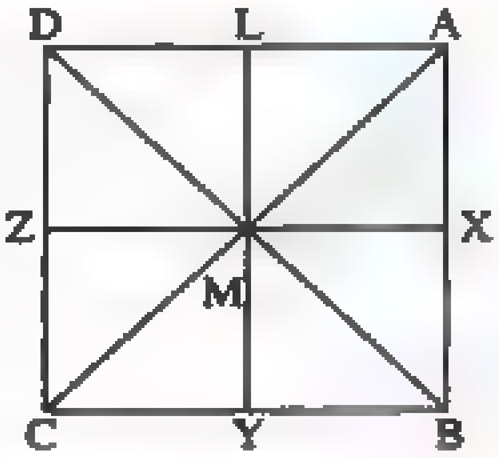
Remark

From the previous figure , the image of the point A by rotation R (A , – 120°) is itself because it is the centre of rotation.

Properties of rotation

Through our study of rotation , we found that the rotation is a geometric transformation that maps the figure to another congruent figure to it.
Therefore it is said that the rotation in the plane is **isometric** , thus we can deduce some of properties and add other properties through our study of the following illustrated example.

Illustrated example



In the opposite figure :

ABCD is a square whose diagonals intersect at M ,
X , Y , Z and L are the midpoints of its sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Find :

1 The image of Δ AXM by rotation R (M , 90°) , then mention what you observe.

2 The image of each of $\overline{AB}$ and $\overline{DC}$ by rotation R (M , – 90°) , then mention what you observe.

3 The image of each of B , Y and C by rotation R (M , 180°) , then mention what you observe.

Solution

1 $\because$ D is the image of A by rotation R (M , 90°) , L is the image of X by rotation R (M , 90°) and M is the image of itself (The centre of rotation).

$\therefore$ Δ DLM is the image of Δ AXM by rotation R (M , 90°)

Notice that :

- DL = AX , LM = XM and DM = AM

i.e.: Rotation in the plane reserves the lengths of the line segments.

- m ($\angle$ DLM) = m ($\angle$ AXM), m ($\angle$ LDM) = m ($\angle$ XAM) and m ($\angle$ DML) = m ($\angle$ AMX)

i.e.: Rotation in the plane reserves the measures of angles.

- Reading Δ AXM is in the clockwise direction and reading its image Δ DLM is in the clockwise direction also.

i.e.: Rotation in the plane reserves the orientation of vertices of the figure.

2 $\because$ B is the image of A by rotation R (M , – 90°) , C is the image of B by rotation R (M , – 90°)

$\therefore$ $\overline{BC}$ is the image of $\overline{AB}$ by rotation R (M , – 90°)

$\because$ A is the image of D by rotation R (M , – 90°) , D is the image of C by rotation R (M , – 90°)

$\therefore$ $\overline{AD}$ is the image of $\overline{DC}$ by rotation R (M , – 90°)

We notice that :

- $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$

i.e.: Rotation in the plane reserves parallelism.

3 • D is the image of B , L is the image of Y and A is the image of C by rotation (M , 180°)

We notice that :

- $Y \in \overline{BC}$ and L (The image of Y) $\in \overline{AD}$

i.e.: Rotation in the plane reserves the betweenness.

- B , Y , C are collinear , D , L , A are also collinear.

i.e.: Rotation in the plane reserves the collinearity.

Example (1)

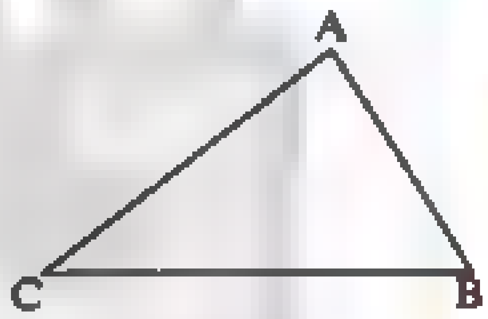
In the opposite figure :

ABC is a triangle Find :

1 The point D as the image of B by rotation R $(A , m(\angle B))$

2 The point E as the image of C by rotation R $(A , -m(\angle C))$

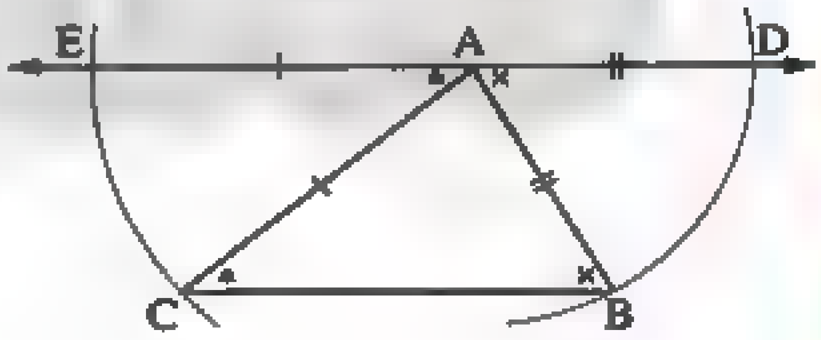
Then prove that : The points D , A and E are collinear.



Solution

$\because$ D is the image of B by the rotation R $(A , m(\angle B))$

$\therefore m(\angle DAB) = m(\angle B)$ (1)



, $\because$ E is the image of C by the rotation R $(A , -m(\angle C))$

$\therefore m(\angle EAC) = m(\angle C)$ (2)

Adding (1) and (2), we deduce that :

$m(\angle DAB) + m(\angle EAC) = m(\angle B) + m(\angle C)$

Adding $m(\angle BAC)$ to both sides,

$\therefore m(\angle DAB) + m(\angle EAC) + m(\angle BAC) = m(\angle B) + m(\angle C) + m(\angle BAC)$

$\because$ The sum of measure of the interior angles of the triangle = 180°

$\therefore m(\angle B) + m(\angle C) + m(\angle BAC) = 180°$

$\therefore m(\angle DAB) + m(\angle EAC) + m(\angle BAC) = 180°$

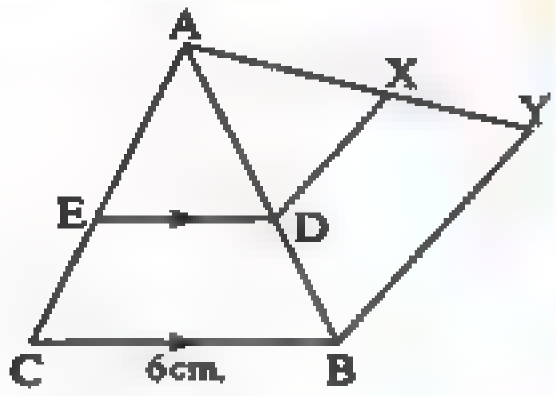
$\therefore$ D , A and E are collinear. (Q.E.D.)

Example 2

In the opposite figure :

If the figure XYBD is the image of the figure DBCE by the rotation R (A , 50°) ,

BC = 6 cm. , $\overline{DE}$ // $\overline{BC}$



1 Find the length of : $\overline{BY}$

2 **Prove that :** $\overline{DX}$ // $\overline{BY}$

Solution

1 ∵ The figure XYBD is the image of the figure DBCE by the rotation R (A , 50°)

∴ B is the image of C and Y is the image of B by this rotation.

∴ $\overline{BY}$ is the image of $\overline{CB}$ by this rotation.

∴ BY = CB = 6 cm. (First req.)

2 ∵ The figure XYBD is the image of the figure DBCE by rotation R (A , 50°)

∴ $\overline{XD}$ and $\overline{YB}$ are the images of $\overline{DE}$ and $\overline{BC}$ by this rotation respectively.

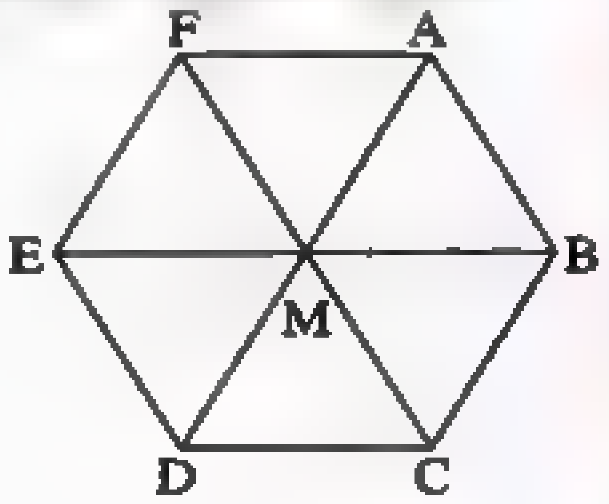
, ∵ $\overline{DE}$ // $\overline{BC}$

∴ $\overline{XD}$ // $\overline{BY}$ (Second req.)

Try by yourself

In the opposite figure :

ABCDEF is a regular hexagon. Complete the following :



1 The image of the point A by rotation around M with an angle of measure 180° is ………

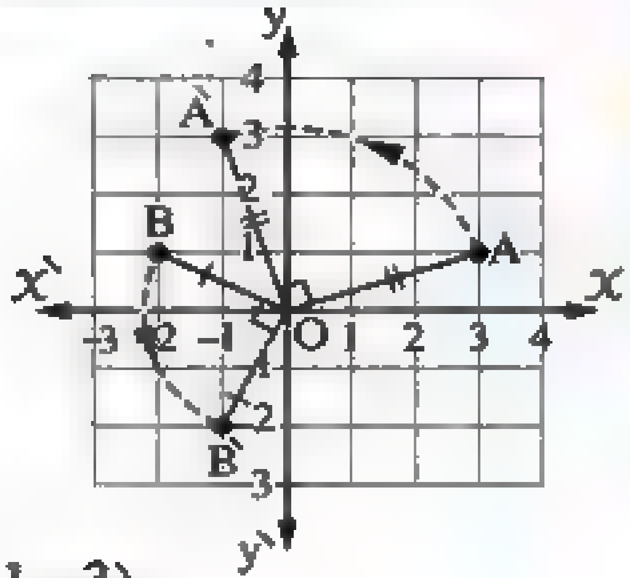
2 The image of $\overline{AB}$ by rotation around M with an angle of measure (– 60°) is ………

3 The image of Δ CMD by rotation around M with an angle of measure 120° is ………

Unit 3

Rotation in the Cartesian plane

First Rotation about the origin point (O) with an angle of measure 90° :



The opposite figure :

shows the two images of the two points

A (3 , 1) and B (– 2 , 1)

by rotation R (O , 90°)

By noticing the figure , we find that :

- The image of the point A (3 , 1) $\xrightarrow[\text{R (O , 90°)}]{\text{by rotation}}$ the point $\grave{A}$ (– 1 , 3)
- The image of the point B (– 2 , 1) $\xrightarrow[\text{R (O , 90°)}]{\text{by rotation}}$ the point $\grave{B}$ (– 1 , – 2)

From the previous ; we deduce the following rule :

The image of the point (x , y) $\xrightarrow[\text{R (O , 90°)}]{\text{by rotation}}$ the point (– y , x)

Remark

- The image of the point (x , y) $\xrightarrow[\text{R (O , – 90°)}]{\text{by rotation}}$ the point (y , – x)

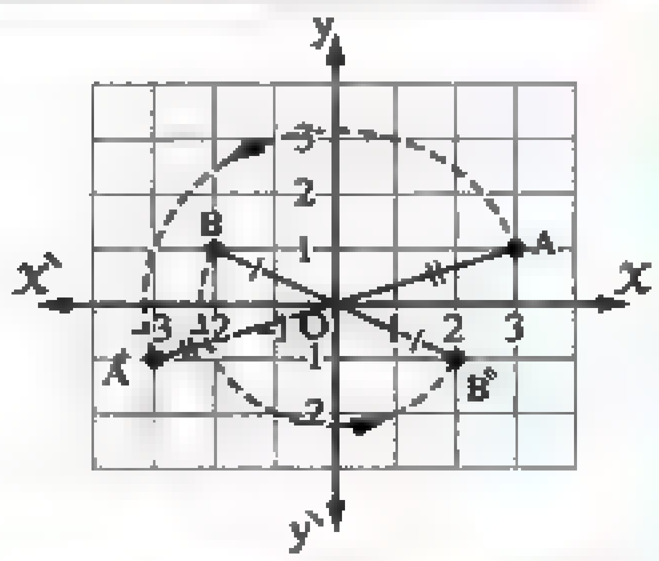
For example :

- The image of the point (2 , – 3) $\xrightarrow[\text{R (O , – 90°)}]{\text{by rotation}}$ the point (– 3 , – 2)
- Rotation about the origin point with an angle of measure 270° is equivalent to rotation about the origin point with an angle of measure (– 90°)

For example :

- The image of the point (2 , – 3) $\xrightarrow[\text{R (O , 270°)}]{\text{by rotation}}$ the point (– 3 , – 2)

Second Rotation about the origin point (O) with an angle of measure 180° :



The opposite figure :

shows the two images of the

two points A (3 , 1) and B (– 2 , 1)

by rotation R (O , 180°)

By noticing the figure, we find that :

- The image of the point A (3 , 1) $\xrightarrow[R(O,\,180^\circ)]{\text{by rotation}}$ the point A` (– 3 , – 1)
- The image of the point B (– 2 , 1) $\xrightarrow[R(O,\,180^\circ)]{\text{by rotation}}$ the point B` (2 , – 1)

From the previous ; we deduce the following rule :

The image of the point (x , y) $\xrightarrow[R(O,\,180^\circ)]{\text{by rotation}}$ the point (– x , – y)

Remarks

1. The image of the point A (x , y) by rotation R (O , 180°) is the same image of the point A by rotation R (O , – 180°)
2. The image of the point A (x , y) about the origin point with an angle of measure ± 360° is the same point A (x , y)
3. Rotation with an angle of measure 90° is called a $\frac{1}{4}$ turn.
4. Rotation with an angle of measure 180° is called a $\frac{1}{2}$ turn.
5. Rotation with an angle of measure 360° is called the identity rotation because it returns the figure to its original position.

Example (3)

Complete the following table :

	The point	Its image by rotation R (O , ± 180°)	Its image by rotation R (O , 90°)
1	(3 , 2)	....	....
2	(– 3 , 4)	....	....
3	(– 2 , – 1)	....	....
4	....	(5 , – 2)	....
5	....	....	(6 , 0)

Solution

1. (– 3 , – 2) , (– 2 , 3)
2. (3 , – 4) , (– 4 , – 3)
3. (2 , 1) , (1 , – 2)
4. (– 5 , 2) , (– 2 , – 5)
5. (0 , – 6) , (0 , 6)

Example 4

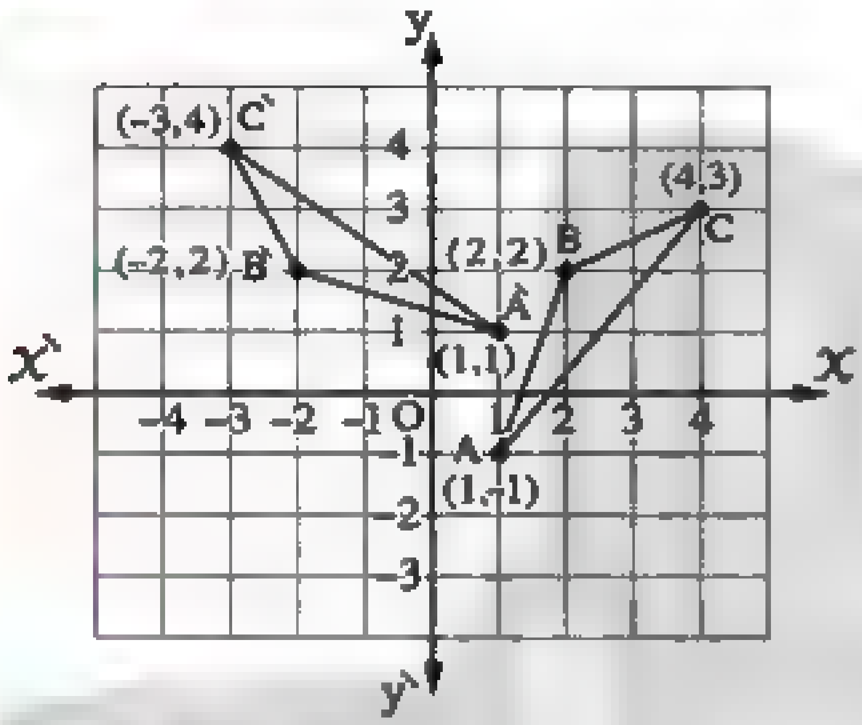
On a square lattice , draw Δ ABC where A (1 , – 1) , B (2 , 2) and C (4 , 3) :

1 Draw Δ ÀB̀C̀ which is the image of Δ ABC by rotation R (O , 90°)

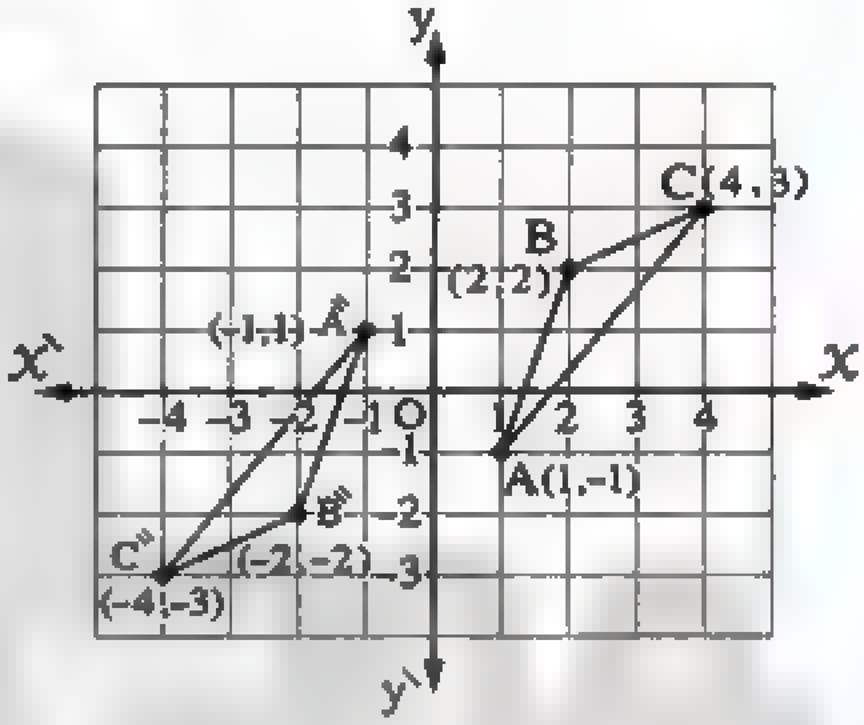
2 Draw Δ A̖B̖C̖ which is the image of Δ ABC by rotation R (O , 180°)

Solution

1



2



Try by yourself

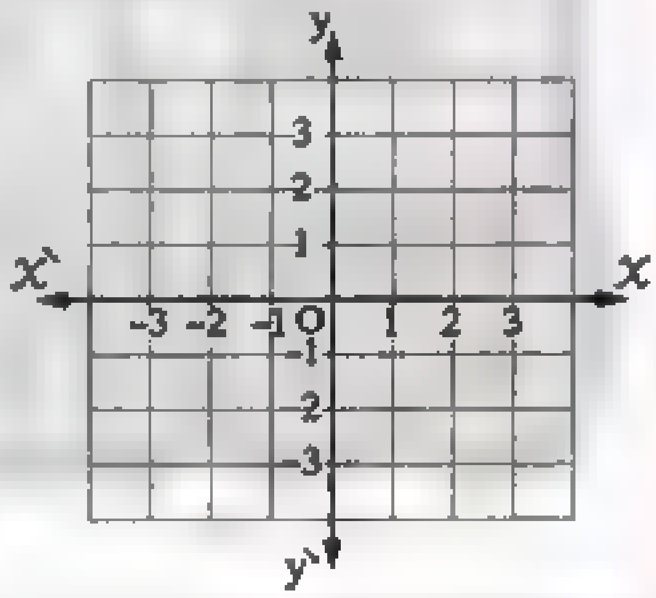
In the opposite figure :

on the square lattice ,

draw $\overline{AB}$ where A (2 , 1) and B (1 , 3),

then draw its image by rotation :

1 R (O , 90°) **2** R (O , 180°)



Lesson Twelve

Summary for geometrical transformations (reflection , translation , rotation) in the Cartesian plane :

The image of the point (x , y)

by reflection in the x-axis	The point $(x , -y)$
by reflection in the y-axis	The point $(-x , y)$
by reflection in the origin point	The point $(-x , -y)$
by translation $(x, y) \Rightarrow (x + k , y + \ell)$	The point $(x + k , y + \ell)$
by rotation R $(O , 90°)$ $(\frac{1}{4}$ turn$)$	The point $(-y , x)$
by rotation about O with an angle of measure $(-90°)$ or $(270°)$	The point $(y , -x)$
by rotation R $(O , \pm 180°)$ $(\frac{1}{2}$ turn$)$	The point $(-x , -y)$
by rotation R $(O , \pm 360°)$ (identity rotation)	The point (x , y)

Try by yourself

The image of the point $(2 , -1)$

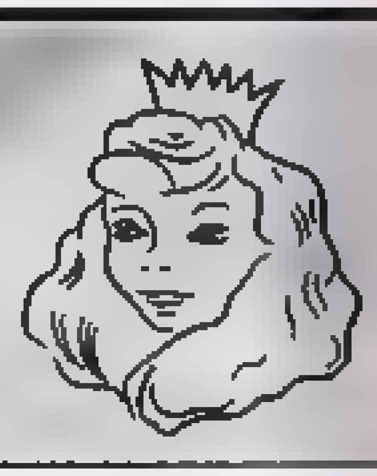
by reflection in the x-axis	The point (……… , ………)
by reflection in the y-axis	The point (……… , ………)
by reflection in the origin point	The point (……… , ………)
by translation $(x , y) \Rightarrow (x - 3 , y + 4)$	The point (……… , ………)
by rotation R $(O , 90°)$	The point (……… , ………)
by rotation R $(O , -90°)$	The point (……… , ………)
by rotation R $(O , \pm 180°)$	The point (……… , ………)
by rotation R $(O , \pm 360°)$	The point (……… , ………)

Optical illusion

Look at the picture.

Turn the book with an angle of measure 180° and look at it again.

What do you notice ?



Unit 3

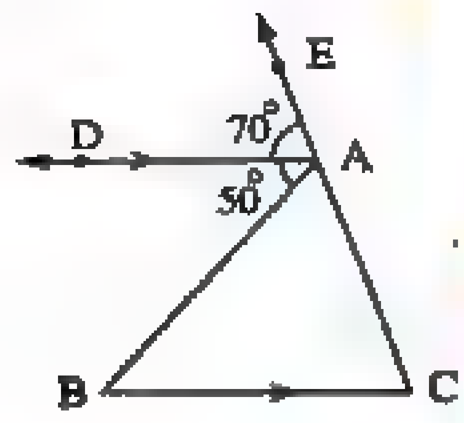
Geometry and Measurement

Unit 3

From the school book

Exercise 1 On deductive proof

1 **In the opposite figure :**



$\overrightarrow{AD} // \overline{BC}$,

m (∠ DAB) = 50°

and m (∠ DAE) = 70°

Find the measures of the angles of Δ ABC

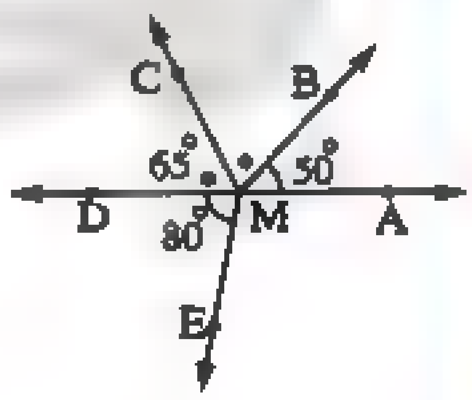
Complete the following table by writting the reason of each step of the solution steps :

Mathematical Statement	The reason
m (∠ DAB) = 50° , m (∠ DAE) = 70°	Given
m (∠ CAB) = 180° − (50° + 70°) = 60°	(1) ....
$\overrightarrow{AD} // \overline{BC}$	(2) ....
m (∠ C) = m (∠ DAE) = 70°	(3) ....
m (∠ B) = m (∠ DAB) = 50°	(4) ....

2 **In the opposite figure :**

m (∠ AMB) = 50° , m (∠ EMD) = 80° , $\overrightarrow{MC}$ bisects ∠ BMD and m (∠ CMD) = 65°

Complete the following proof to find m (∠ AME)



Given ....

R.T.F. ....

Proof ∵ $\overrightarrow{MC}$ bisects ∠ .... (given)

∴ m (∠ BMC) = m (∠ ....) = ...°

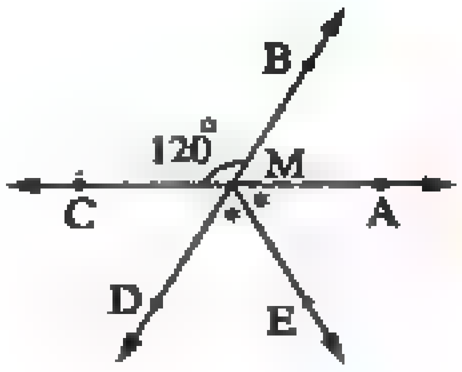
∵ m (∠ AMB) + m (∠ BMC) + m (∠ CMD) + m (∠ DME) + m (∠ AME) = ...°

∴ m (∠ AME) = ...° − ....° = ...° (The req.)

3 **In the opposite figure :**

$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$, $m(\angle BMC) = 120°$

and $\overrightarrow{ME}$ bisects $\angle AMD$



Complete the following solution steps to find $m(\angle EMC)$

Given ....

R.T.F. ....

Proof $\because \overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$

$\therefore m(\angle BMC) = m(\angle ....)$ (V.O.A.)

$\therefore m(\angle ....) = 120°$

$\because \overrightarrow{ME}$ bisects $\angle AMD$

$\therefore m(\angle ....) = m(\angle ....)$

$\therefore m(\angle EMD) = \frac{..........}{..........} = ...°$

$\because M \in \overleftrightarrow{BD}$

$\therefore m(\angle BMC) + m(\angle ....) = 180°$

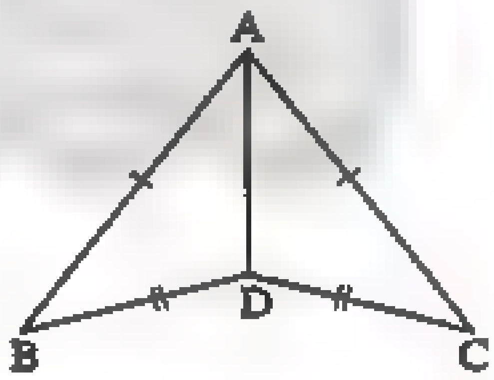
$\therefore m(\angle DMC) = ...° - ....° = ...°$

$\because m(\angle EMC) = m(\angle ....) + m(\angle ....)$

$\therefore m(\angle EMC) = ...° + ...° = ...°$ (The req.)

4 **In the opposite figure :**

AB = AC , BD = CD



Complete the following proof to prove that $\overrightarrow{AD}$ bisects $\angle BAC$

Given ....

R.T.P. ....

Proof $\because$ In $\Delta\Delta$ ADB , ... :

AB = ... (given)

.... = CD (given)

$\overline{AD}$ ....

$\therefore \Delta ADB \equiv \Delta$ .... then we deduce that :

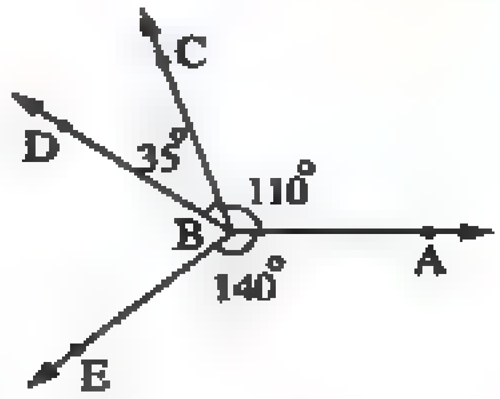
$m(\angle ....) = m(\angle ....)$

$\therefore \overrightarrow{AD}$ bisects $\angle$ .... (Q.E.D.)

Unit 3

5 **In the opposite figure :**

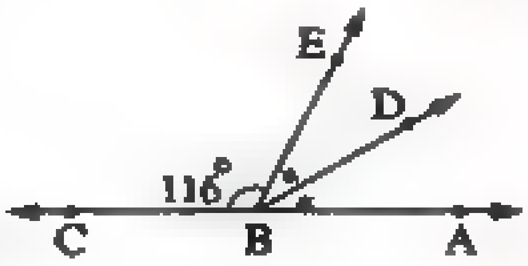
m (∠ ABC) = 110° , m (∠ CBD) = 35° and
m (∠ ABE) = 140°
Find : m (∠ EBD)



« 75° »

6 **In the opposite figure :**

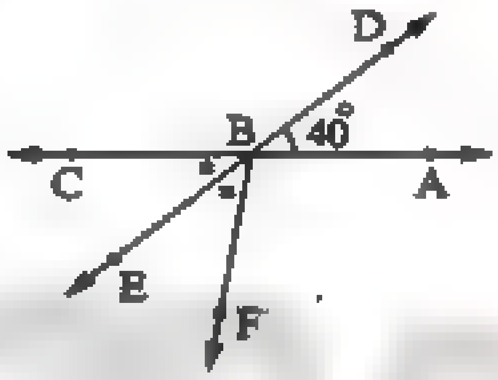
B ∈ $\overleftrightarrow{AC}$, m (∠ CBE) = 116° and
$\overrightarrow{BD}$ bisects ∠ ABE
Find : m (∠ ABD)



« 32° »

7 **In the opposite figure :**

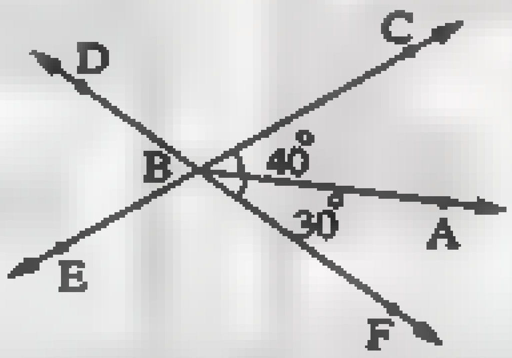
$\overleftrightarrow{AC}$ ∩ $\overleftrightarrow{DE}$ = {B} , m (∠ ABD) = 40° and
$\overrightarrow{BE}$ bisects ∠ CBF
Find : m (∠ ABF)



« 100° »

8 **In the opposite figure :**

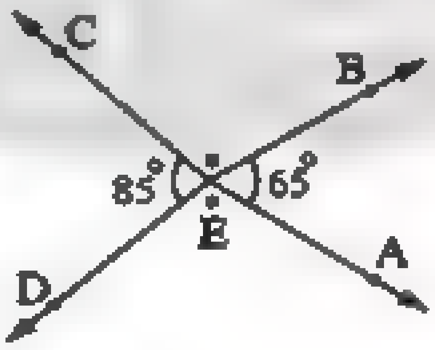
$\overleftrightarrow{CE}$ ∩ $\overleftrightarrow{FD}$ = {B} ,
m (∠ ABC) = 40° and m (∠ ABF) = 30°
Find : m (∠ DBC)



« 110° »

9 **In the opposite figure :**

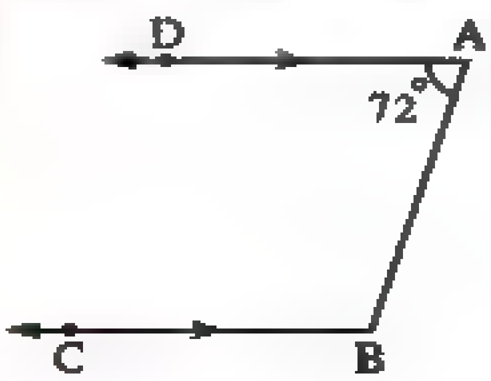
$\overrightarrow{EA}$ ∩ $\overrightarrow{EB}$ ∩ $\overrightarrow{EC}$ ∩ $\overrightarrow{ED}$ = {E} ,
If m (∠ BEC) = m (∠ AED)
Find : m (∠ BEC)
Are A , E , and C on the same straight line ? Why ?

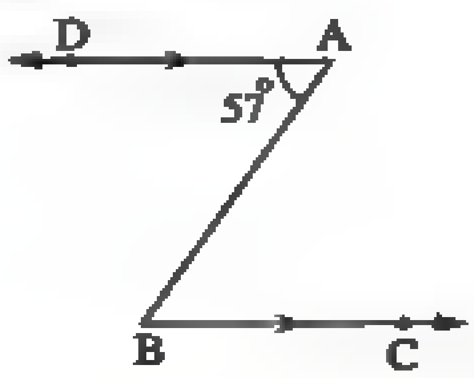


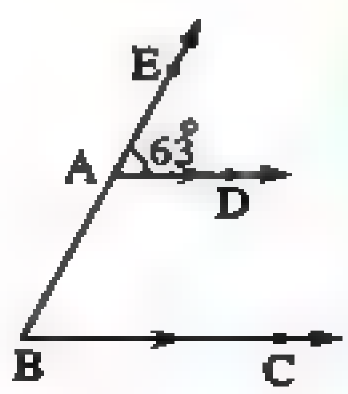
« 105° »

10 **In each of the following figures ,**

If $\overleftrightarrow{AD}$ // $\overleftrightarrow{BC}$, **Find :** m (∠ ABC) , giving reason.







11 In the opposite figure :

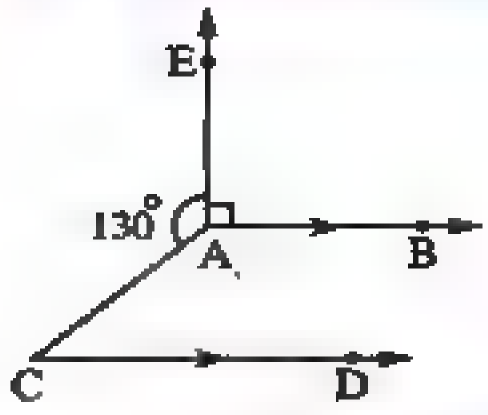
$\overrightarrow{AB} \parallel \overrightarrow{CD}$,

, m ($\angle$ EAC) = 130° and

m ($\angle$ EAB) = 90°

Find : (1) m ($\angle$ BAC)

(2) m ($\angle$ C)



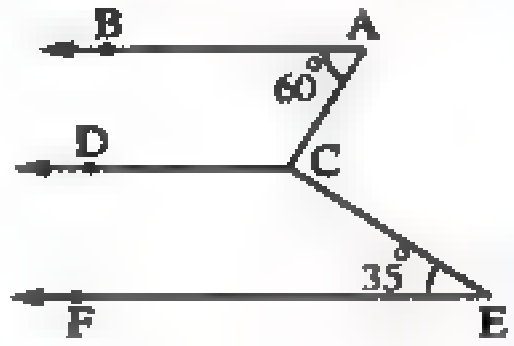
« 140° , 40° »

12 In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AB} \parallel \overrightarrow{EF}$

, m ($\angle$ A) = 60° and m ($\angle$ E) = 35°

Find : m ($\angle$ ACE)



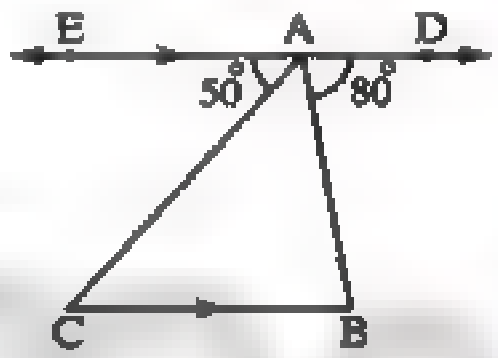
« 95° »

13 In the opposite figure :

$\overleftrightarrow{DE} \parallel \overrightarrow{BC}$, A $\in \overleftrightarrow{DE}$, m ($\angle$ DAB) = 80° and

m ($\angle$ EAC) = 50°

Find the measures of the angles of Δ ABC



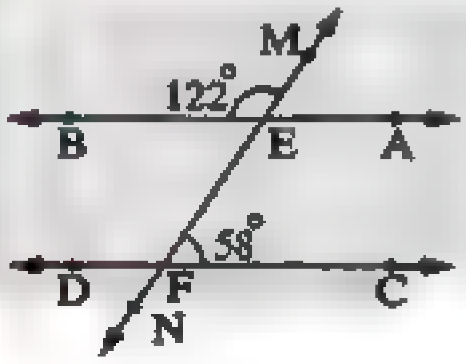
« m ($\angle$ BAC) = 50° , m ($\angle$ B) = 80° , m ($\angle$ C) = 50° »

14 In each of the following figures ,

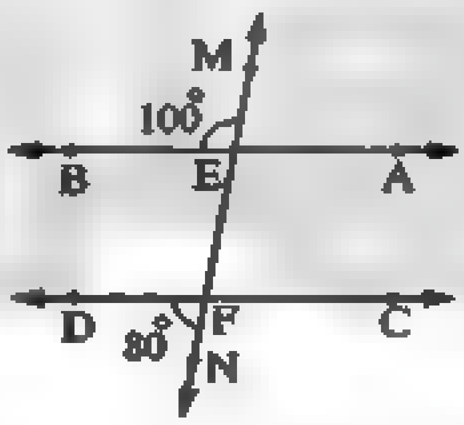
If $\overleftrightarrow{MN}$ intersects $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ at E and F respectively ,

Prove that : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

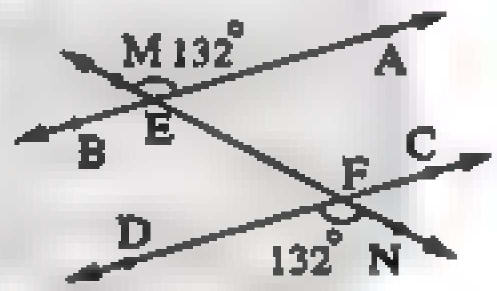
(1)



(2)



(3)

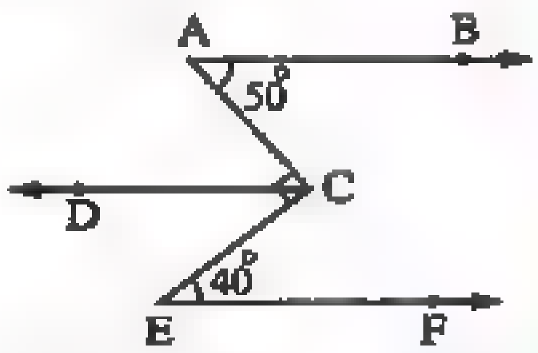


15 In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, m ($\angle$ A) = 50° ,

$\angle$ ACE is right and m ($\angle$ E) = 40°

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{EF}$

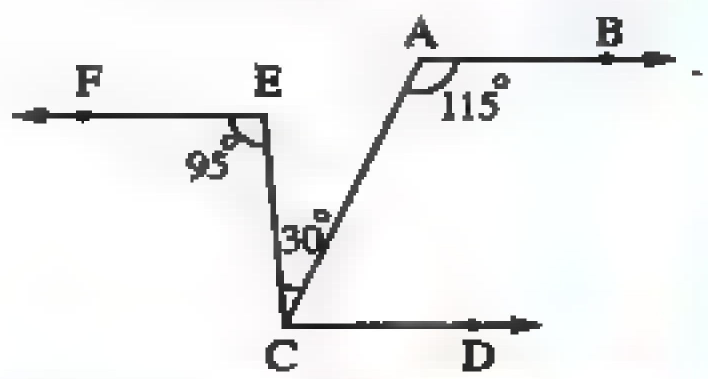


16 In the opposite figure :

$\overrightarrow{EF} \parallel \overrightarrow{CD}$, m ($\angle$ CEF) = 95° ,

m ($\angle$ ACE) = 30° , m (BAC) = 115°

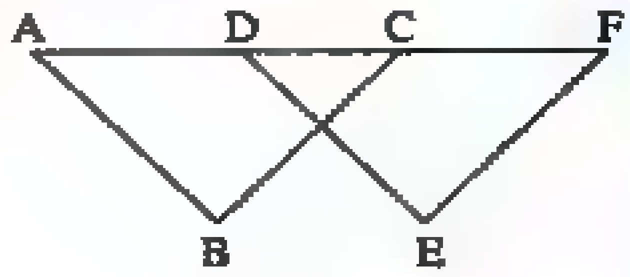
Prove that : $\overrightarrow{AB} \parallel \overrightarrow{EF}$



17 **In the opposite figure :**

The two triangles are congruent

Prove that : $\overline{BC} \parallel \overline{EF}$



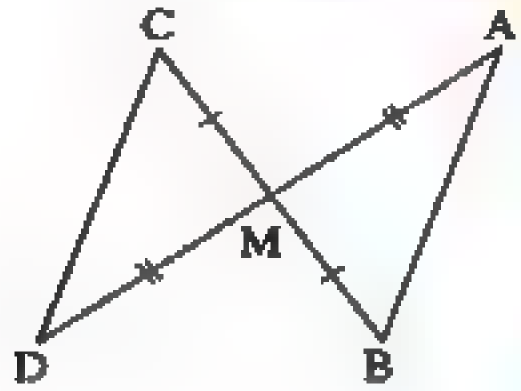
18 **In the opposite figure :**

$\overline{AD} \cap \overline{BC} = \{M\}$,

MA = MD and MB = MC

Prove that :

(1) AB = CD (2) $\overline{AB} \parallel \overline{CD}$



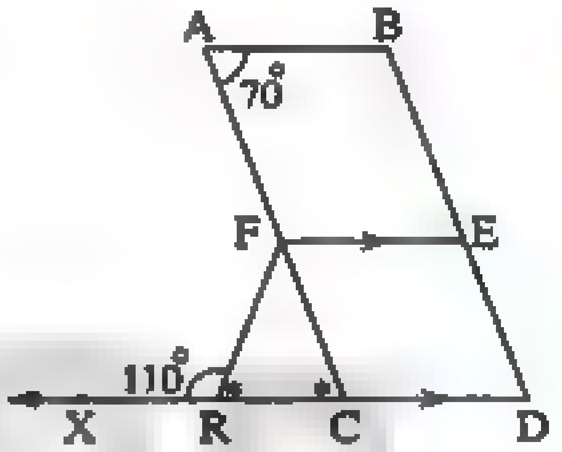
19 **In the opposite figure :**

$\overline{FE} \parallel \overline{CD}$, m (∠ FRC) = m (FCR),

m (∠ A) = 70° and m (∠ FRX) = 110°

(1) **Prove that :** $\overline{CD} \parallel \overline{AB}$

(2) **Find :** m (∠ AFE) in three ways.



« 110° »

20 **Prove that :**

(1) A straight line which is perpendicular to one of two parallel lines is also perpendicular to the other.

(2) A straight line that is parallel to one of two parallel lines is also parallel to the other.

21 **In the opposite figure :**

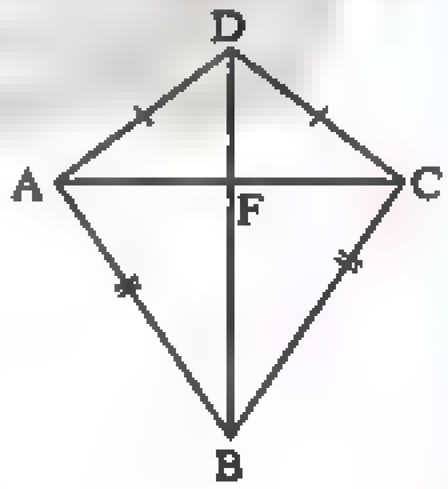
AD = CD and AB = BC

Use the properties of congruent triangles

to show that :

(1) $\overrightarrow{DB}$ bisects ∠ ADC

(2) $\overline{AC}$ and $\overline{DB}$ are perpendicular to each other.



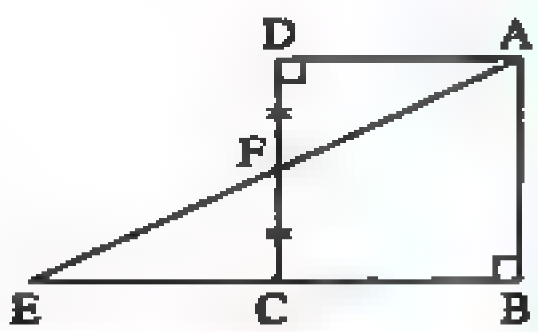
22 **In the opposite figure :**

ABCD is a square in which F

is the midpoint of $\overline{CD}$ and

$\overrightarrow{AF} \cap \overrightarrow{BC} = \{E\}$

Prove that : CE = CB



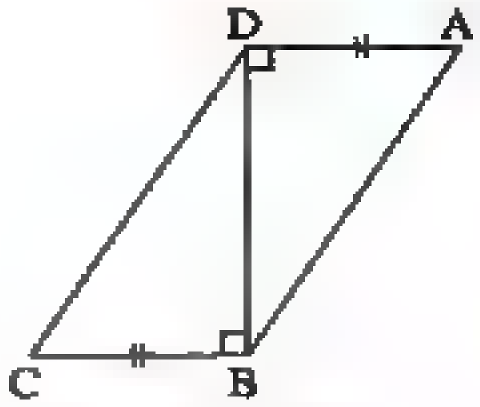
23 **In the opposite figure :**

$AD = BC$ and $m(\angle ADB) = m(\angle DBC) = 90°$

Prove that :

(1) $AB = CD$

(2) $\overline{AB} \parallel \overline{CD}$

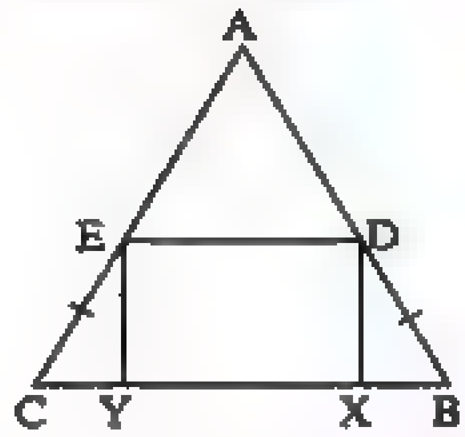


24 **In the opposite figure :**

$EC = DB$ and

DXYE is a rectangle.

Prove that : $m(\angle ADE) = m(\angle AED)$

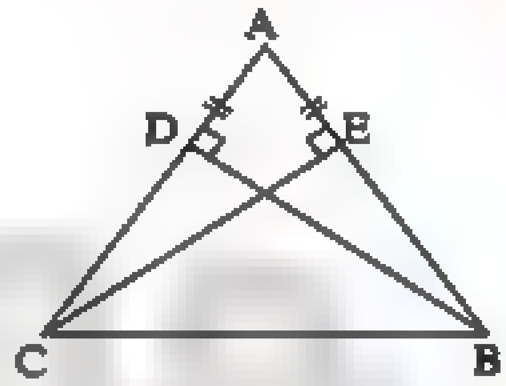


25 **In the opposite figure :**

$\overline{BD} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$ and

$AD = AE$

Prove that : $BD = CE$



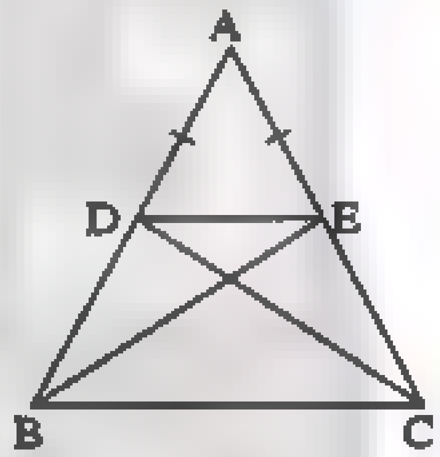
26 **In the opposite figure :**

$AD = AE$ and

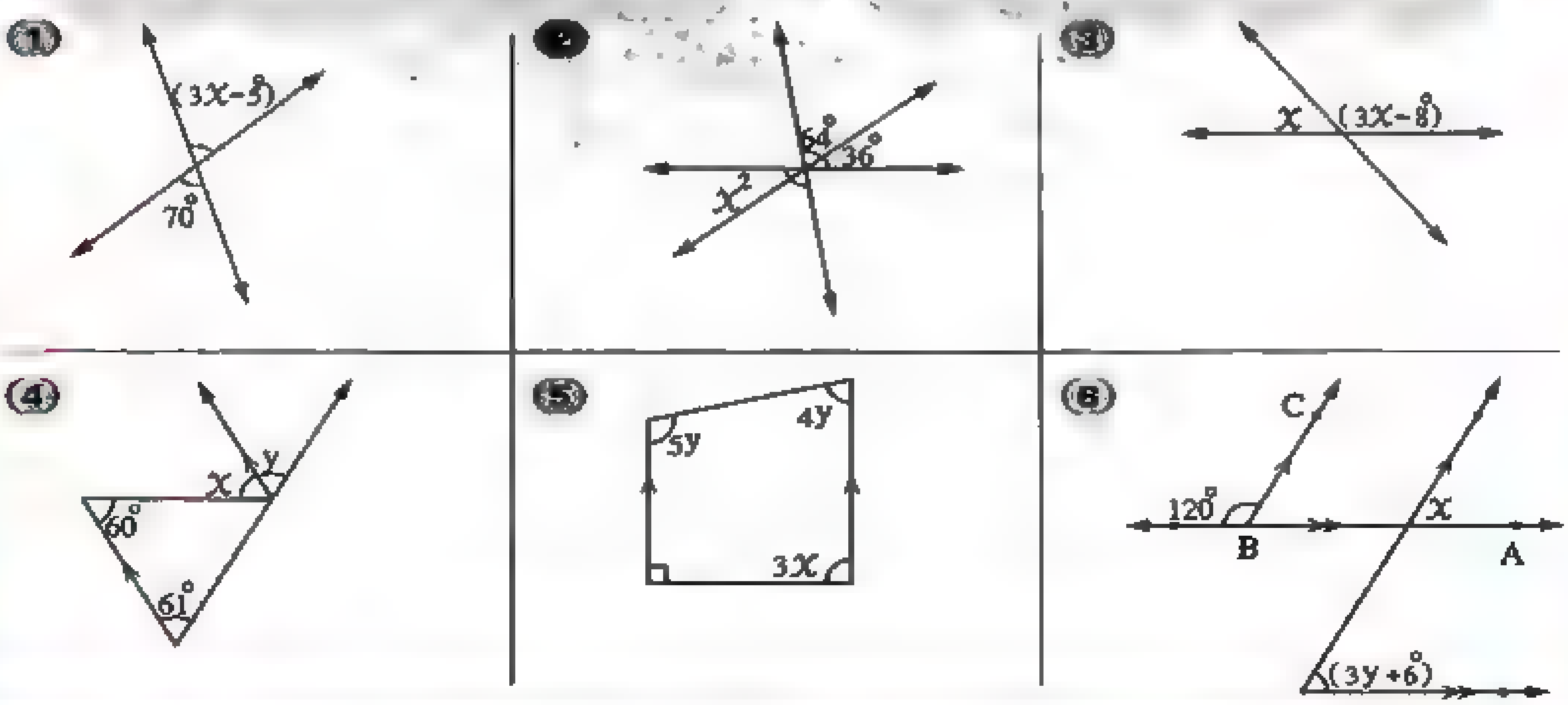
$m(\angle ADC) = m(\angle AEB)$

Show that :

(1) $BE = CD$ (2) $BD = CE$



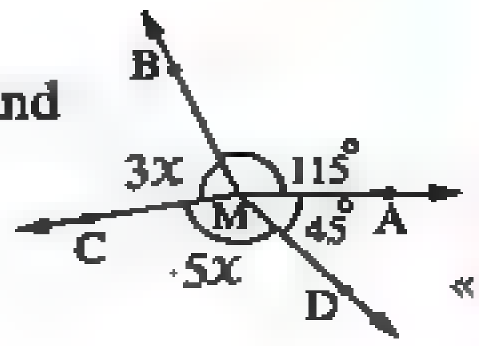
27 **Find the values of x and y in each of the following :**



28 **In the opposite figure :**

m (∠ AMB) = 115° , m (∠ AMD) = 45° , m (∠ BMC) = 3 x and m (∠ CMD) = 5 x What is the value of x?

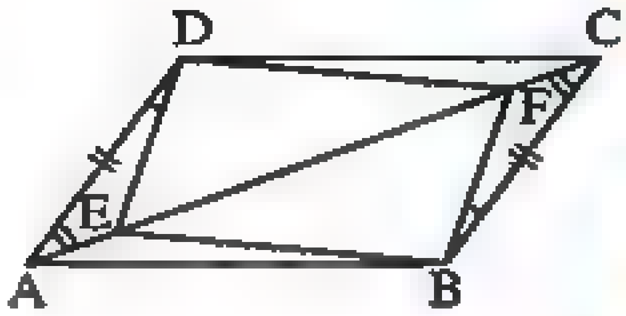
Are A , M and C collinear ? Why ?



« 25° »

For excellent pupils

29 **In the opposite figure :**

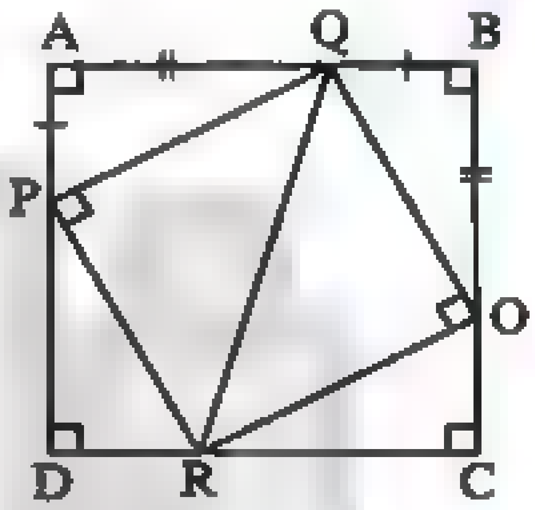


(1) Is Δ ADE congruent to Δ CBF ? Give your reason(s).

(2) **Prove that :**

First : Δ DEF ≡ Δ BFE **Second :** Δ ABE ≡ Δ CDF

30 **In the opposite figure :**



(1) Is Δ PAQ congruent to Δ QBO ? Give your reason (s).

(2) **Show that :**

First : Δ PQR ≡ Δ OQR **Second :** Δ PDR ≡ Δ RCO

Lesson Two

From the school book

Exercise 2 On the polygon

1 Complete the following :

1. The regular polygon is the one in which :
   (a) .... (b) ....
2. The sum of measures of the interior angles of the quadrilateral = ... °
3. The sum of measures of the interior angles of the pentagon = ...°
4. The sum of measures of the interior angles of the hexagon = ...°
5. The sum of measures of the interior angles of the heptagon = ...°
6. The measure of the interior angle of the regular pentagon = ... °
   and the measure of the interior angle of the regular heptagon = ... °
7. The sum of measures of the exterior angles of the hexagon equals .... °
8. If the perimeter of a regular hexagon is 30 cm. , then its side length = ... cm.
   an the measure of each interior angle in it = ...
9. If the perimeter of a regular polygon = 80 cm. and its side length = 10 cm. ,
   then the measure of each interior angle in it = ... °

---

2 Choose the correct answer from those given :

1. The sum of measures of the interior angles of a polygon of n sides = ...
   (a) $n \times 180°$ (b) $(n-2) \times 180°$ (c) $\frac{(n-2) \times 180°}{2}$ (d) $\frac{(n-2) \times 180°}{2\,n}$
2. The measure of the interior angle of a regular polygon of n sides equals ....
   (a) $\frac{(n-2) \times 90°}{n}$ (b) $\frac{(n-2) \times 180°}{2}$ (c) $\frac{(n-2) \times 180°}{n}$ (d) $180° \times (n-1)$
3. The measure of the interior angle of the regular polygon of 10 sides equals ....
   (a) 72° (b) 108° (c) 144° (d) 150°
4. The measure of the interior angle of a regular polygon of 18 sides equals ....
   (a) 130° (b) 140° (c) 150° (d) 160°
5. If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is ....
   (a) 6 (b) 4 (c) 7 (d) 8
6. The sum of measures of the exterior angles of the triangle equals ....
   (a) 90° (b) 180° (c) 360° (d) 720°
7. In the quadrilateral ABCD , if m (∠ A) = 2 m (∠ B) = m (∠ C) = 96° ,
   then m (∠ D) = ...
   (a) 96° (b) 48° (c) 120° (d) 144°

Unit 3

3 **Find the number of the diagonals of each of the following figures :**

(1) Triangle.

(2) Quadrilateral.

(3) Pentagon.

Hint : The number of diagonals of the polygon of n sides $= \frac{n(n-3)}{2}$

4 **In each of the following , find the measure of the angle marked by (?) :**

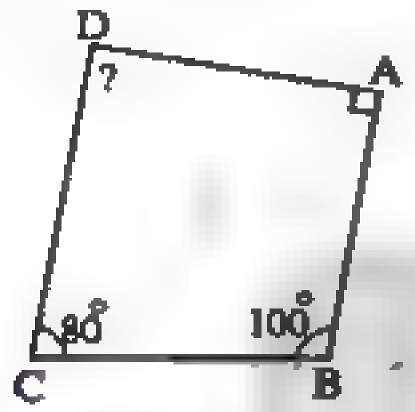


Fig. (1)

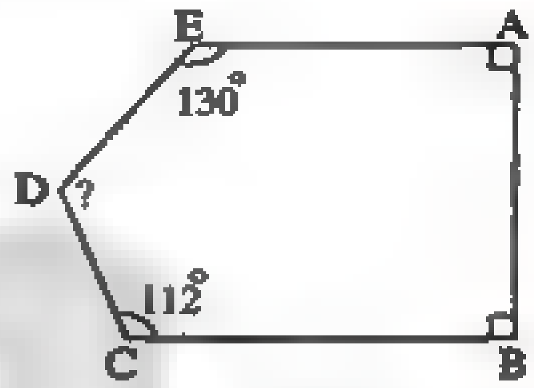
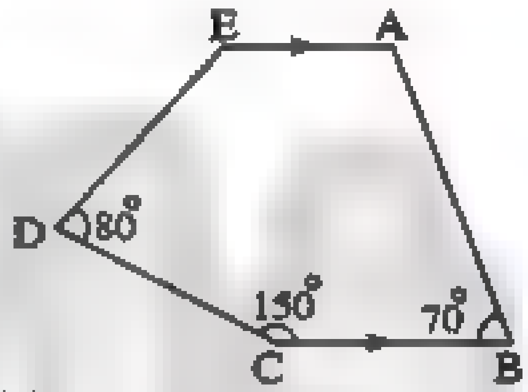


Fig. (2)

« 90° , 118° »

5 **In the opposite figure :**



$\overline{AE} \parallel \overline{BC}$, m (∠ B) = 70° , m (∠ C) = 150° and

m (∠ D) = 80°

Complete the following proof to find m (∠ E)

Given ....

R.T.F. ....

Proof ∵ $\overline{AE} \parallel \overline{BC}$ and $\overleftrightarrow{\cdots\cdots}$ is a transversal to them.

∴ m (∠ A) + m (∠ ....) = 180°

(Two interior angles in the same side of the transversal)

∵ m (∠ ....) = 70°

∴ m (∠ A) = ... − 70° = ...°

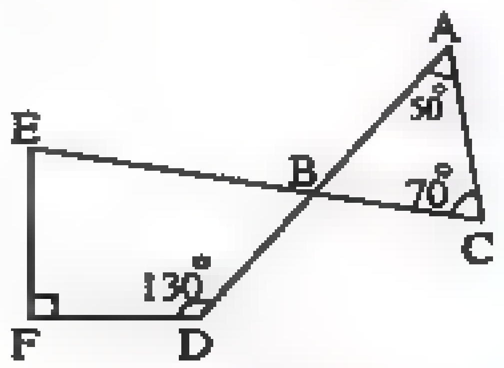
∵ ABCDE is a pentagon.

∴ The sum of measures of its interior angles

= (5 − ....) × .... = ... °

∴ m (∠ E) = ... − (70° + 150° + 80° + 110°) = ... ° (The req.)

6 **In the opposite figure :**



$\overline{CE} \cap \overline{AD} = \{B\}$, m (∠ A) = 50°

, m (∠ C) = 70° , m (∠ D) = 130° and

m (∠ F) = 90°

Complete the following proof to find m (∠ E)

Given ……………………………………………………………………

R.T.F. ……………………………………………………………………

Proof

∵ The sum of measures of the interior angles of the triangle = ……°

∴ In Δ ABC : m ($\angle$ ABC) = 180° – (……… + ………) = ………°

∵ $\overline{CE} \cap \overline{AD} = \{B\}$

∴ m ($\angle$ ………) = m ($\angle$ ………) = ………° (V.O.A.)

∵ The sum of measures of the interior angles of the quadrilateral = ………°

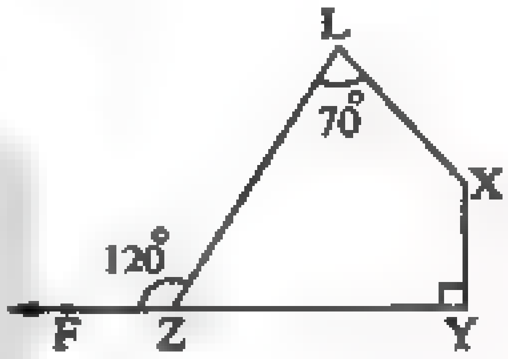
∴ m ($\angle$ E) = ……… – (……… + ……… + ………) = ………° (The req.)

7 **In the opposite figure :**

F $\in \overrightarrow{YZ}$, m ($\angle$ L) = 70° ,

m ($\angle$ Y) = 90° and m ($\angle$ LZF) = 120°

Find : m ($\angle$ X)



« 140° »

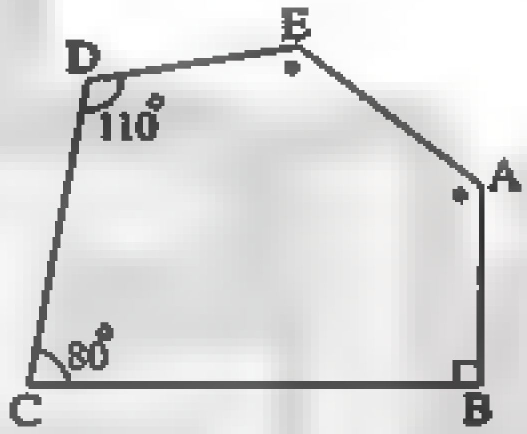
8 **In the opposite figure :**

If $\overline{AB} \perp \overline{BC}$, m ($\angle$ C) = 80° ,

m ($\angle$ D) = 110° and

m ($\angle$ A) = m ($\angle$ E)

Find : m ($\angle$ A)



« 130° »

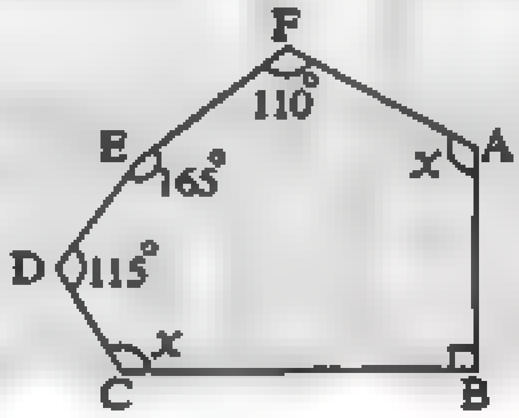
9 **In the opposite figure :**

ABCDEF is a hexagon.

m ($\angle$ FAB) = m ($\angle$ DCB)

Find :

The value of x



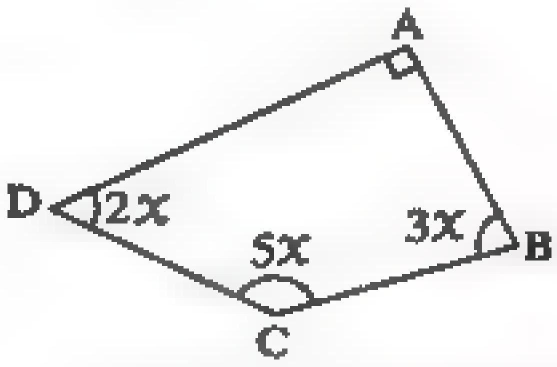
« 120° »

10 **In the opposite figure :**

ABCD is a quadrilateral

in which : m ($\angle$ A) = 90°

Find : The value of x



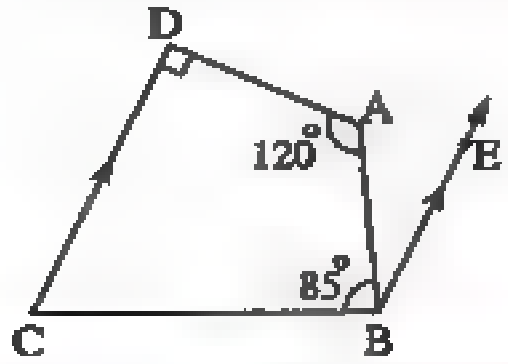
« 27° »

11 **In the opposite figure :**

m ($\angle$ A) = 120° , m ($\angle$ D) = 90° ,

m ($\angle$ ABC) = 85° and $\overrightarrow{BE} \parallel \overrightarrow{CD}$

Find : m ($\angle$ ABE)



« 30° »

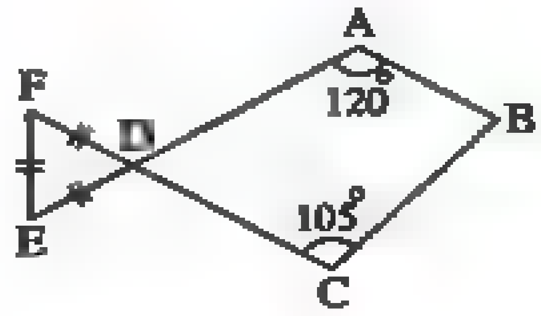
12 **In the opposite figure :**

$\overline{AE} \cap \overline{CF} = \{D\}$,

Δ DEF is an equilateral triangle ,

m ($\angle$ A) = 120° and m ($\angle$ C) = 105°

Find : m ($\angle$ B)



« 75° »

---

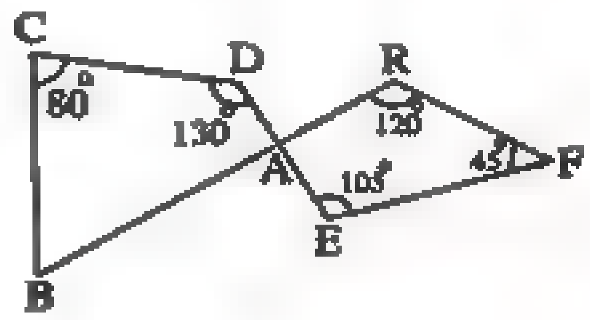
13 **In the opposite figure :**

$\overline{ED} \cap \overline{RB} = \{A\}$, m ($\angle$ F) = 45° ,

m ($\angle$ R) = 120° , m ($\angle$ E) = 105° ,

m ($\angle$ D) = 130° and m ($\angle$ C) = 80°

Find : m ($\angle$ B)



« 60° »

---

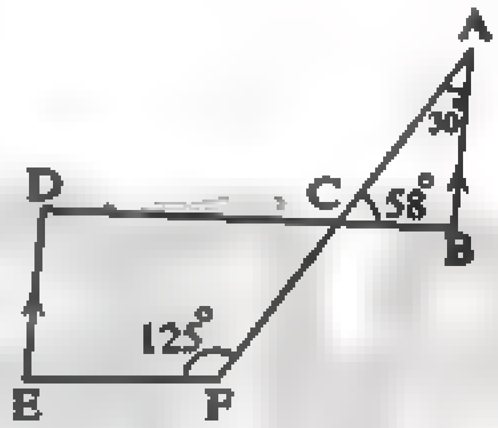
14 **In the opposite figure :**

$\overline{BD} \cap \overline{AF} = \{C\}$, $\overline{AB} \parallel \overline{ED}$,

m ($\angle$ A) = 30° and m ($\angle$ ACB) = 58° ,

m ($\angle$ CFE) = 125°

Find : m ($\angle$ E)



« 85° »

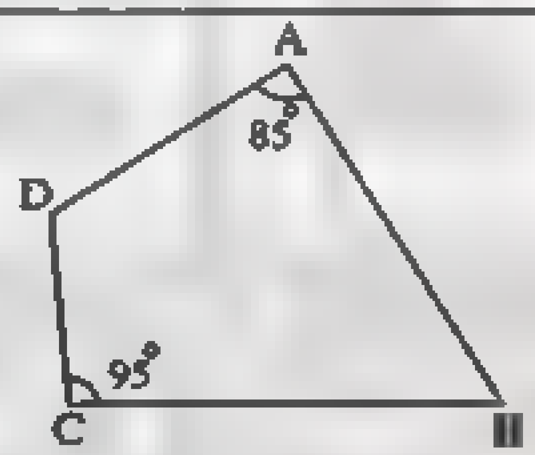
---

15 **In the opposite figure :**

m ($\angle$ A) = 85° , m ($\angle$ C) = 95° and

m ($\angle$ B) = $\frac{1}{2}$ m ($\angle$ D)

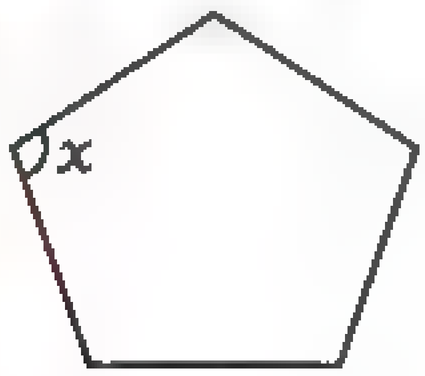
Find the measure of each of them.

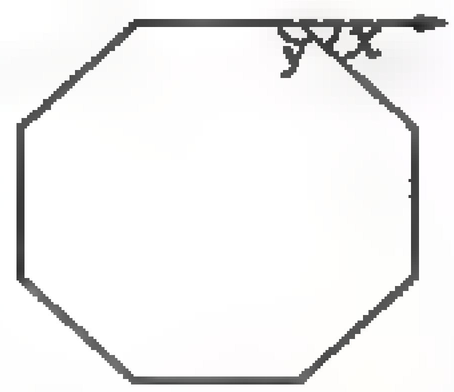


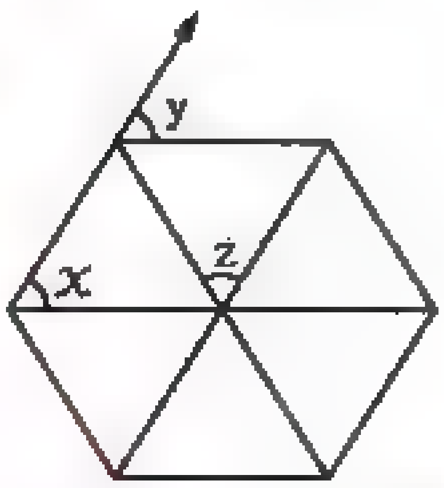
« 60° , 120° »

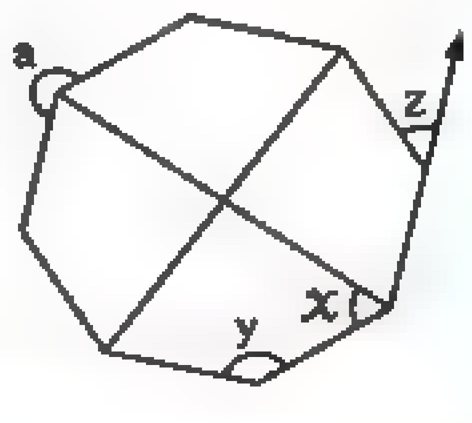
---

16 **In each of the following , if the polygon is regular , find the measure of the unknown angles.**

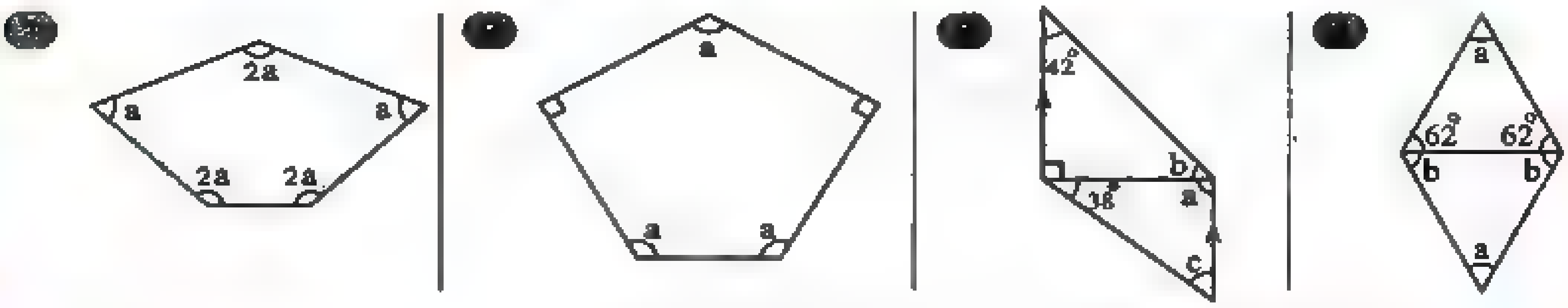








17 In each of the following , find the values of the unknown symbols :

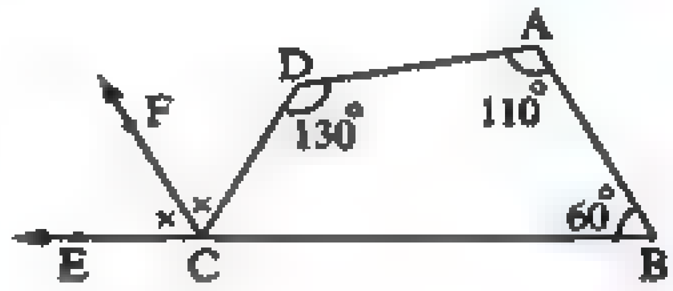


18 In the opposite figure :

m (∠ A) = 110° , m (∠ B) = 60° ,

m (∠ D) = 130° , $\overrightarrow{CF}$ bisects ∠ DCE and C ∈ $\overrightarrow{BE}$

Prove that : $\overrightarrow{CF}$ // $\overline{AB}$

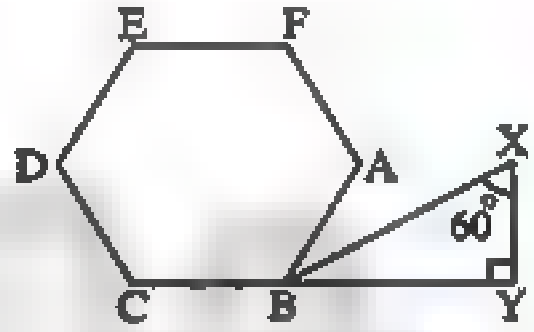


19 In the opposite figure :

ABCDEF is a regular hexagon ,

Y ∈ $\overrightarrow{CB}$, $\overline{XY}$ ⊥ $\overline{YB}$ and m (∠ X) = 60°

Prove that : $\overrightarrow{BX}$ bisects ∠ ABY



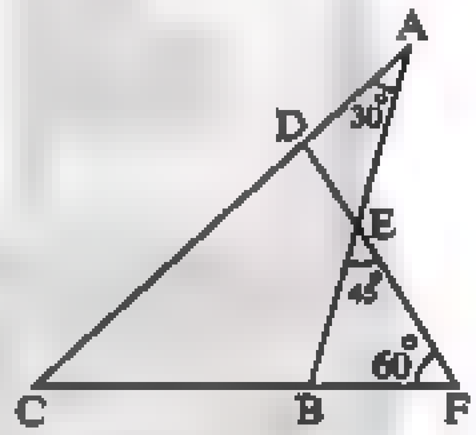
20 In the opposite figure :

ABC is a triangle in which : m (∠ A) = 30° ,

$\overrightarrow{DE}$ ∩ $\overrightarrow{CB}$ = {F} , m (∠ F) = 60° and

m (∠ BEF) = 45°

Find the measure of each angle of the figure DEBC



« m (∠ EBC) = 105° , m (∠ C) = 45° , m (∠ BED) = 135° , m (∠ EDC) = 75° »

21 If the ratio among the measures of the angles of a pentagon is 3 : 3 : 2 : 3 : 4 find the greatest measure of the angles of this pentagon. « 144° »

22 If the measure of the exterior angle of a regular polygon is 30° , how many sides does it have ? What is the sum of the measures of its interior angles ? « 12 , 1800° »

23 Is it possible that a regular polygon has an interior angle of measure 100° ? Why ?

24 A polygon of 9 sides. The sum of measures of eight angles of it is 1140°

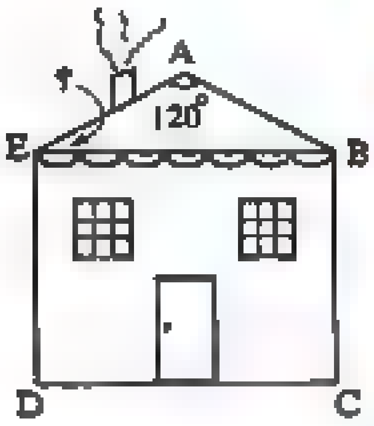
- Find the measure of the remained angle. « 120° »
- Is it possible that this polygon is regular ? Explain your answer.

25 A polygon has 15 sides :

(1) Calculate the sum of the measures of its interior angles. « 2340° »

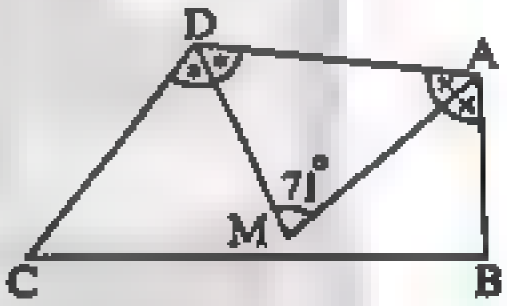
(2) If the sum of the measures of five of its exterior angles is 200°, calculate the sum of the measures of the ten interior angles which are not adjacent to the five exterior angles. « 1640° »

Life Application

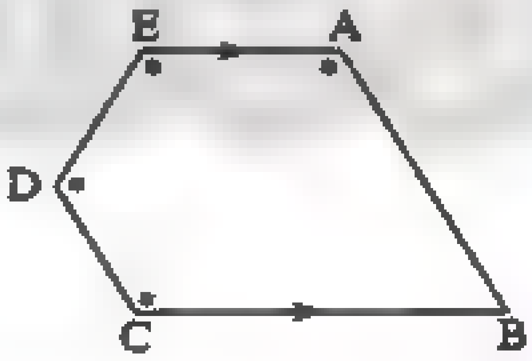


26 The opposite figure represents a design of the front of one of the house.
The measure of the angle of the ceiling = 120° ,
$\angle ABC \equiv \angle AED$
Without using the geometrical tools , find the measure of the angle of ceiling slope on the horizontal line. « 30° »

For excellent pupils



27 In the opposite figure :
$\overrightarrow{AM}$ bisects $\angle BAD$, $\overrightarrow{DM}$ bisects $\angle ADC$ and
m ($\angle AMD$) = 71°
Prove that : m ($\angle B$) + m ($\angle C$) = 142°



28 In the opposite figure :
$\overline{AE} \parallel \overline{BC}$,
m ($\angle A$) = m ($\angle E$) = m ($\angle D$) = m ($\angle C$)
Find : m ($\angle B$) « 60° »

From the school book

Exercise 3 On the parallelogram and its properties

1 Complete the following :

(1) In a parallelogram , every two opposite sides are ……… , ………

(2) In a parallelogram , every two opposite angles are ………

(3) In a parallelogram , every two consecutive angles are ………

(4) In a parallelogram , the two diagonals ………

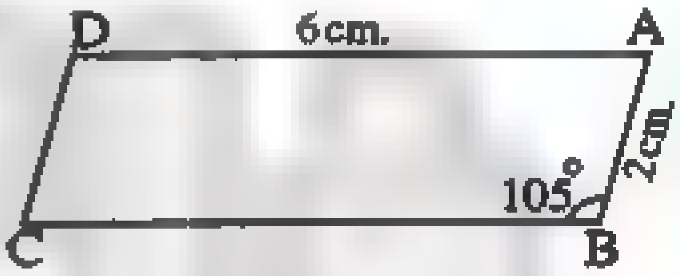
(5) 📖 The quadrilateral in which two sides are parallel is called ………

(6) A quadrilateral represents a parallelogram if ……… (write only one answer)

(7) 📖 ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = ……… °

(8) 📖 In the parallelogram XYZL , if m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then m (∠ Y) = ……… °

2 In the opposite figure :



ABCD is a parallelogram in which AB = 2 cm. ,
AD = 6 cm. and m (∠ B) = 105°

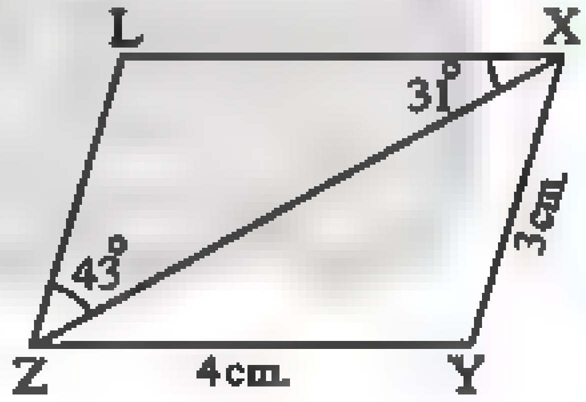
Complete the following :

(1) BC = ……… cm. , DC = ……… cm.

(2) m (∠ D) = ………° , m (∠ A) = ………° and m (∠ C) = ………°

(3) The perimeter of the parallelogram ABCD = ……… cm.

3 In the opposite figure :



XYZL is a parallelogram in which :
XY = 3 cm. , YZ = 4 cm. , m (∠ LXZ) = 31°
and m (∠ LZX) = 43°

Complete the following proof to find :

(1) m (∠ Y) (2) The perimeter of the parallelogram XYZL

Given ………………………………………………

R.T.F. ………………………………………………

Proof ∵ Δ XLZ in which : m (∠ LXZ) = ………° , m (∠ LZX) = ………°

∴ m (∠ L) = 180° − (………° + ………°) = ………°

, ∵ The figure XYZL is a parallelogram.

∴ m (∠ Y) = m (∠ ………) = ………° (First req.)

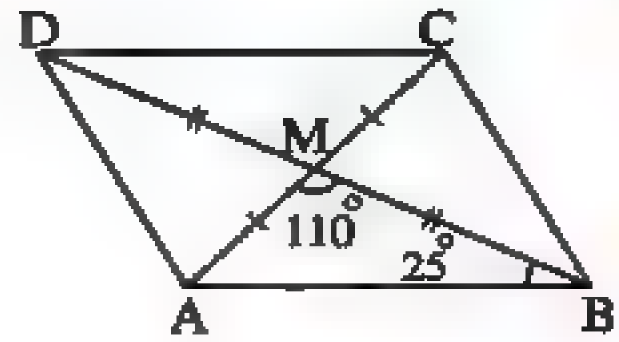
, The perimeter of the parallelogram XYZL

= (XY + ………) × 2 = (3 + ………) × ……… = ……… × ……… = ……… cm.

(Second req.)

Unit 3

4 **In the opposite figure :**



ABCD is a quadrilateral whose diagonals intersect at M , MA = MC , MB = MD , m (∠ AMB) = 110° and m (∠ MBA) = 25°

Complete the following proof :

(a) To prove that the figure ABCD is a parallelogram.

(b) To find m (∠ ACD)

Given ....

R.T.P. ....

R.T.F. ....

Proof In the figure ABCD :

∵ MA = ... (Given) , MB = ... (given)

∴ Its diagonals .... each other.

∴ The figure ABCD is .... (First req.)

InΔ MBA :

∵ m (∠ AMB) = ...° , m (∠ MBA) = ...°

∴ m (∠ MAB) = 180° – (..........° + ...°) = ...°

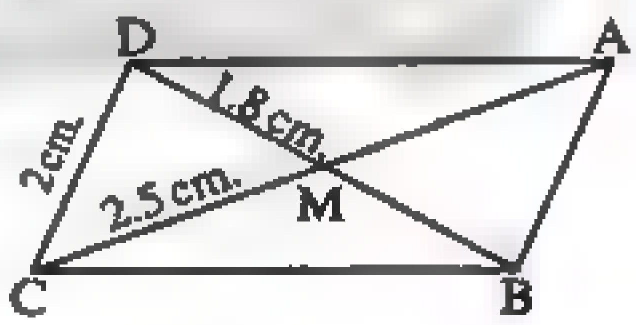
, ∵ The figure ABCD is ....

∴ $\overline{AB}$ // ....

, ∵ $\overleftrightarrow{\quad}$ .... is a transversal to them.

∴ m (∠ ACD) = m (∠ ....) = ...° (.......... angles) (Second req.)

5 **In the opposite figure :**



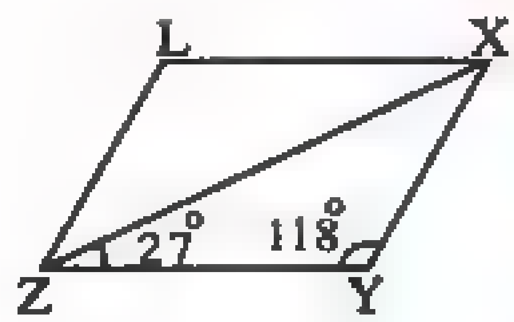
ABCD is a parallelogram such that :

$\overline{AC} \cap \overline{BD} = \{M\}$ If CD = 2 cm. ,

MC = 2.5 cm. and MD = 1.8 cm.

Calculate the perimeter of Δ AMB « 6.3 cm. »

6 **In the opposite figure :**



XYZL is a parallelogram in which :

m (∠ Y) = 118° , m (∠ XZY) = 27°

Find :

(a) m (∠ YXZ) (b) m (∠ LZX)

(c) m (∠ LXZ) (d) m (∠ L) « 35° , 35° , 27° , 118° »

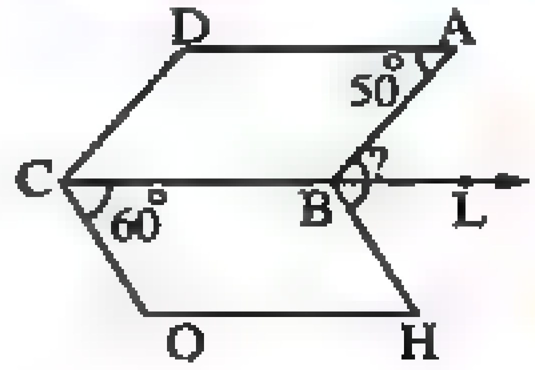
7 **In the opposite figure :**

ABCD is a parallelogram in which :

m (∠ BAD) = 50° and BHOC is a parallelogram

in which m (∠ BCO) = 60° , L ∈ $\overrightarrow{CB}$

Find with proof : m (∠ ABH) « 110° »



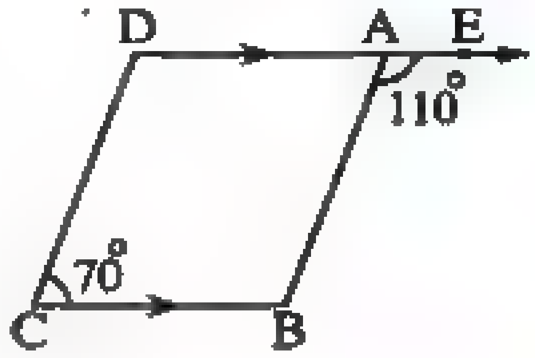
8 **In the opposite figure :**

ABCD is a quadrilateral in which :

$\overline{AD}$ // $\overline{BC}$, E ∈ $\overline{DA}$, m (∠ BAE) = 110°

and m (∠ DCB) = 70°

Prove that : ABCD is a parallelogram



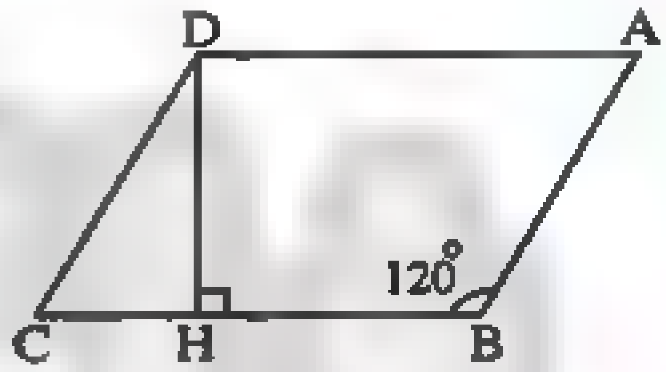
9 **In the opposite figure :**

ABCD is a parallelogram in which :

m (∠ B) = 120° and $\overline{DH}$ ⊥ $\overline{BC}$

where $\overline{DH}$ ∩ $\overline{BC}$ = {H}

Find : m (∠ HDC) « 30° »

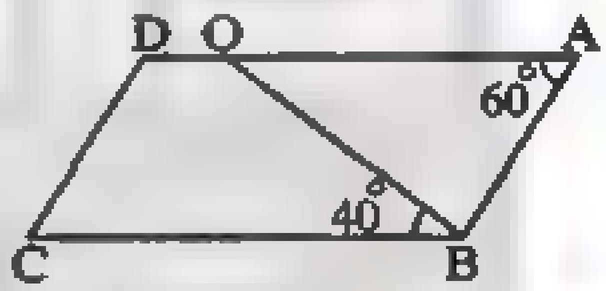


10 **In the opposite figure :**

ABCD is a parallelogram , in which :

m (∠ A) = 60° , m (∠ OBC) = 40° where O ∈ $\overline{AD}$

Find : m (∠ ABO) « 80° »

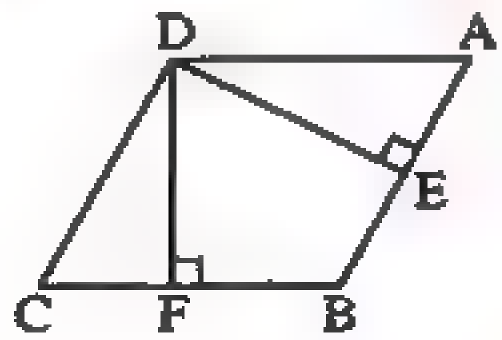


11 **In the opposite figure :**

ABCD is a parallelogram in which :

$\overline{DE}$ ⊥ $\overline{AB}$ and $\overline{DF}$ ⊥ $\overline{BC}$

Prove that : m (∠ EDF) = m (∠ A)



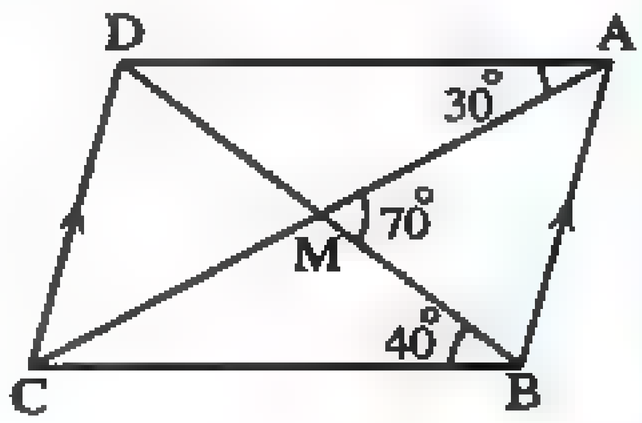
12 📖 **In the opposite figure :**

ABCD is a quadrilateral where : $\overline{AC}$ ∩ $\overline{BD}$ = {M} ,

$\overline{AB}$ // $\overline{DC}$, m (∠ AMB) = 70° , m (∠ MBC) = 40°

and m (∠ MAD) = 30°

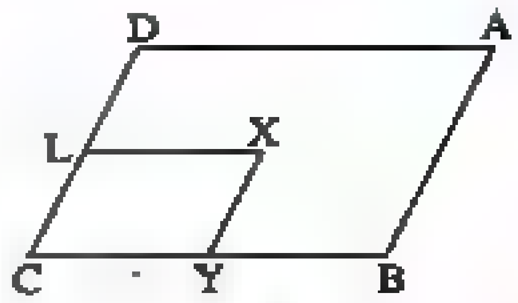
Prove that : ABCD is a parallelogram.



13 **In the opposite figure :**

If ABCD and XYCL are two parallelograms,

Prove that : m (∠ A) = m (∠ X)

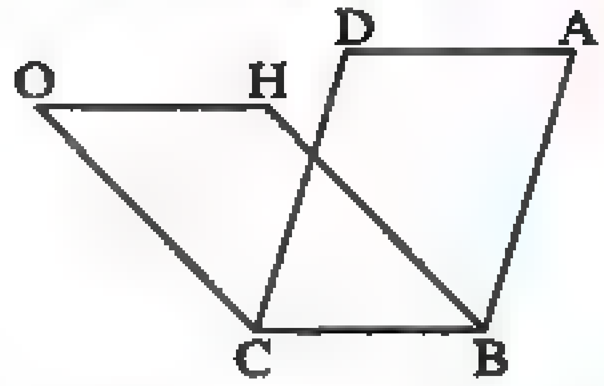


14 **In the opposite figure :**

Each of ABCD

and HBCO is a parallelogram

Prove that : AD = HO

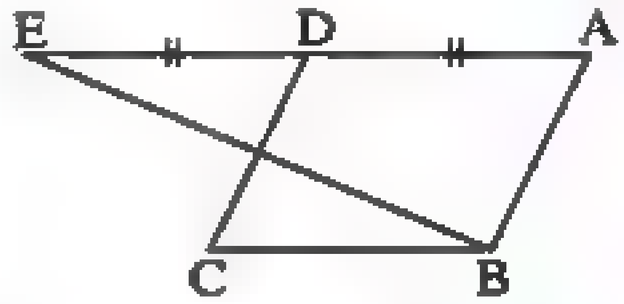


15 **In the opposite figure :**

ABCD is a parallelogram,

E ∈ $\overrightarrow{AD}$ in which : AD = DE

Prove that : $\overline{DC}$ and $\overline{BE}$ bisect each other.

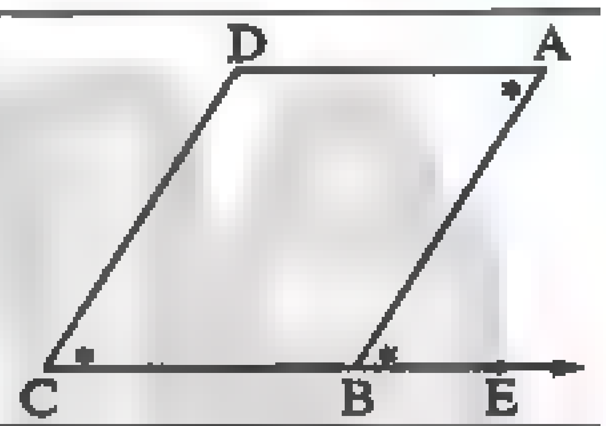


16 **In the opposite figure :**

ABCD is a quadrilateral,

E ∈ $\overrightarrow{CB}$ and m (∠ BCD) = m (∠ EBA) = m (∠ A)

Prove that : ABCD is a parallelogram.



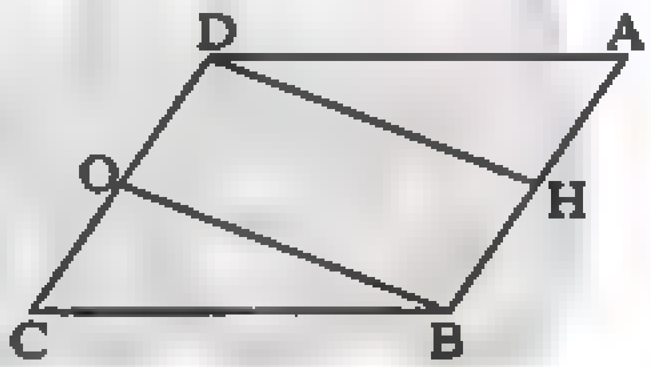
17 **In the opposite figure :**

ABCD is a parallelogram ,

H is the midpoint of $\overline{AB}$

and O is the midpoint of $\overline{DC}$

Prove that : HBOD is a parallelogram.

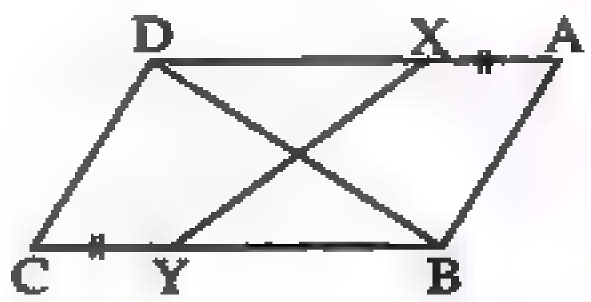


18 **In the opposite figure :**

ABCD is a parallelogram ,

X ∈ $\overline{AD}$, Y ∈ $\overline{BC}$, if AX = CY

Prove that : $\overline{XY}$ and $\overline{BD}$ bisect each other.



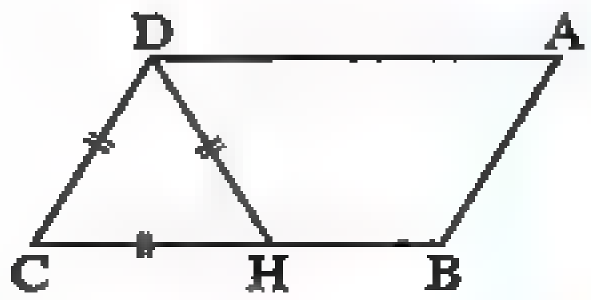
19 **In the opposite figure :**

ABCD is a parallelogram , H ∈ $\overline{BC}$ where :

Δ DHC is an equilateral triangle.

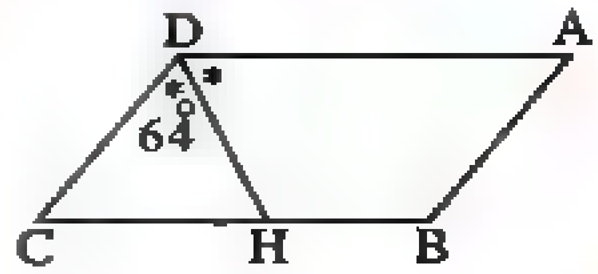
1 **Prove that :** HC = AB

2 **Find :** m (∠ B) and m (∠ HDA)



« 120° , 60° »

20 **In the opposite figure :**



ABCD is a parallelogram , $H \in \overline{BC}$

, $\overrightarrow{DH}$ bisects $\angle$ ADC and m ($\angle$ HDC) = 64°

Find : (1) m ($\angle$ DHB) (2) m ($\angle$ ABC) « 116° , 128° »

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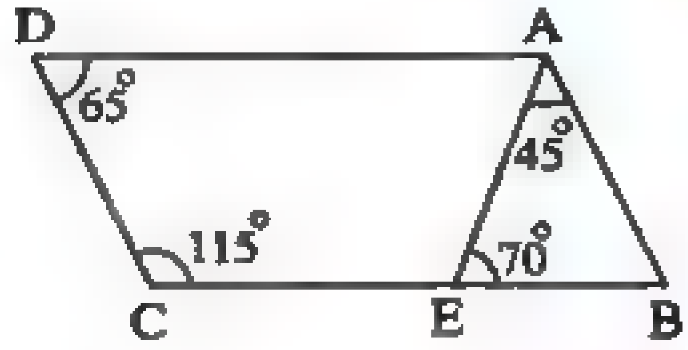
21 **In the opposite figure :**



$E \in \overline{BC}$, m ($\angle$ BAE) = 45°,

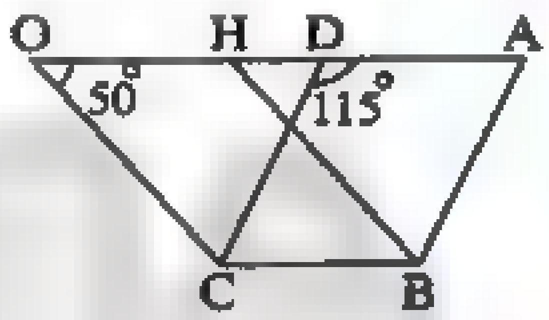
m ($\angle$ AEB) = 70° , m ($\angle$ D) = 65°

and m ($\angle$ C) = 115°

Prove that : ABCD is a parallelogram.

---

22 **In the opposite figure :**



ABCD and HBCO are two parallelograms

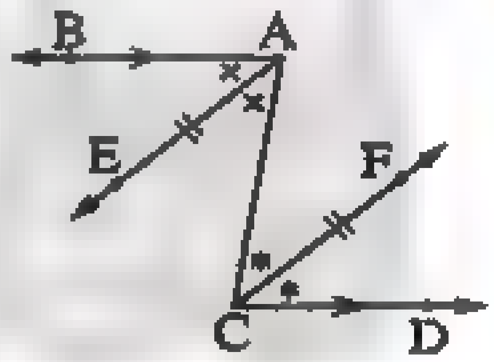
such that m ($\angle$ O) = 50°

and m ($\angle$ ADC) = 115°

Find : m ($\angle$ ABH) « 65° »

---

23 **In the opposite figure :**



$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AE}$ bisects $\angle$ BAC ,

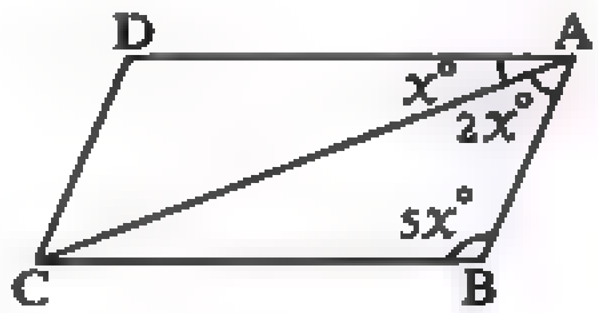
$\overrightarrow{CF}$ bisects $\angle$ ACD

If AE = CF ,

Prove that : AECF is a parallelogram.

---

24 **In the opposite figure :**



ABCD is a parallelogram in which :

m ($\angle$ DAC) = x° , m ($\angle$ BAC) = 2 x°

and m ($\angle$ ABC) = 5 x°

Find : m ($\angle$ BCD) and m ($\angle$ ADC) in degrees. « 67.5° , 112.5° »

---

25 **Choose the correct answer from the given ones :**

(1) ABCD is a parallelogram in which : m ($\angle$ A) = 50° , then m ($\angle$ C) = ...

(a) 50° (b) 60° (c) 130° (d) 150°

Unit 3

ABCD is a parallelogram in which : m (∠ A) + m (∠ C) = 140°
, then m (∠ B) = ...

(a) 70° (b) 40° (c) 110° (d) 220°

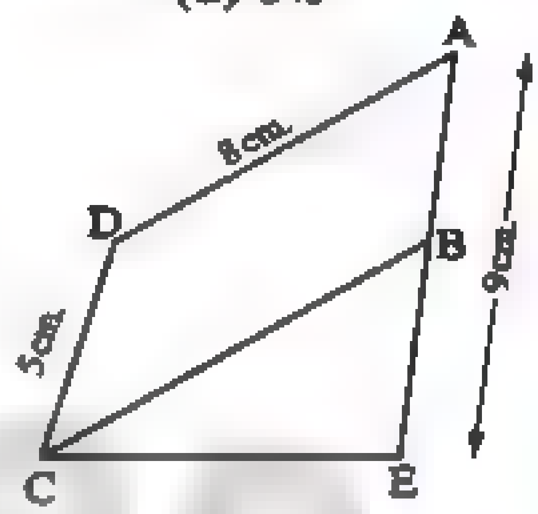
If the lengths of two consecutive sides of a parallelogram are 3 cm. and 5 cm. , then its perimeter equals .... cm.

(a) 12 (b) 14 (c) 16 (d) 18

If the perimeter of a parallelogram is 25 cm. and if one of its sides is of length 7 cm. , then the consecutive side is of length .... cm.

(a) 7 (b) 18 (c) 12.5 (d) 5.5

In the opposite figure :

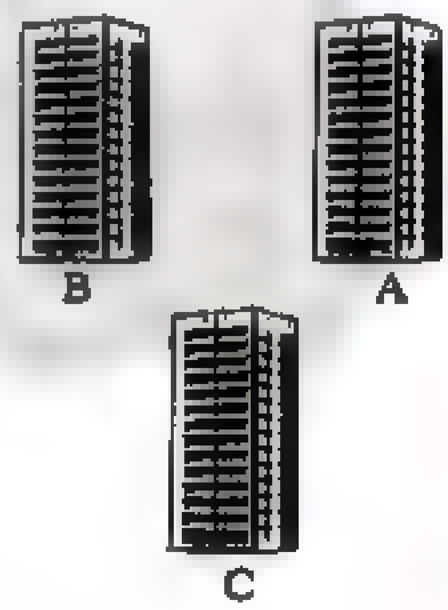


If ABCD is a parallelogram,
E ∈ $\overrightarrow{AB}$, CD = 5 cm., AE = 9 cm.
AD = 8 cm., the perimeter of Δ BEC = 18 cm.
, then the length of $\overline{EC}$ = ... cm.

(a) 8 (b) 6 (c) 5 (d) 4

Life Application

26 In one of the new cities , each four buildings are put like A , B , C and D according to the following :



- The distance between the two buildings A and B equals the distance between the two buildings C and D
- $\overline{AB}$ // $\overline{CD}$ Explain how we can decide the place where the building D will be built.

For excellent pupils

27 ABCD is a parallelogram in which : E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{CD}$, if $\overline{AF}$ ∩ $\overline{DE}$ = {M} , $\overline{BF}$ ∩ $\overline{CE}$ = {N}

Prove that : ❶ $\overline{ED}$ // $\overline{FB}$ ❷ FMEN is a parallelogram.

---

28 XYZL is a parallelogram in which : m (∠ Y) = 3 m (∠ X)

Find the measures of the interior angles of XYZL

« m (∠ Y) = m (∠ L) = 135° and m (∠ X) = m (∠ Z) = 45° »

From the school book

Exercise 4 On the special cases of the parallelogram

1 Complete the following :

1. A parallelogram whose two diagonals are perpendicular is called ....
2. The parallelogram whose two diagonals are .... is called a rectangle.
3. The parallelogram whose two diagonals are equal in length and perpendicular is called ....
4. The quadrilateral whose sides are equal in length is called ....
5. The quadrilateral whose diagonals bisect each other is called ....
6. The rectangle is a .... with a right angle.
7. The rhombus is a .... in which its diagonals are perpendicular.
8. The square is a .... with a right angle.
9. The rhombus whose two diagonals are equal in length is called ....
10. The rectangle in which its two diagonals are perpendicular is called ....
11. The rectangle in which its two adjacent sides have the same length is called ....
12. If $\overline{XY} // \overline{ZL}$, $XY = ZL$, then the quadrilateral XYZL is called ....
13. If ABCD is a rhombus, then .... $\perp$ ....
14. The perimeter of the square = ... ,
    The perimeter of the rectangle = ... and
    The perimeter of the rhombus = ...
15. The rhombus whose perimeter is 42 cm., its side length = ... cm.

2 Choose the correct answer from the given ones :

1. The two diagonals of a rectangle ....
   (a) are perpendicular. (b) are equal in length.
   (c) are perpendicular and equal in length. (d) bisect its interior angles.
2. The two diagonals of a rhombus are ....
   (a) perpendicular and are not equal.
   (b) equal in length and are not perpendicular.
   (c) perpendicular and equal in length.
   (d) not equal in length and are not perpendicular.

Unit 3

(3) The two diagonals of the square , are ....

(a) just perpendicular.

(b) just equal in length.

(c) perpendicular and equal in length.

(d) not equal in length and are not perpendicular.

(4) If two adjacent sides are equal in length in a parallelogram, then the figure is a ....

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

(5) If : ABCD is a rectangle in which AC = 5 cm., then : BD = ... cm.

(a) 2.5 (b) 5 (c) 10 (d) 20

(6) If : ABCD is a square, then : m (∠ CAB) = ...

(a) 90° (b) 45° (c) 60° (d) 30°

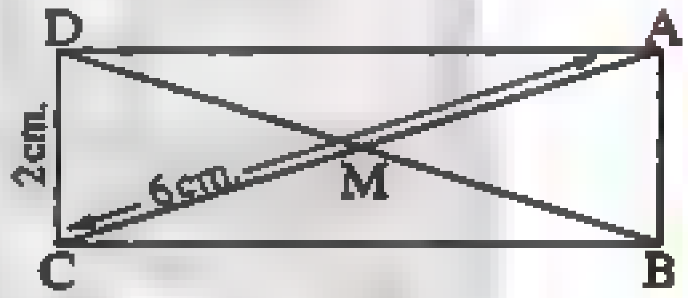
(7) If : ABCD is a parallelogram in which m (∠ A) = m (∠ B), then : ABCD is a ....

(a) rectangle. (b) rhombus. (c) square. (d) trapezium.

(8) If : ABCD is a rhombus in which m (∠ ACB) = 32°, then : m (∠ D) = ...

(a) 32° (b) 64° (c) 116° (d) 26°

3 **In the opposite figure :**



ABCD is a rectangle in which :

AC = 6 cm. , CD = 2 cm.

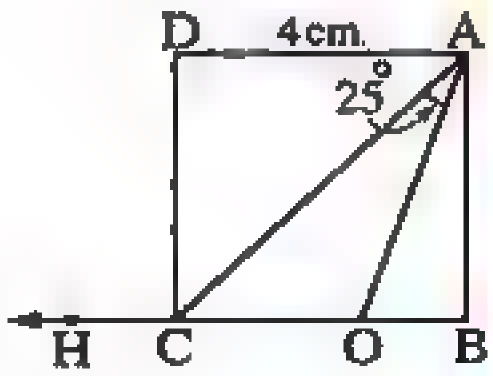
and $\overline{AC} \cap \overline{BD} = \{M\}$

Complete : (1) AB = ... cm.

(2) DM = ... cm.

(3) The perimeter of Δ ABM = ... cm.

4 **In the opposite figure :**



ABCD is a square in which AD = 4 cm. ,

$O \in \overline{BC}$ such that : m (∠ OAC) = 25°

and $H \in \overrightarrow{BC}$

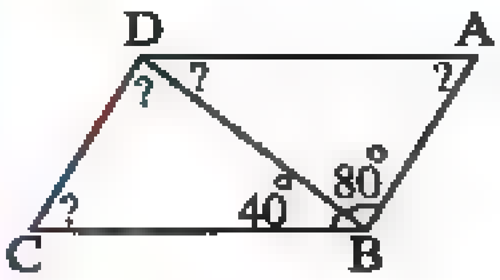
Complete the following :

(1) The perimeter of the square = ... cm.

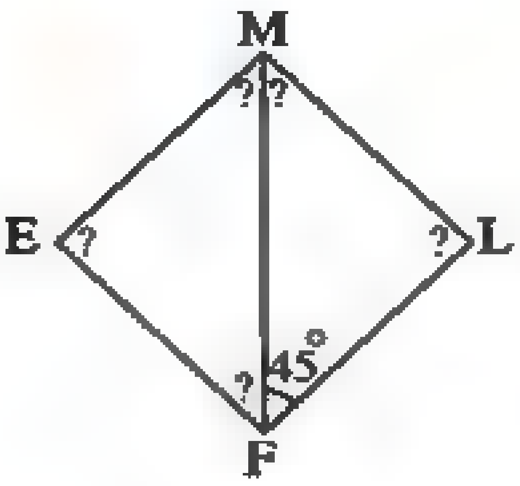
(2) m (∠ ACH) = ...°

(3) m (∠ AOC) = ...°

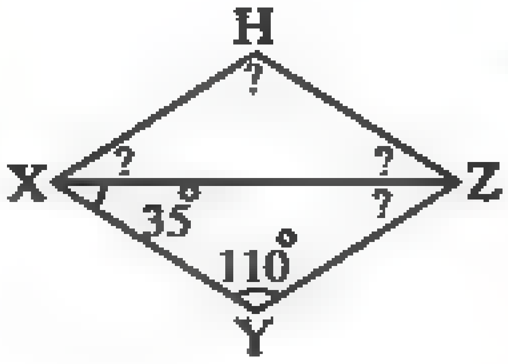
5 **Find the measures of the angles marked by (?) in each of the following figures :**



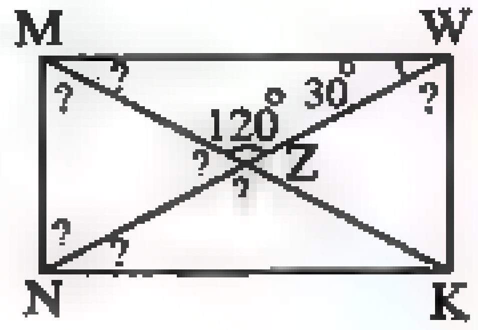
A parallelogram
Fig. (1)



a square
Fig. (2)

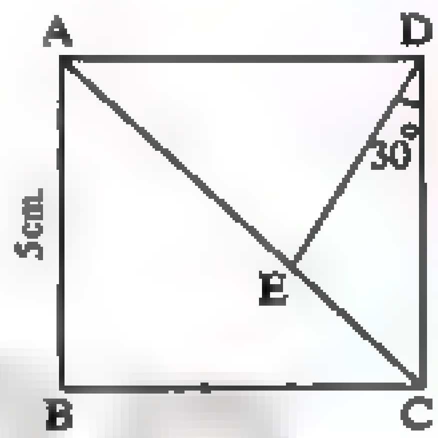


a rhombus
Fig. (3)



a rectangle
Fig. (4)

6 **In the opposite figure :**



ABCD is a square whose side length = 5 cm.,

E ∈ $\overline{AC}$ in which m (∠ EDC) = 30°

Complete the following proof to find :

(1) The perimeter of the square ABCD

(2) m (∠ AED)

Given ....

R.T.F. ....

Proof ∵ The perimeter of the square = side length × ....

∴ The perimeter of the square ABCD

= .... × .... = ... cm. (First req.)

, ∵ ABCD is a square , $\overline{AC}$ is a diagonal

∴ m (∠ ACD) = ...°

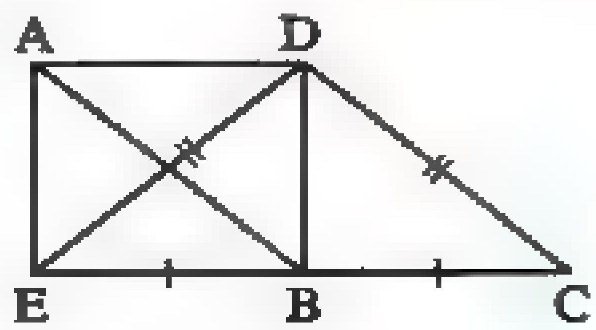
In Δ DEC :

m (∠ DEC) = 180° – (..........° + ...°) = ...°

, ∵ E ∈ $\overline{AC}$

∴ m (∠ AED) = ...° – ....° = ...° (Second req.)

7 **In the opposite figure :**



ABCD is a parallelogram,

E ∈ $\overrightarrow{CB}$ where BC = BE , if DE = DC

Complete the following proof to prove that :

The figure AEBD is a rectangle.

Unit 3

Given ....

R.T.P. ....

Proof ∵ ABCD is a parallelogram.

∴ AD = ... , ... // $\overline{BC}$

, ∵ EB = ... , E ∈ $\overrightarrow{CB}$ (given)

∴ AD = ... , ... // $\overline{EB}$

∴ The figure AEBD is a parallelogram.

, ∵ DE = ... (given)

, AB = ... (properties of parallelogram)

∴ DE = ...

∴ The two diagonals of the parallelogram AEBD are ....

∴ The figure AEBD is a .... (Q.E.D.)

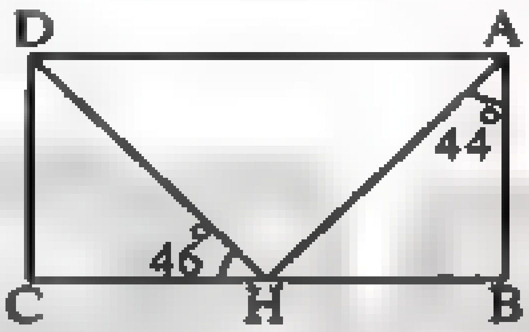
---

8 **In the opposite figure :**

ABCD is a rectangle , H ∈ $\overline{BC}$ such that :

m (∠ DHC) = 46° and m (∠ BAH) = 44°

Calculate : m (∠ AHD)



« 88° »

---

9 **In the opposite figure :**

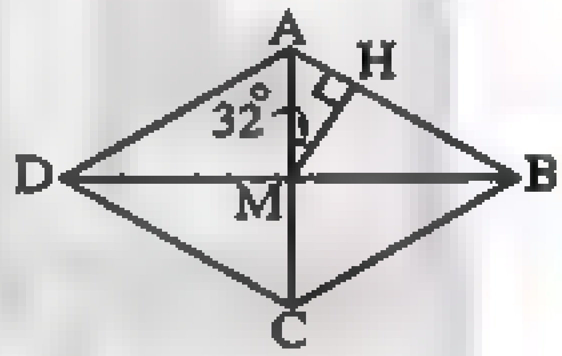
ABCD is a rhombus in which :

$\overline{AC}$ ∩ $\overline{BD}$ = {M} ,

H ∈ $\overline{AB}$ where $\overline{MH}$ ⊥ $\overline{AB}$

If m (∠ AMH) = 32° ,

then calculate the measures of the angles of the rhombus ABCD « 116° , 64° , 116° , 64° »



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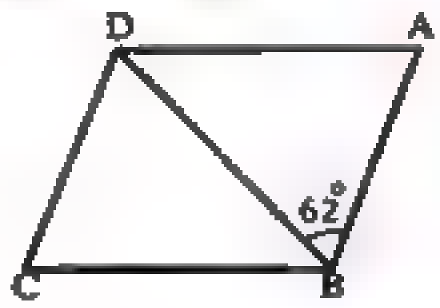
10 **In the opposite figure :**

ABCD is a rhombus,

$\overline{BD}$ is a diagonal in it ,

m (∠ ABD) = 62°

Find with proof : m (∠ A)



« 56° »

---

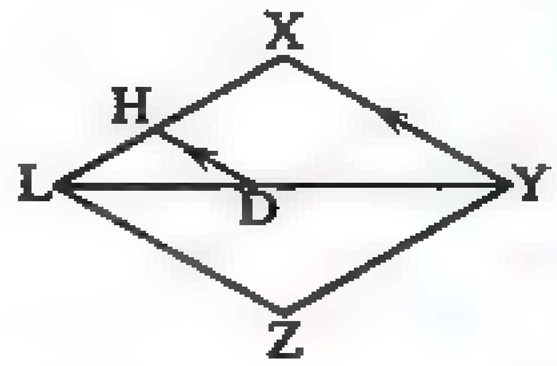
11 **In the opposite figure :**

XYZL is a rhombus , D ∈ $\overline{YL}$

Draw $\overrightarrow{DH}$ // $\overrightarrow{YX}$ such that :

$\overrightarrow{DH}$ ∩ $\overline{XL}$ = {H}

Prove that : m (∠ HDL) = m (∠ HLD)

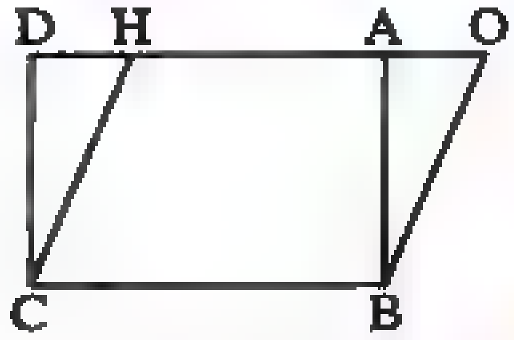


12 **In the opposite figure :**

ABCD is a rectangle

and OBCH is a parallelogram

Prove that : DH = AO

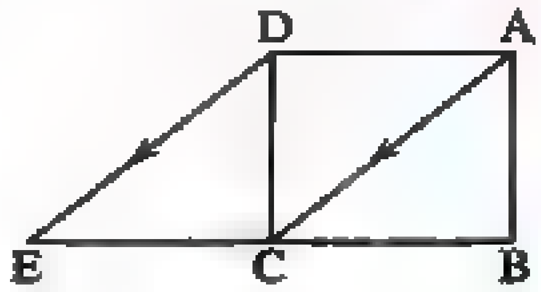


13 **In the opposite figure :**

ABCD is a square, $E \in \overrightarrow{BC}$, $\overline{AC} // \overline{DE}$

Prove that : ACED is a parallelogram.

Find : m (∠ ACE)



« 135° »

14 ABCD is a rhombus in which m (∠ BAC) = 45°

Prove that : ABCD is a square.

15 **In the opposite figure :**

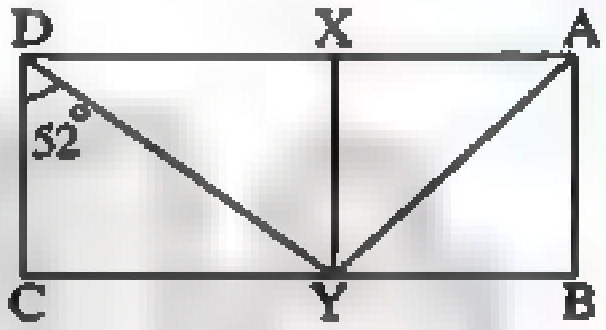
ABCD is a rectangle, $X \in \overline{AD}$

and $Y \in \overline{BC}$ such that :

AXYB is a square.

If m (∠ YDC) = 52°,

then find with proof : m (∠ AYD)



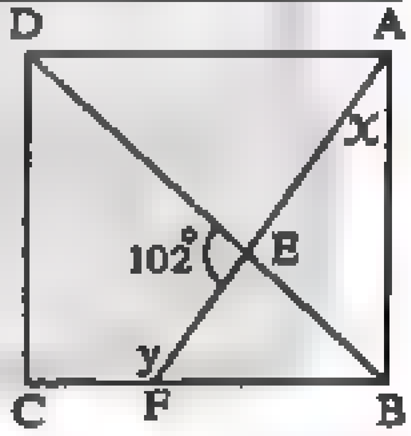
« 97° »

16 **In the opposite figure :**

ABCD is a square,

Find in degrees the value

of each of x and y



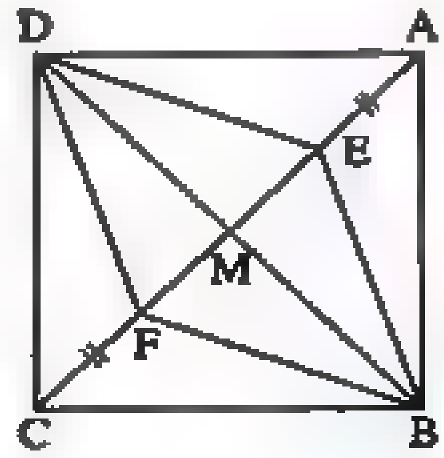
« 33°, 123° »

17 **In the opposite figure :**

ABCD is a square, its diagonals intersect at M,

$E \in \overline{AC}$, $F \in \overline{AC}$ such that : AE = CF

Prove that : EBFD is a rhombus.



18 **In the opposite figure :**

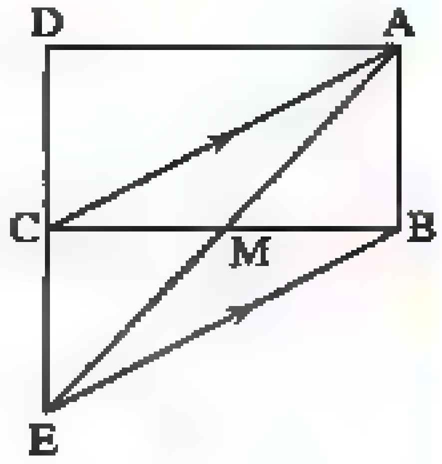
ABCD is a rectangle,

$\overrightarrow{BE} \parallel \overline{AC}$, $\overrightarrow{BE} \cap \overrightarrow{DC} = \{E\}$,

$\overline{BC} \cap \overline{AE} = \{M\}$

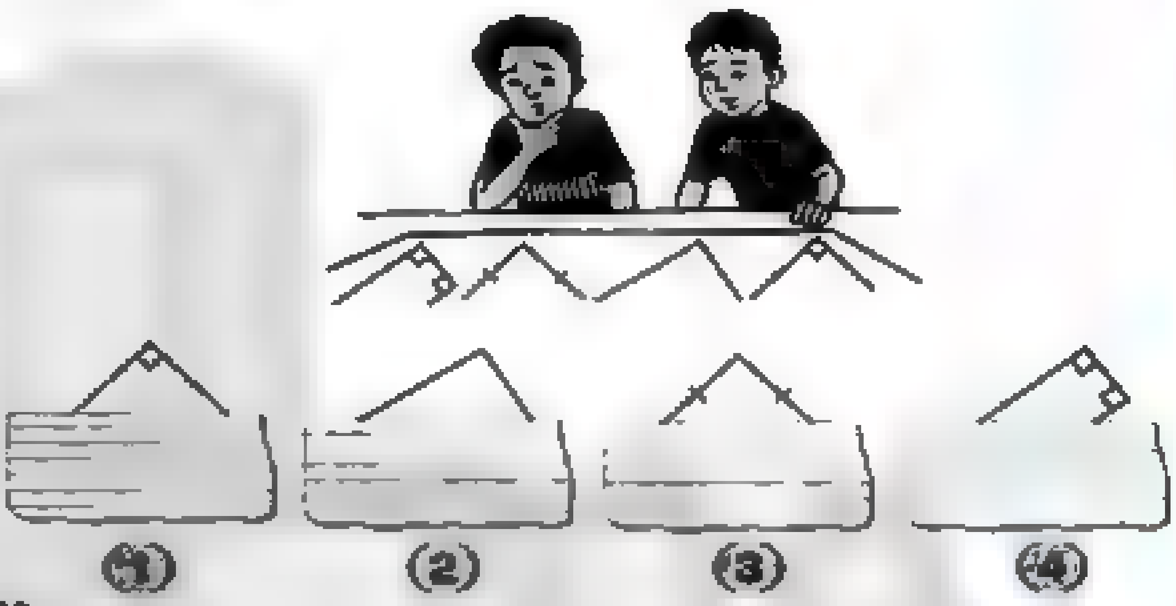
Prove that :

(1) DC = CE (2) $MC = \frac{1}{2}$ AD



Life Application

19 Eslam drew a parallelogram ,
a rhombus , a rectangle
and a square , then he hid parts
of them as in the opposite
figure and he asked his friend
Bassem to recognize each figure.
Help Bassem to name each drawn figure.



For excellent pupils

20 **Use "some" or "all" to get a correct statement :**

(1) ········· squares are rectangles.

(2) ········· quadrilaterals are parallelograms.

(3) ········· squares are rhombuses.

(4) ········· parallelograms are rectangles.

(5) ········· rectangles are parallelograms.

(6) ········· rhombuses are squares.

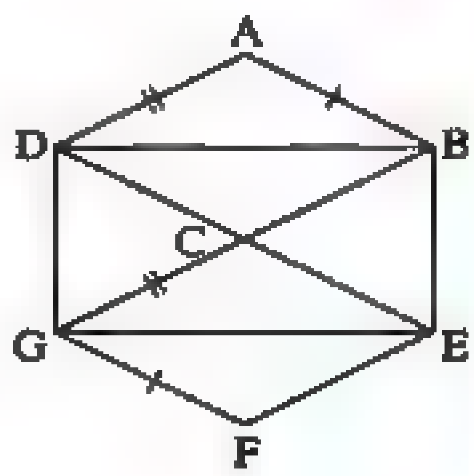
21 **In the opposite figure :**

ABCD is a parallelogram,

CEFG is a rhombus,

if AB = GF , AD = CG

Prove that : BEGD is a rectangle.



From the school book

Exercise 5 On the triangle

1 Complete the following :

(1) The sum of measures of the inteior angles of a triangle = ...°

(2) The measure of the exterior angle of a triangle is equal to the sum of ....

(3) If the measure of an angle in a triangle equals the sum of measures of the other two angles in the triangle , then the triangle is ....

(4) If the measure of an angle in a triangle is greater than the sum of measures of the other two angles , then the triangle is ....

(5) In Δ ABC : If m ($\angle$ A) + m ($\angle$ C) = m ($\angle$ B) , then m ($\angle$ B) = ...°

(6) In Δ ABC : If m ($\angle$ B) > m ($\angle$ A) + m ($\angle$ C) , then $\angle$ B is ....

(7) It is possible to find a triangle each of its interior angles is of measure ....°

2 Choose the correct answer from the given ones :

(1) The triangle contains two .... angles at least.

(a) acute (b) obtuse (c) right (d) reflex

(2) The sum of measures of the interior angles of a triangle equals the measure of .... angle.

(a) a right (b) a straight (c) an acute (d) a reflex

(3) In Δ XYZ , if : m ($\angle$ X) = 50° , m ($\angle$ Y) = 100° , then : m ($\angle$ Z) = ...

(a) 30° (b) 50° (c) 80° (d) 100°

(4) In Δ ABC , if : m ($\angle$ A) + m ($\angle$ B) = 110° , then : m ($\angle$ C) = ...

(a) 110° (b) 90° (c) 70° (d) 55°

(5) If the measures of two angles in a triangle are 35° and 45° , then the triangle is ....

(a) acute-angled (b) right-angled (c) obtuse-angled (d) equilateral

(6) The measure of the exterior angle of the equilateral triangle at any one of its vertices equals ....

(a) 60° (b) 120° (c) 150° (d) 30°

Unit 3

3 In each of the following figures , find the measure of the angle marked by (?) :

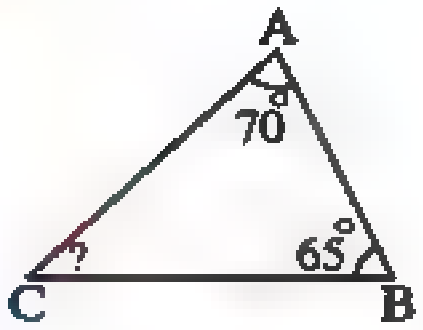


fig. (1)

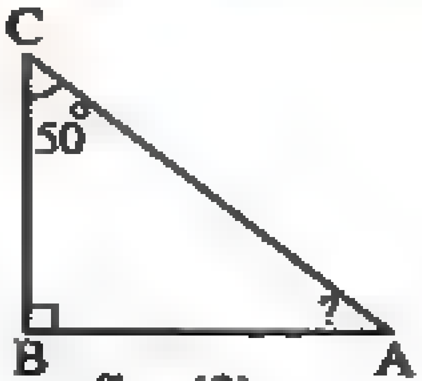


fig. (2)

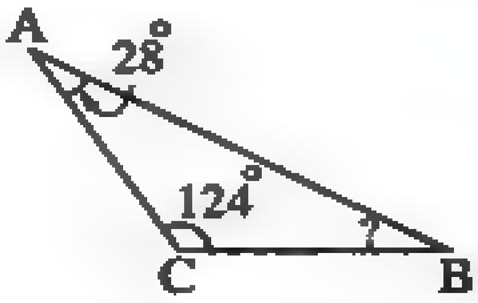


fig. (3)

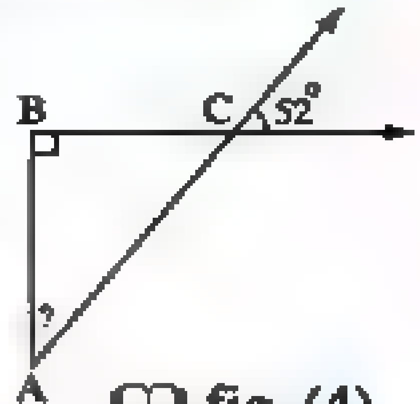


fig. (4)

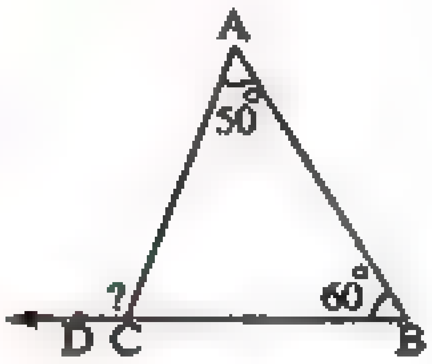


fig. (5)

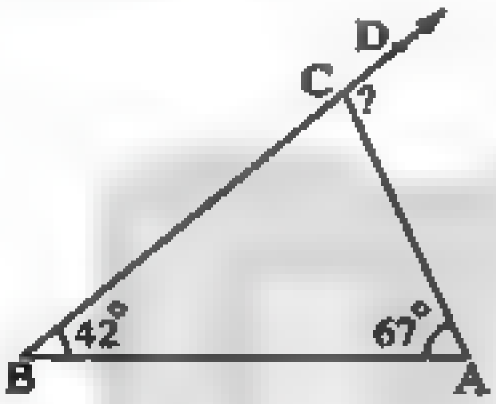


fig. (6)

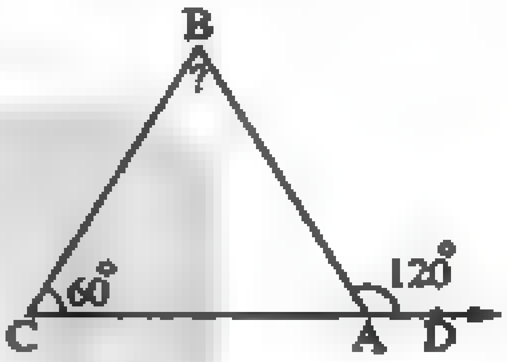


fig. (7)

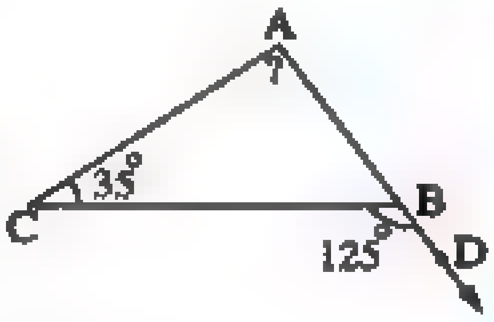


fig. (8)

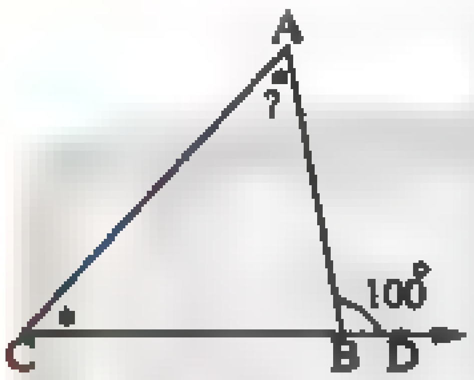


fig. (9)

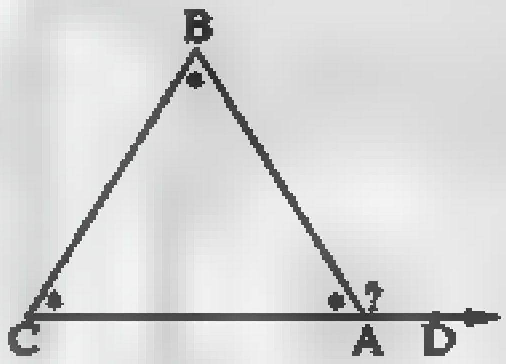


fig. (10)

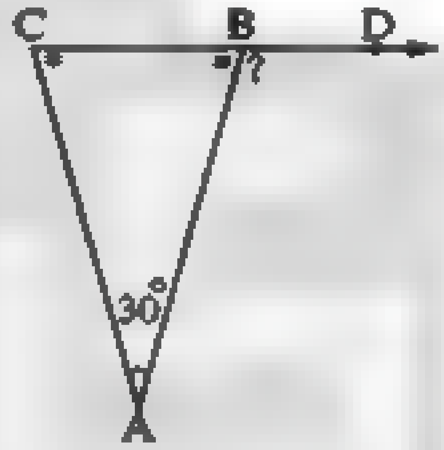


fig. (11)

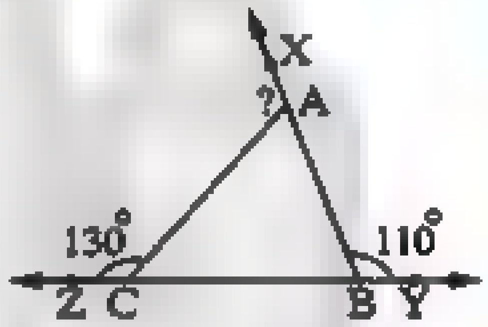


fig. (12)

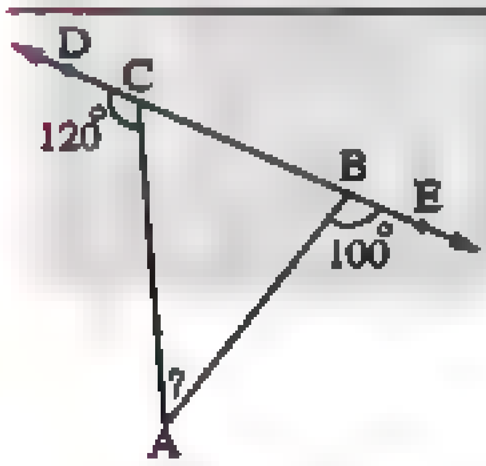


fig. (13)

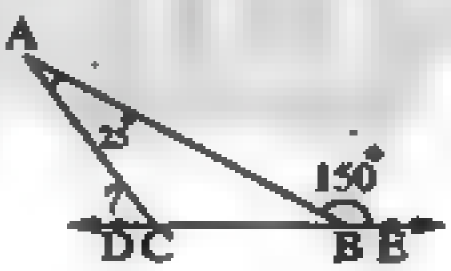


fig. (14)

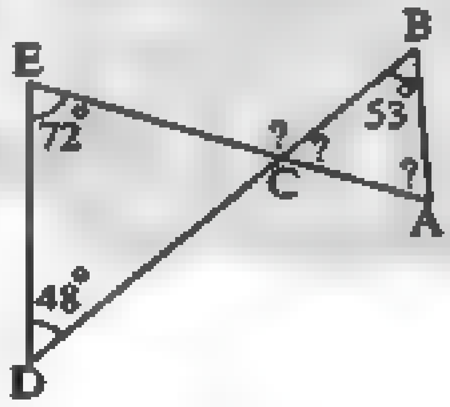


fig. (15)

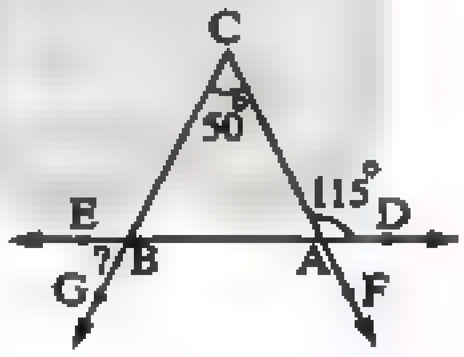


fig. (16)

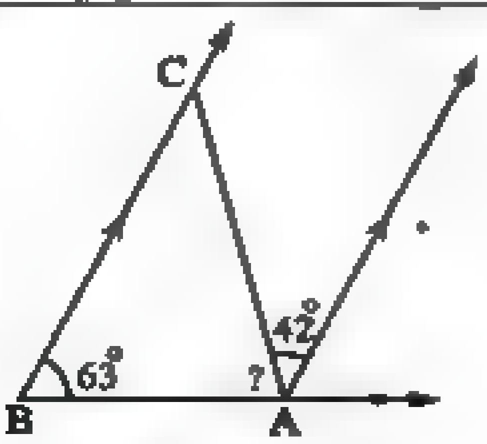


fig. (17)

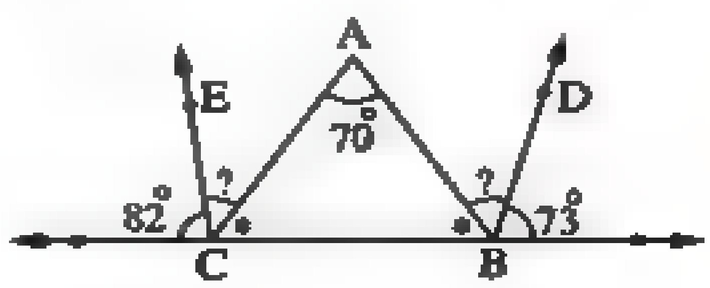
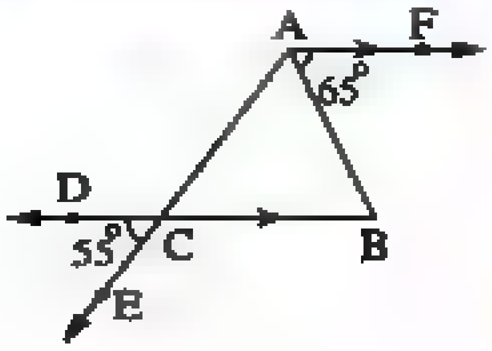


fig. (18)

4 **In the opposite figure :**



$\overrightarrow{BD} \cap \overrightarrow{AE} = \{C\}$, $\overrightarrow{AF} \parallel \overrightarrow{BC}$, m (∠ BAF) = 65°

, m (∠ DCE) = 55°

Complete the following proof to find :

the measures of the interior angles of Δ ABC

Given ....

R.T.F. ....

Proof ∵ $\overrightarrow{BD} \cap \overrightarrow{AE} = \{C\}$ (given)

∴ m (∠ ACB) = m (∠ ....) = ...° (..........)

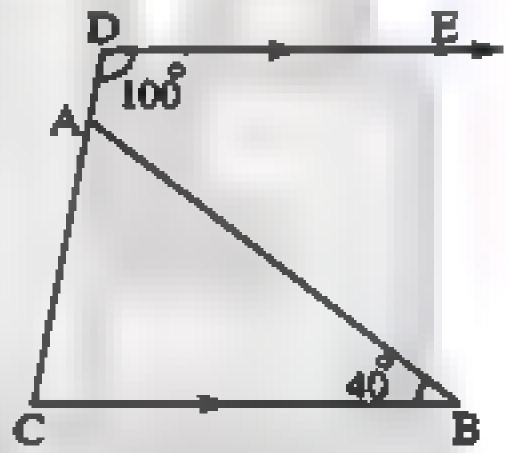
, ∵ $\overrightarrow{AF} \parallel \overrightarrow{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

∴ m (∠ FAB) = m (∠ ....) = ...° (...... ... angles)

, ∵ The sum of measures of the interior anlges of the triangle = ...°

∴ m (∠ BAC) = ...° − (..........° + ...°) = ...° (The req.)

5 **In the opposite figure :**



A ∈ $\overline{DC}$, $\overrightarrow{DE} \parallel \overrightarrow{CB}$, m (∠ D) = 100° and

m (∠ B) = 40°

Complete the following proof to find : m (∠ BAD)

Given ....

R.T.F. ....

Proof ∵ $\overrightarrow{DE} \parallel \overline{..........}$, $\overleftrightarrow{..........}$ is a transversal.

∴ m (∠ D) + m (∠ C) = ...°

(Two interior angles in the same side of the transversal)

∵ m (∠ D) = ...°

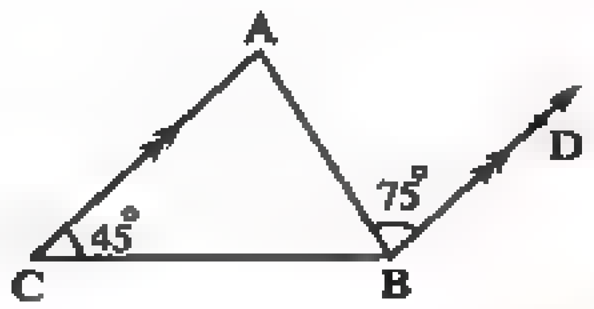
∴ m (∠ C) = ... − .... = ...°

∵ ∠ BAD is an exterior angle of Δ ....

∴ m (∠ BAD) = m (∠ ....) + m (∠ ....)

= ....° + ...° = ...° (The req.)

6 **In the opposite figure :**



$\overrightarrow{BD} \parallel \overrightarrow{CA}$, m (∠ C) = 45° and

m (∠ ABD) = 75°

Find : m (∠ ABC) « 60° »

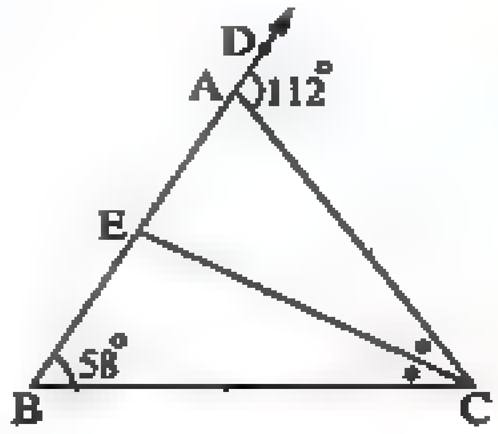
7 **In the opposite figure :**

ABC is a triangle in which : m (∠ B) = 58° ,

E ∈ $\overline{AB}$ such that $\overrightarrow{CE}$ bisects ∠ ACB ,

D ∈ $\overrightarrow{BA}$ and m (∠ CAD) = 112°

Find : m (∠ AEC)



« 85° »

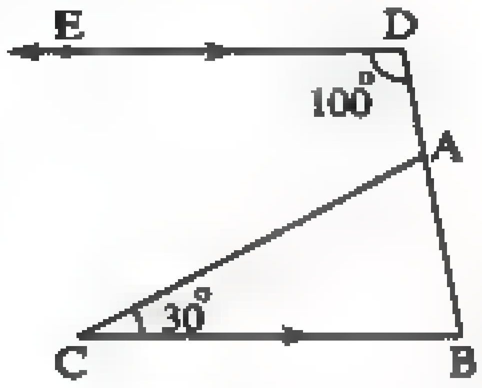
8 **In the opposite figure :**

$\overrightarrow{DE}$ // $\overline{BC}$, m (∠ D) = 100° ,

m (∠ C) = 30° and

A ∈ $\overline{DB}$

Find : m (∠ BAC)



« 70° »

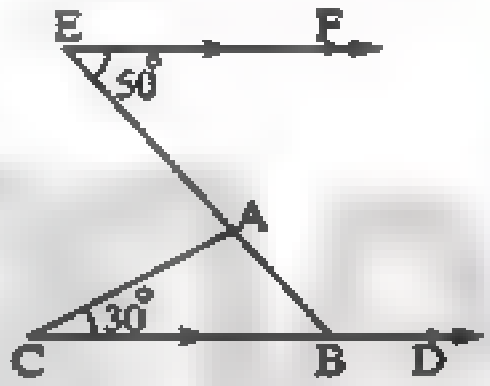
9 **In the opposite figure :**

$\overrightarrow{EF}$ // $\overrightarrow{CD}$, m (∠ E) = 50° and

m (∠ C) = 30°

Find the measures of the angles

of Δ ABC and m (∠ ABD)



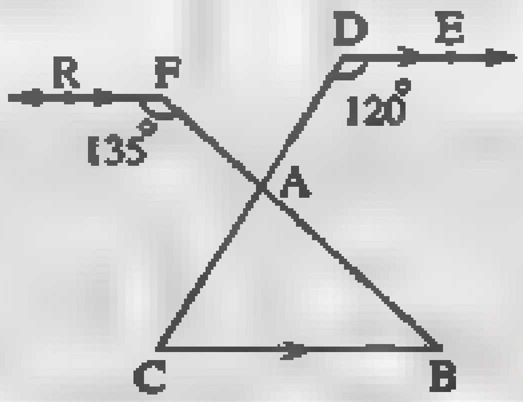
« m (∠ ABC) = 50° , m (∠ BAC) = 100° , m (∠ ABD) = 130° »

10 **In the opposite figure :**

$\overrightarrow{DE}$ // $\overrightarrow{FR}$ // $\overline{BC}$,

m (∠ CDE) = 120° and m (∠ RFB) = 135°

Calculate the measures of the angles of Δ ABC



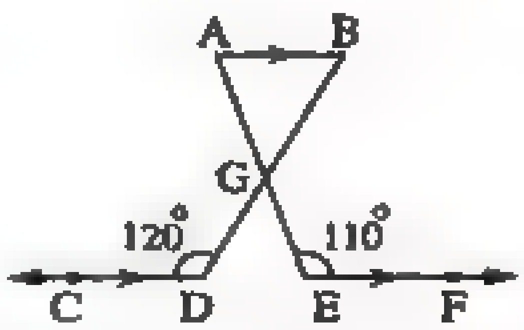
« m (∠ B) = 45° , m (∠ C) = 60° , m (∠ A) = 75° »

11 **In the opposite figure :**

$\overline{AB}$ // $\overleftrightarrow{DC}$ // $\overrightarrow{EF}$, m (∠ E) = 110° and

m (∠ D) = 120°

Find : m (∠ EGD)



« 50° »

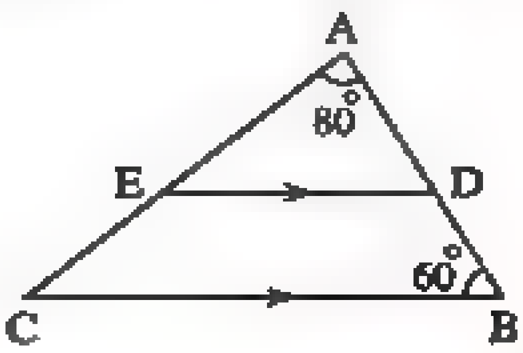
12 **In the opposite figure :**

ABC is a triangle in which : m (∠ A) = 80° and

m (∠ B) = 60°

$\overline{DE}$ // $\overline{BC}$ where : D ∈ $\overline{AB}$ and E ∈ $\overline{AC}$

Find : m (∠ AED) and m (∠ DEC)



« 40° , 140° »

13 **In the opposite figure :**

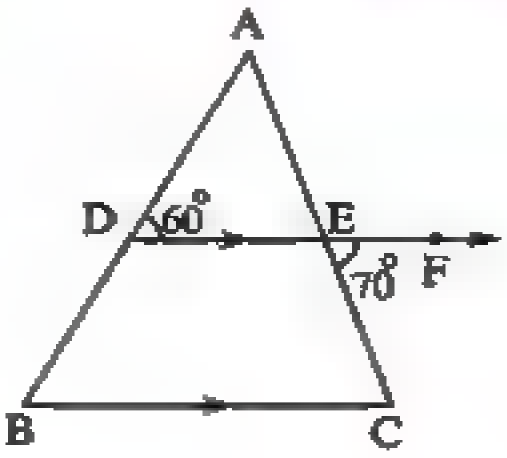
ABC is a triangle , m (∠ ADE) = 60° ,

m (∠ FEC) = 70° ,

D ∈ $\overline{AB}$, $\overrightarrow{DF}$ // $\overline{BC}$ and

$\overline{AC}$ ∩ $\overrightarrow{DF}$ = {E}

Find the measures of the interior angles of Δ ABC



« m (∠ C) = 70° , m (∠ B) = 60° , m (∠ A) = 50° »

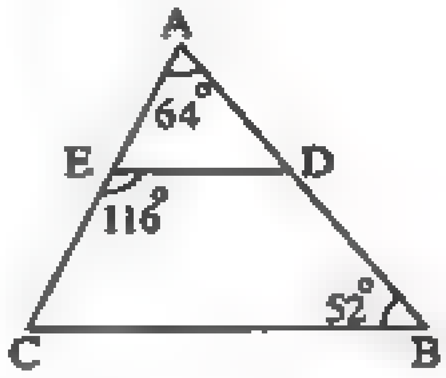
14 **In the opposite figure :**

ABC is a triangle in which m (∠ A) = 64° ,

m (∠ B) = 52° ,

m (∠ DEC) = 116° , E ∈ $\overline{AC}$ and D ∈ $\overline{AB}$

Prove that : $\overline{DE}$ // $\overline{BC}$



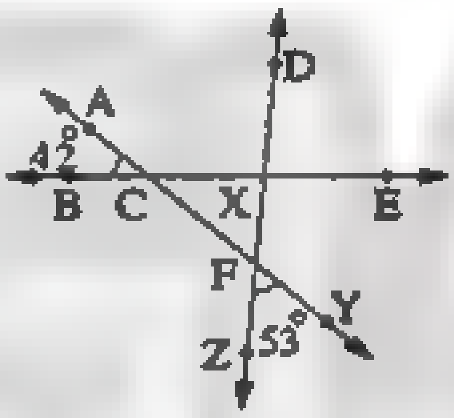
15 📖 **In the opposite figure :**

Prove that :

m (∠ DXE) = 85° ,

then calculate m (∠ DXC)

and m (∠ EXF)



« 95° , 95° »

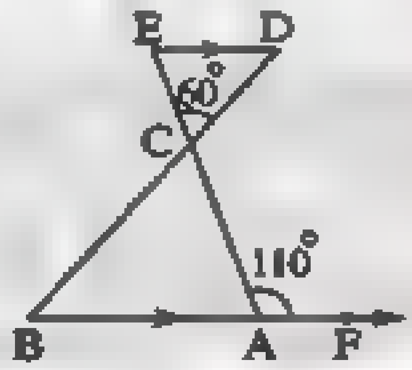
16 **In the opposite figure :**

$\overrightarrow{ED}$ // $\overrightarrow{BA}$, m (∠ CAF) = 110° ,

$\overline{DB}$ ∩ $\overline{AE}$ = {C} ,

m (∠ DCE) = 60° and F ∈ $\overrightarrow{BA}$

Find the measures of the angles of the two triangles DCE and ABC



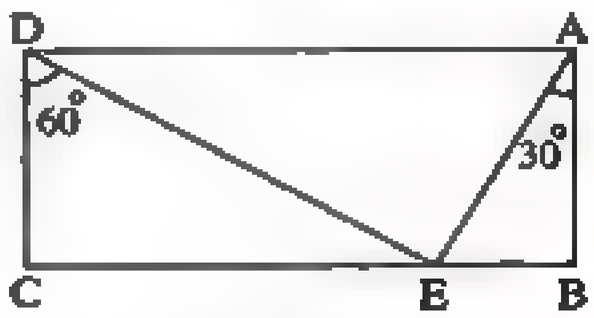
« m (∠ E) = 70° , m (∠ D) = 50° , m (∠ B) = 50° , m (∠ ACB) = 60° , m (∠ BAC) = 70° »

17 **In the opposite figure :**

ABCD is a rectangle , E ∈ $\overline{BC}$ where :

m (∠ BAE) = 30° and m (∠ EDC) = 60°

Find : m (∠ AED)



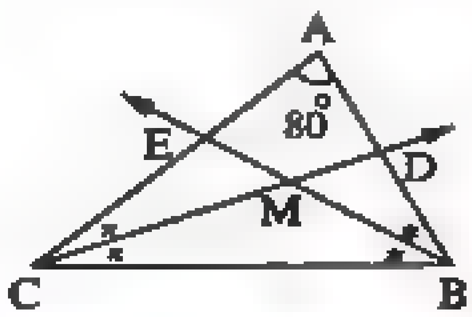
« 90° »

18 **In the opposite figure :**

$\overrightarrow{BM}$ bisects ∠ ABC and $\overrightarrow{CM}$ bisects ∠ ACB

If m (∠ A) = 80° ,

find : m (∠ EMD)



« 130° »

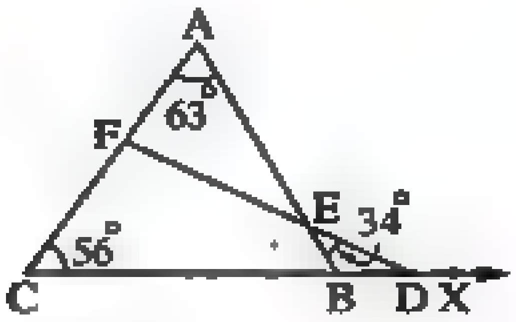
19 **In the opposite figure :**

ABC is a triangle , $D \in \overrightarrow{CB}$, $X \in \overrightarrow{CB}$,

$m(\angle A) = 63°$, $m(\angle C) = 56°$ and

$m(\angle DEB) = 34°$

Find : $m(\angle EDX)$



« 153° »

---

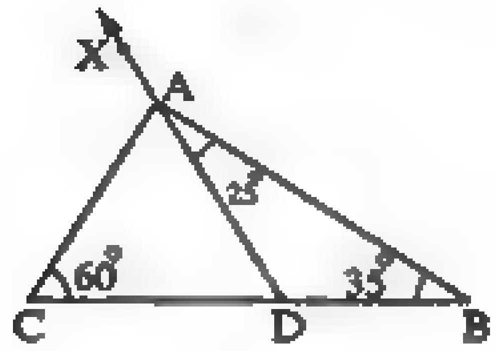
20 **In the opposite figure :**

ABC is a triangle , $m(\angle B) = 35°$,

$m(\angle C) = 60°$, $m(\angle BAD) = 25°$,

$D \in \overline{BC}$ and $X \in \overrightarrow{DA}$

Find : $m(\angle XAC)$



« 120° »

---

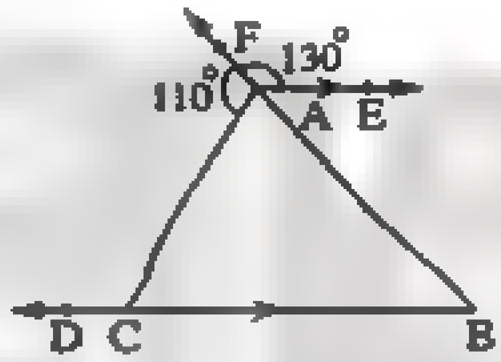
21 **In the opposite figure :**

ABC is a triangle , $\overrightarrow{AE} \mathbin{//} \overline{BC}$, $D \in \overleftrightarrow{BC}$,

$F \in \overrightarrow{BA}$, $m(\angle FAE) = 130°$ and

$m(\angle FAC) = 110°$

Find : $m(\angle ACD)$



« 120° »

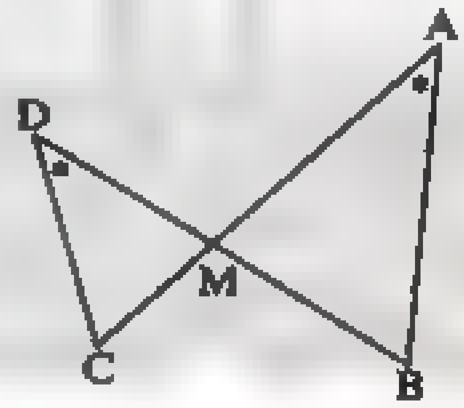
---

22 **In the opposite figure :**

$\overline{AC} \cap \overline{BD} = \{M\}$ and

$m(\angle A) = m(\angle D)$

Prove that : $m(\angle B) = m(\angle C)$



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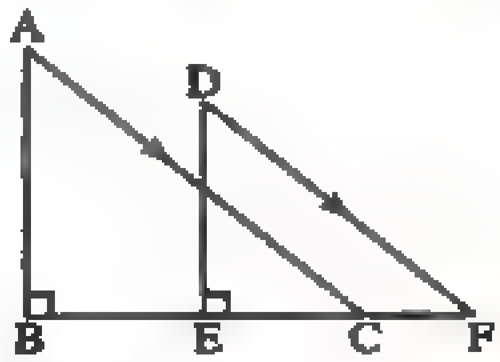
23 **In the opposite figure :**

The points F , C , E and B are collinear ,

$m(\angle B) = m(\angle DEC) = 90°$ and $\overline{AC} \mathbin{//} \overline{DF}$

Prove that : $m(\angle A) = m(\angle D)$



---

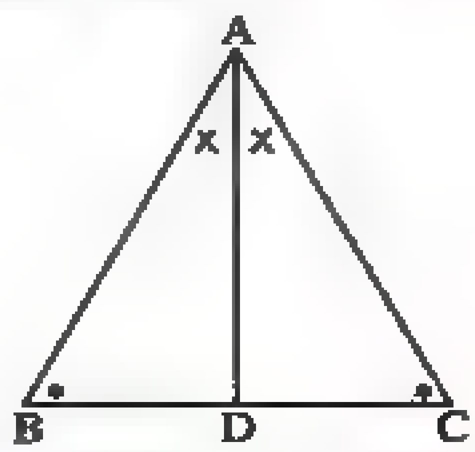
24 📖 **In the opposite figure :**

ABC is a triangle ,

$m(\angle B) = m(\angle C)$ and

$\overrightarrow{AD}$ is the bisector of $\angle A$

Prove that : $AB = AC$

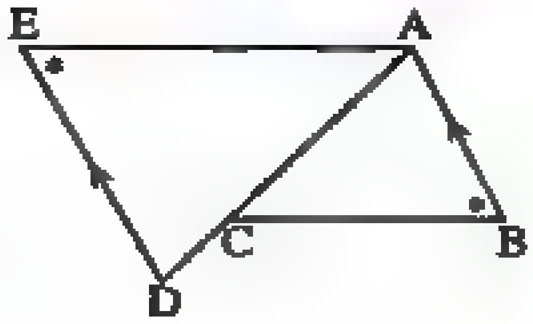


25 **In the opposite figure :**

$\overline{AB} \parallel \overline{ED}$ and

$m(\angle ABC) = m(\angle AED)$

Prove that : $\overline{BC} \parallel \overline{AE}$



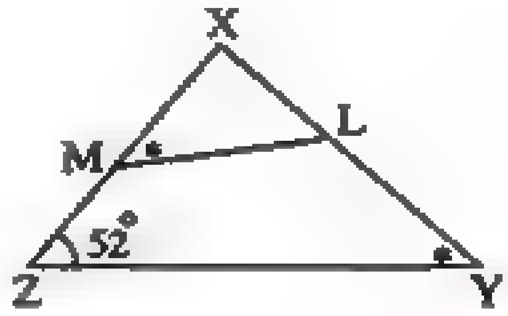
26 **In the opposite figure :**

XYZ is a triangle in which $m(\angle Z) = 52°$,

$L \in \overline{XY}$ and $M \in \overline{XZ}$ such that :

$m(\angle Y) = m(\angle XML)$

Find : $m(\angle XLM)$ « 52° »

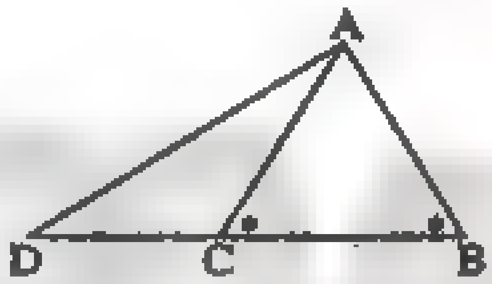


27 **In the opposite figure :**

ABC is a triangle in which :

$m(\angle B) = m(\angle ACB)$ and $D \in \overrightarrow{BC}$

Prove that : $m(\angle B) > m(\angle D)$



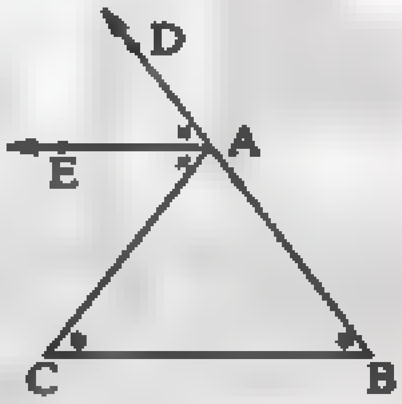
28 **In the opposite figure :**

ABC is a triangle in which :

$m(\angle B) = m(\angle C)$, $D \in \overrightarrow{BA}$ and

$\overrightarrow{AE}$ bisects $\angle DAC$

Prove that : $\overrightarrow{AE} \parallel \overline{BC}$



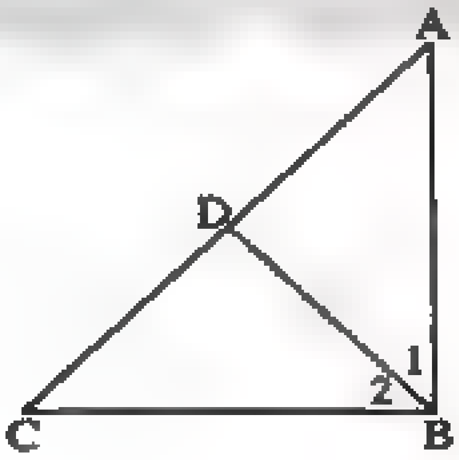
29 **In the opposite figure :**

ABC is a triangle in which : $D \in \overline{AC}$,

$m(\angle 1) = m(\angle A)$ and

$m(\angle 2) = m(\angle C)$

Prove that : $\angle ABC$ is a right angle.



Unit 3

For excellent pupils

30 ABC is a triangle in which : m ($\angle$ A) = 2 m ($\angle$ C) and m ($\angle$ B) = 4 m ($\angle$ C)

Prove that : $\angle$ B is an obtuse angle.

---

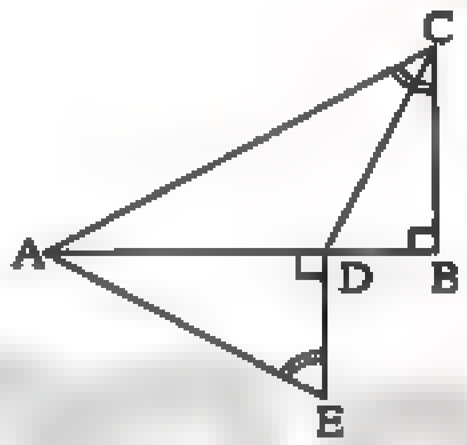
31 ABC is a triangle in which : m ($\angle$ C) = 28° , m ($\angle$ A) = $4X$°

, m ($\angle$ B) = $(2X + 2)$°

Find : m ($\angle$ A) and m ($\angle$ B) « 100° , 52° »

---

32 **In the opposite figure :**



ABC is a triangle in which : D $\in \overline{AB}$

m ($\angle$ B) = 90° , m ($\angle$ ADE) = 90° and

m ($\angle$ ACB) = m ($\angle$ E)

Prove that : m ($\angle$ BDC) > m ($\angle$ DAE)

From the school book

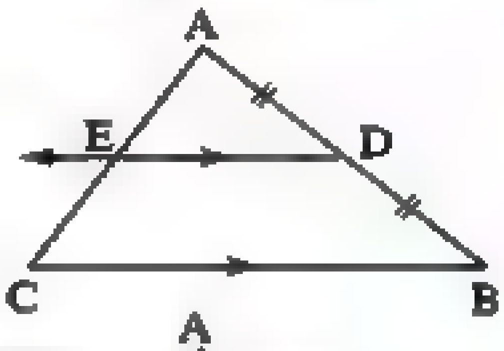
Exercise 6 On theorem 2 and its corollary, and theorem 3

1 Complete the following :

(1) The ray drawn from the midpoint of a side of a triangle parallel to another side ....

(2) The line segment joining the midpoints of two sides of a triangle is .... the third side.

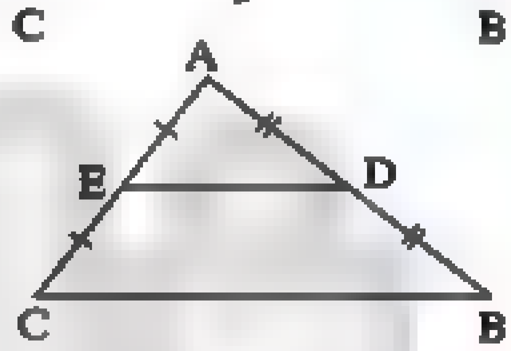
(3) The length of the line segment joining the midpoints of two sides of a triangle equals ....



(4) **In the opposite figure :**

If D is the midpoint of $\overline{AB}$, $\overleftrightarrow{DE}$ // $\overline{BC}$

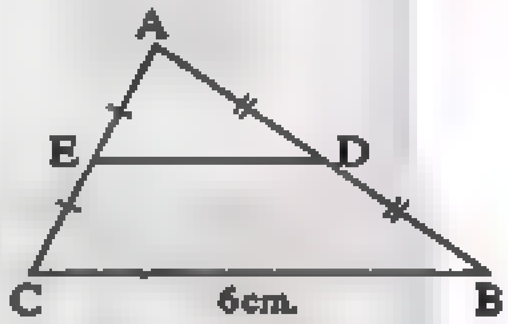
, then : .... is the midpoint of ....



(5) **In the opposite figure :**

If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, then : .... // ....

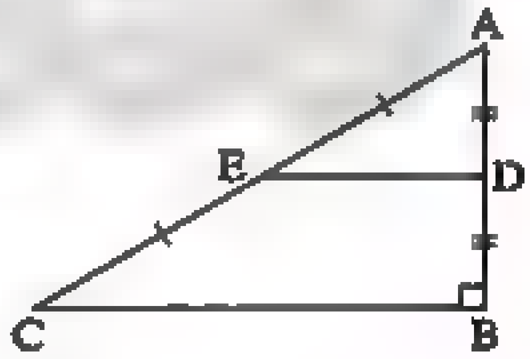


(6) **In the opposite figure :**

∵ D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, BC = 6 cm.

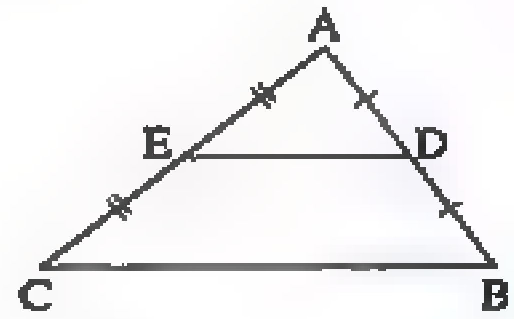
∴ DE = ... cm



(7) **In the opposite figure :**

If : m (∠ B) = 90° , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, then : m (∠ ADE) = ...°

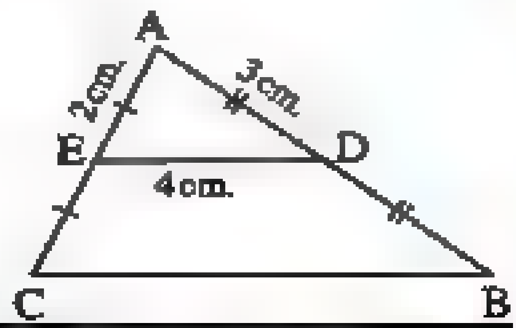


(8) **In the opposite figure :**

If : D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, and the perimeter of the triangle ABC = 24 cm.

, then the perimeter of the triangle ADE = ... cm.



(9) **In the opposite figure :**

∵ D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, AD = 3 cm. , AE = 2 cm. and DE = 4 cm.

∴ The perimeter of the figure DBCE = ... cm.

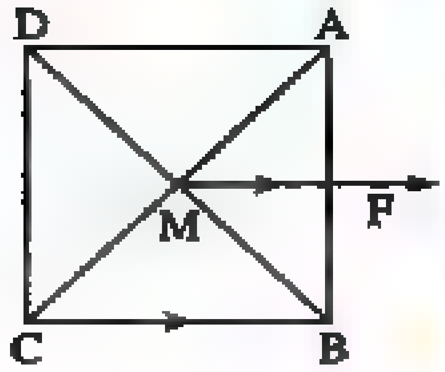
Unit 3

- **In the opposite figure :**

If the perimeter of the square ABCD = 20 cm.

, $\overrightarrow{MF}$ // $\overline{CB}$ where F $\in \overline{AB}$

, then : AF = ... cm.

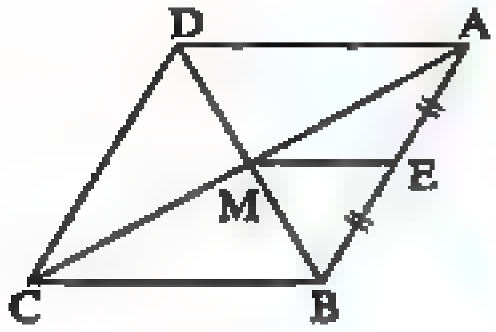


- **In the opposite figure :**

$\because$ The perimeter of the rhombus ABCD = 24 cm.,

E is the midpoint of $\overline{AB}$

$\therefore$ ME = ... cm.

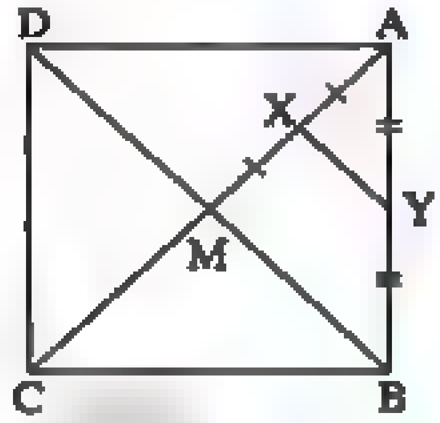


- **In the opposite figure :**

$\because$ ABCD is a square, X and Y are the midpoints

of $\overline{AM}$ and $\overline{AB}$ respectively and AC = 12 cm.

$\therefore$ XY = ... cm., m ($\angle$ AYX) = ... °



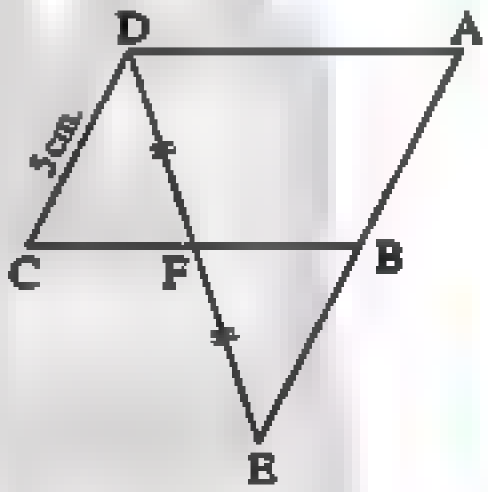
2 In the opposite figure :

ABCD is a parallelogram, DC = 5 cm.

, F $\in \overline{BC}$, $\overrightarrow{DF} \cap \overrightarrow{AB}$ = {E},

if : DF = FE

Complete the following proof to find the length of $\overline{BE}$



Given ....

R.T.F. ....

Proof

$\because$ ABCD is a ....

$\therefore$ $\overline{AD}$ // .... $\therefore$ $\overline{BF}$ // ....

, In Δ AED :

$\because$ F is the midpoint of ...., $\overline{BF}$ // ....

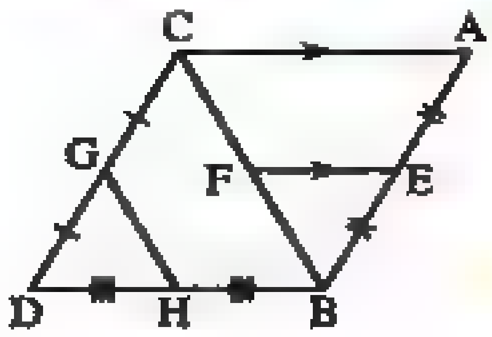
$\therefore$ B is the midpoint of ....

$\therefore$ AB = ...

, $\because$ AB = ... = ... cm. (Properties of ....)

$\therefore$ BE = ... cm. (The req.)

3 **In the opposite figure :**



ABC is a triangle in which CA = CB ,

E is the midpoint of $\overline{AB}$, $\overline{EF}$ // $\overline{AC}$,

H and G are the two midpoints of $\overline{BD}$ and $\overline{CD}$ respectively.

Complete the following proof to prove that : EF = GH

Given ....

R.T.P. ....

Proof

In Δ ABC :

∵ E is the midpoint of .... , $\overline{EF}$ // ....

∴ F is the midpoint of ....

∴ EF = $\frac{1}{2}$ ....

, In Δ BDC :

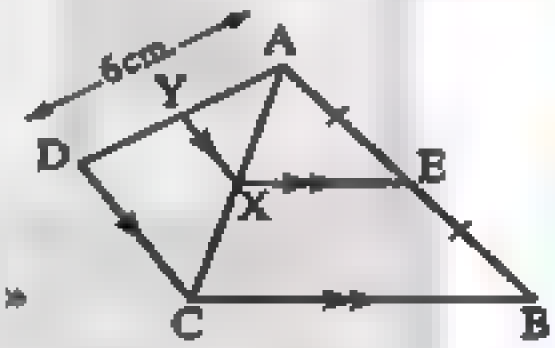
∵ H is the midpoint of ....

, ... is the midpoint of $\overline{CD}$

∴ GH = $\frac{1}{2}$ ....

, ∵ CA = ... ∴ EF = ... (Q.E.D.)

4 **In the opposite figure :**



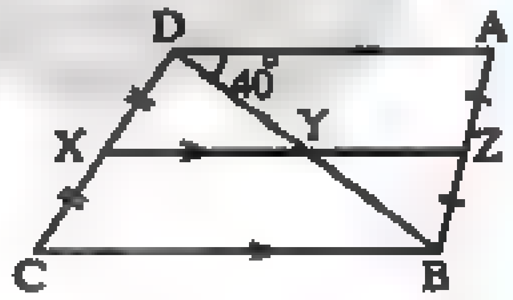
AE = EB , $\overline{EX}$ // $\overline{BC}$, $\overline{XY}$ // $\overline{CD}$

and AD = 6 cm.

Find the length of : $\overline{AY}$

« 3 cm. »

5 **In the opposite figure :**



X is the midpoint of $\overline{CD}$

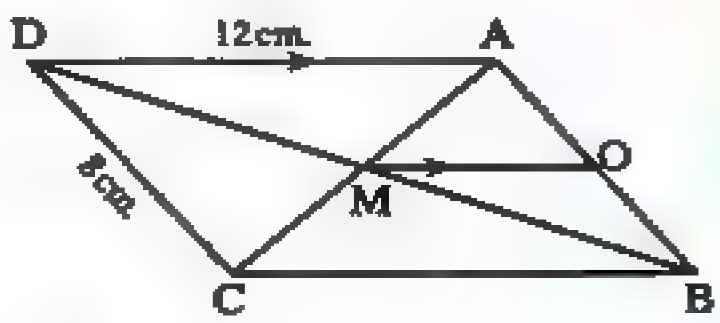
, Z is the midpoint of $\overline{AB}$

, $\overline{XY}$ // $\overline{CB}$, m (∠ ADB) = 40°

Find : m (∠ ZYB)

« 40° »

6 **In the opposite figure :**



ABCD is a parallelogram , $\overline{AC}$ ∩ $\overline{BD}$ = {M}

Draw $\overline{MO}$ // $\overline{AD}$ to cut $\overline{AB}$ at O

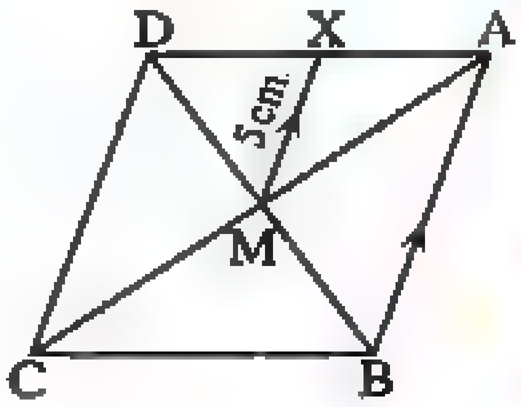
If : AD = 12 cm. and DC = 8 cm.

, **then find :** ● The perimeter of ABCD

● The length of $\overline{AO}$

« 40 cm. , 4 cm. »

7 **In the opposite figure :**



ABCD is a parallelogram , its diagonals intersect at M

Draw $\overrightarrow{MX}$ // $\overline{BA}$ to intersect $\overline{AD}$ at X

(1) **Prove that :** X is the midpoint of $\overline{AD}$

(2) If MX = 5 cm. , then find the length of $\overline{CD}$

« 10 cm. »

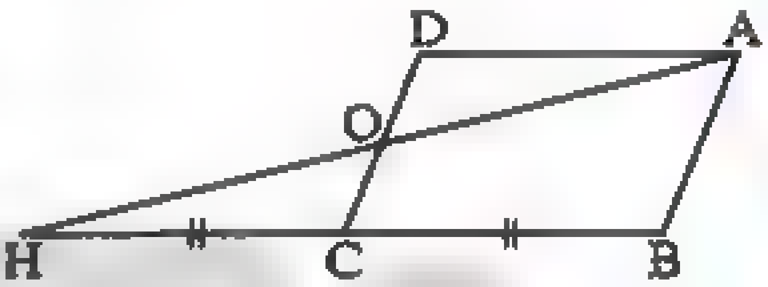
---

8 ABCD is a parallelogram , its diagonals intersect at M , $\overrightarrow{ME}$ // $\overline{BA}$ and cuts $\overline{AD}$ at E

Prove that : ME = $\frac{1}{2}$ DC

---

9 **In the opposite figure :**



ABCD is a parallelogram ,

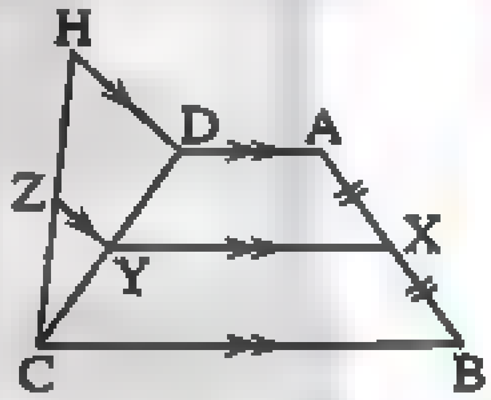
BC = CH , H ∈ $\overrightarrow{BC}$

Draw $\overline{AH}$ to cut $\overline{DC}$ at O

Prove that : AO = OH

---

10 **In the opposite figure :**



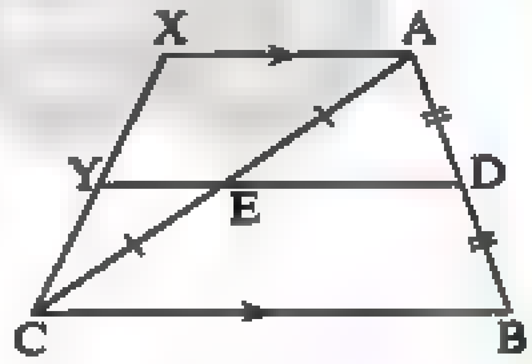
ABCD is a trapezium , $\overline{AD}$ // $\overline{BC}$, X is the midpoint of $\overline{AB}$

If $\overline{AD}$ // $\overline{XY}$ where Y ∈ $\overline{DC}$, $\overline{YZ}$ // $\overline{DH}$

Prove that : CZ = ZH

---

11 **In the opposite figure :**



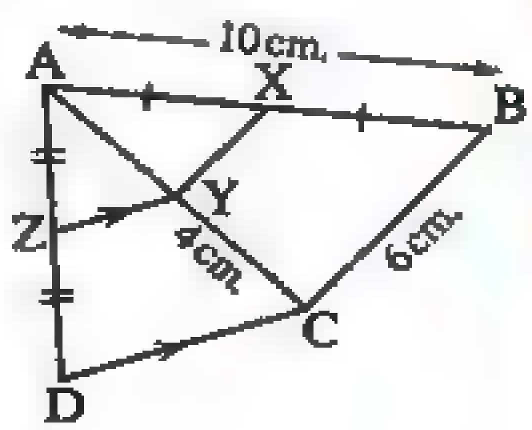
AD = DB , AE = EC ,

$\overline{AX}$ // $\overline{BC}$, $\overrightarrow{DE}$ ∩ $\overline{XC}$ = {Y}

Prove that : Y is the midpoint of $\overline{XC}$

---

12 **In the opposite figure :**



ABCD is a quadrilateral in which :

X and Z are the midpoints of $\overline{AB}$

and $\overline{AD}$ respectively and Y ∈ $\overline{AC}$ such that :

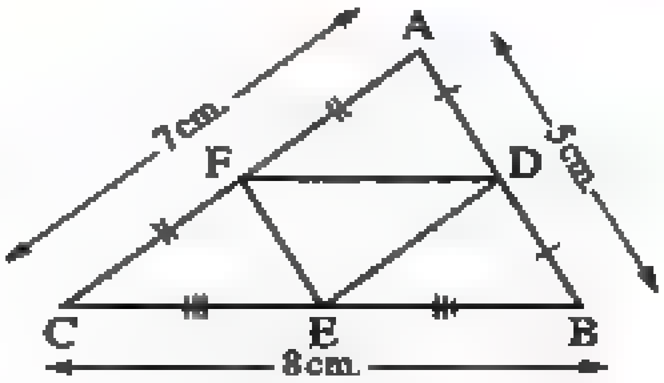
$\overline{YZ}$ // $\overline{CD}$ and YC = 4 cm.

If BC = 6 cm. and AB = 10 cm. , then

find : (1) The length of $\overline{AY}$ (2) The perimeter of Δ AXY « 4 cm. , 12 cm. »

13 📖 **In the opposite figure :**

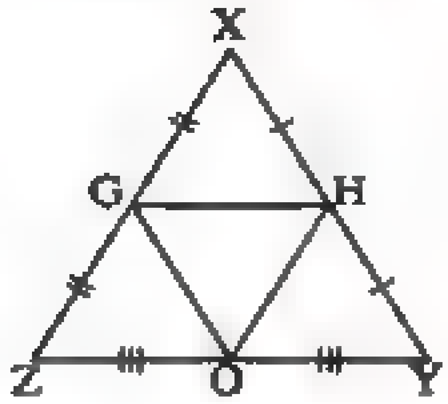
AB = 5 cm. , BC = 8 cm. ,
AC = 7 cm. , D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.
Calculate the perimeter of : Δ DEF



« 10 cm. »

14 **In the opposite figure :**

XYZ is a triangle in which :
H , O and G are the midpoints of $\overline{XY}$, $\overline{YZ}$ and $\overline{ZX}$ respectively.
If the perimeter of Δ HOG is 18 cm. ,
then find the perimeter of : Δ XYZ

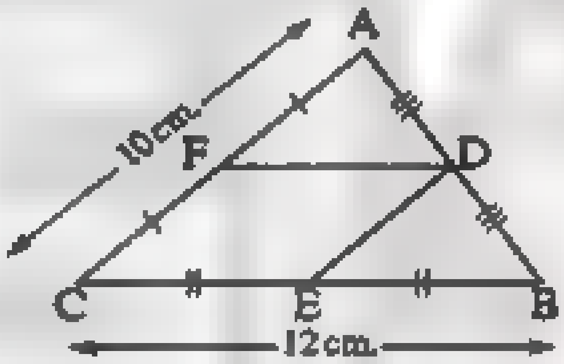


« 36 cm. »

15 ABC is a triangle , if X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively. **Prove that :** the perimeter of Δ XYZ = $\frac{1}{2}$ that of Δ ABC

16 📖 **In the opposite figure :**

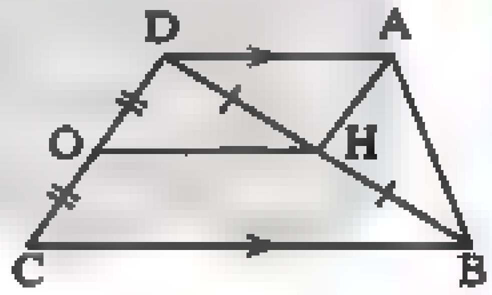
ABC is a triangle in which D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively,
BC = 12 cm. , AC = 10 cm.
Find the perimeter of the figure DECF



« 22 cm. »

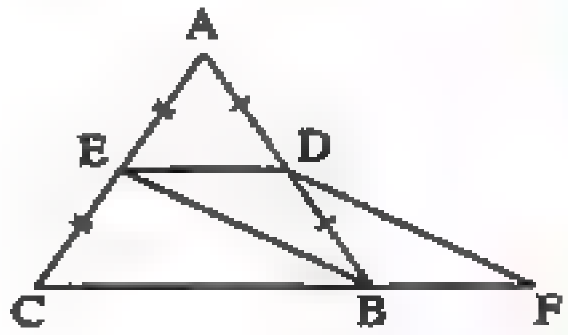
17 📖 **In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, AD = $\frac{1}{2}$ BC ,
H is the midpoint of $\overline{BD}$,
O is the midpoint of $\overline{CD}$
Prove that : AHOD is a parallelogram.



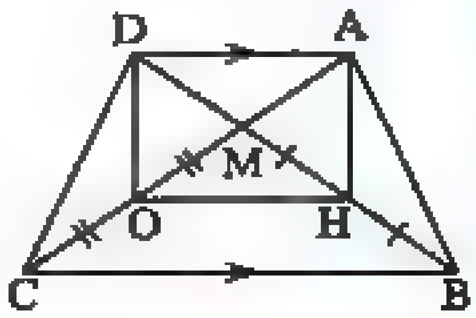
18 📖 **In the opposite figure :**

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively,
F ∈ $\overrightarrow{CB}$ where BF = $\frac{1}{2}$BC
Prove that : BEDF is a parallelogram.



19 **In the opposite figure :**

ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$
and AD = $\frac{1}{2}$ BC and $\overline{AC}$ ∩ $\overline{DB}$ = {M}
Let H and O are the midpoints of $\overline{MB}$ and $\overline{MC}$ respectively.
Prove that : AHOD is a parallelogram.



Unit 3

20 **In the opposite figure :**

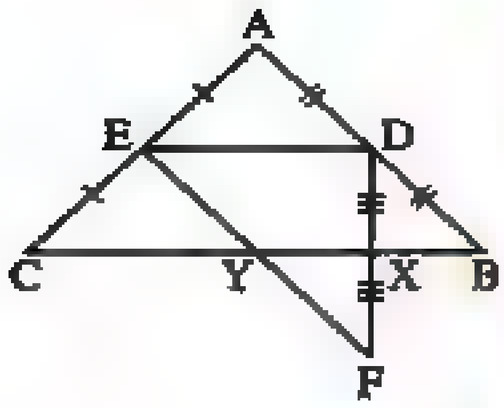
D is the midpoint of $\overline{AB}$,

E is the midpoint of $\overline{AC}$,

$\overline{DF} \cap \overline{BC} = \{X\}$, DX = XF ,

BC = 12 cm.

Find the length of : $\overline{XY}$



« 3 cm. »

---

21 **In the opposite figure :**

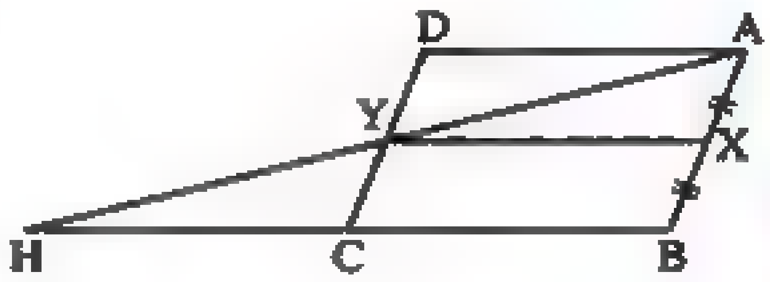
ABCD is a parallelogram ,

X is the midpoint of $\overline{AB}$

Draw $\overrightarrow{XY}$ // $\overline{BC}$ to cut $\overline{DC}$ at Y

Draw $\overrightarrow{AY}$ to cut $\overrightarrow{BC}$ at H

Prove that : C is the midpoint of $\overline{BH}$



---

22 **In the opposite figure :**

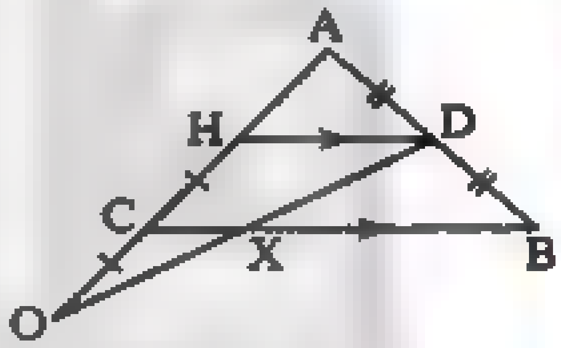
ABC is a triangle , D is the midpoint of $\overline{AB}$,

$\overleftrightarrow{DH}$ // $\overline{BC}$, O $\in \overrightarrow{AC}$ such that HC = CO

Prove that : CO = $\frac{1}{3}$ AO

If we draw $\overline{DO}$ to cut $\overline{BC}$ at X , then

prove that : OX = XD



---

23 **In the opposite figure :**

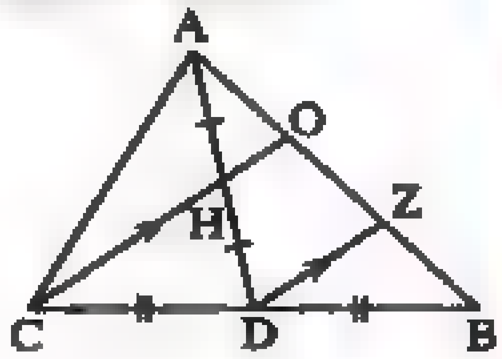
ABC is a triangle , D is the midpoint of $\overline{BC}$

and H is the midpoint of $\overline{AD}$

Draw $\overleftrightarrow{CH}$ to cut $\overline{AB}$ at O ,

then draw $\overrightarrow{DZ}$ // $\overline{CO}$ to cut $\overline{AB}$ at Z

Prove that : AO = OZ = ZB



---

24 ABCD is a parallelogram. M is the intersection point of its diagonals ,

draw $\overleftrightarrow{CE}$ // $\overline{BD}$ to cut $\overrightarrow{AB}$ at E and $\overrightarrow{AD}$ at F

Prove that : ● AB = BE ● AD = DF

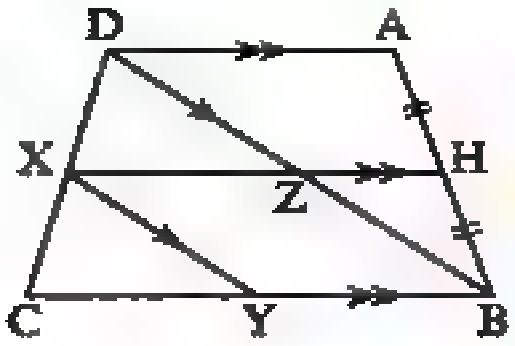
25 **In the opposite figure :**

ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$

Let H be the midpoint of $\overline{AB}$,

$\overline{HX} \parallel \overline{BC}$ and $\overline{XY} \parallel \overline{DB}$

Prove that : Y is the midpoint of $\overline{BC}$



26 ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, E is the midpoint of $\overline{AB}$, draw $\overrightarrow{EX} \parallel \overline{BC}$ to cut $\overline{DB}$ at X , $\overline{DC}$ at Y , and draw $\overrightarrow{YZ} \parallel \overline{DB}$ to cut $\overline{BC}$ at Z

Prove that : XD = YZ

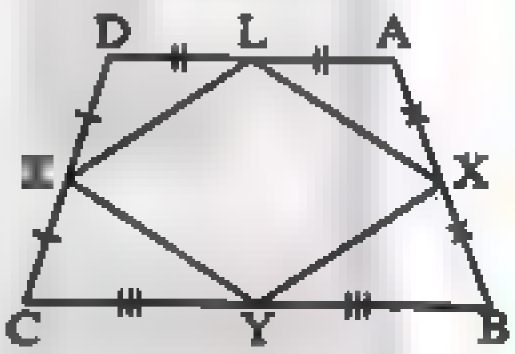
27 ABC is a triangle in which AB = 9 cm. , AC = 8 cm. , D $\in \overline{AB}$, E $\in \overline{AB}$ such that AD = DE = EB and $\overrightarrow{DX}$, $\overrightarrow{EY}$ are drawn parallel to $\overline{BC}$ and cutting $\overline{AC}$ at X and Y respectively , where DX = 4 cm.

Calculate : the perimeter of the shape DEYX « $17\frac{2}{3}$ cm. »

28 **In the opposite figure :**

ABCD is a quadrilateral in which X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Prove that : XYZL is a parallelogram.



29 ABC is a triangle in which AB = AC , X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

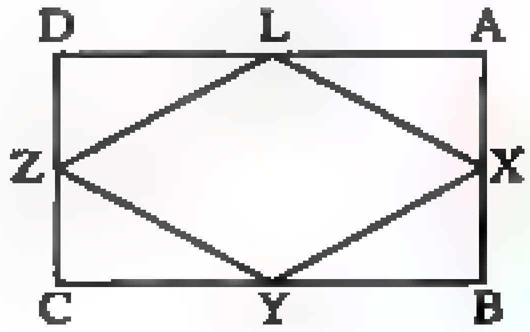
Prove that : AXYZ is a rhombus.

30 **In the opposite figure :**

ABCD is a rectangle and X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Prove that :

- XYZL is a rhombus.
- The perimeter of the rhombus = 2 BD

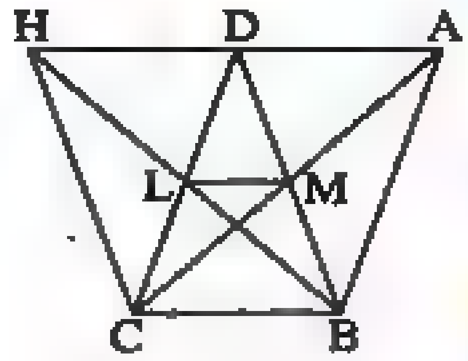


31 **In the opposite figure :**

ABCD and DBCH are two parallelograms having a common base $\overline{BC}$ and on one side of $\overline{BC}$

Let $\overline{AC} \cap \overline{BD} = \{M\}$ and $\overline{DC} \cap \overline{BH} = \{L\}$

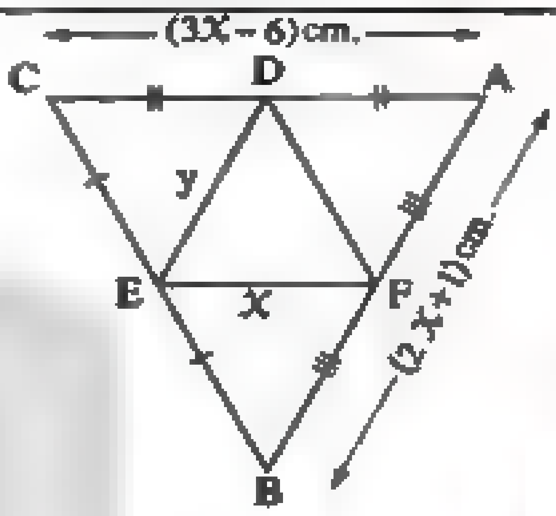
Prove that : (1) $\overline{ML} \parallel \overline{BC}$ (2) $ML = \frac{1}{4} AH$



32 **Connected with algebra :**

In the opposite figure :

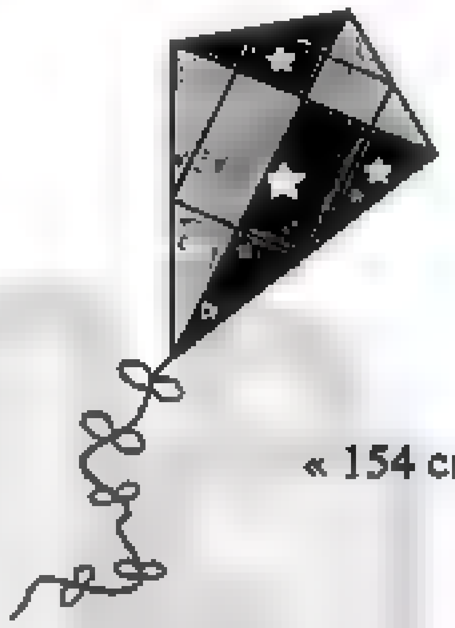
Find the value of each of : X and y



« 6 cm. , 6.5 cm. »

Life Application

33 Sara wants to design a kite whose two diagonals are 64 cm. and 90 cm.
She want to put a stripe to decorate the kite such that the stripe joins between the midpoints of the sides of the kite.
How long is the stripe ?



« 154 cm. »

For excellent pupils

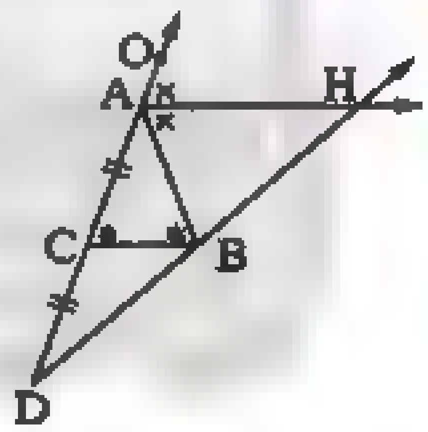
34 **In the opposite figure :**

ABC is a triangle in which : m (∠ ABC) = m (∠ ACB)

, D $\in \overrightarrow{AC}$ such that AC = CD and O $\in \overrightarrow{CA}$

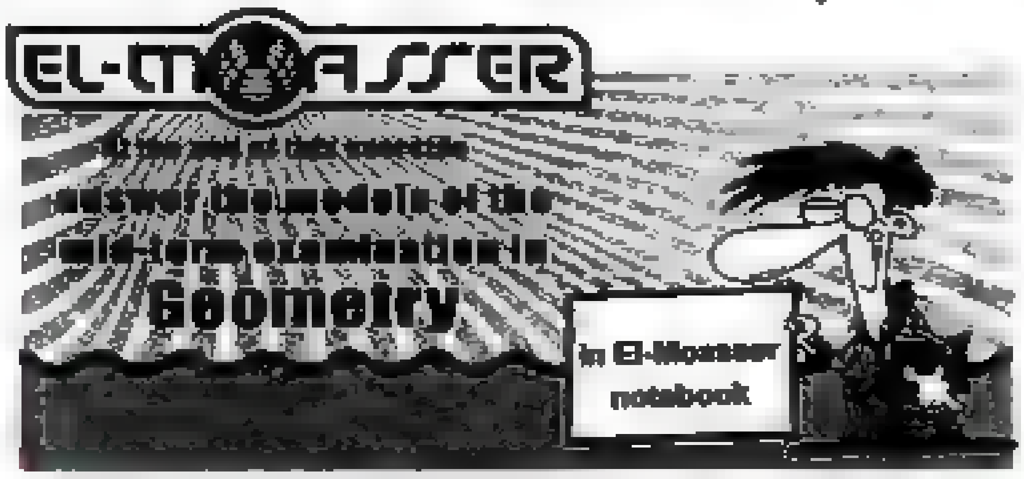
Let $\overrightarrow{AH}$ bisects ∠ BAO such that : $\overrightarrow{AH} \cap \overrightarrow{DB} = \{H\}$

Prove that : DB = BH



35 ABCD is a quadrilateral in which $\overline{AC} \perp \overline{BD}$ and X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Prove that : XYZL is a rectangle whose area equals $\frac{1}{4}$ AC × BD



From the school book

Exercise 7 On Pythagoras' theorem

1 **In each of the following figures , find the length of the unknown side :**

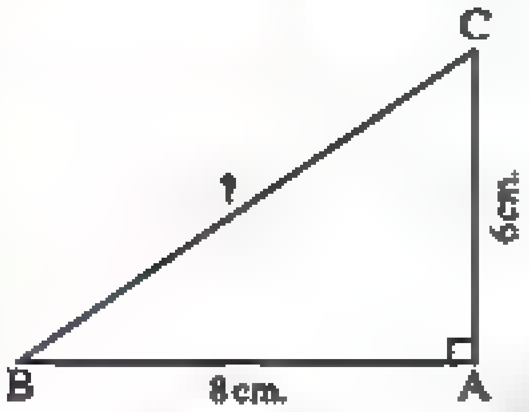


Fig. (1)

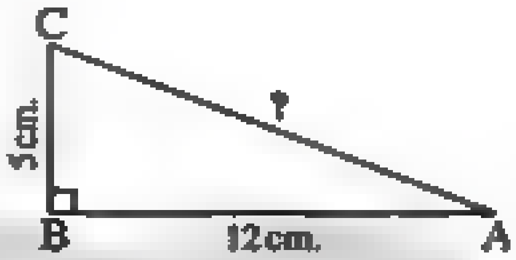


Fig. (2)

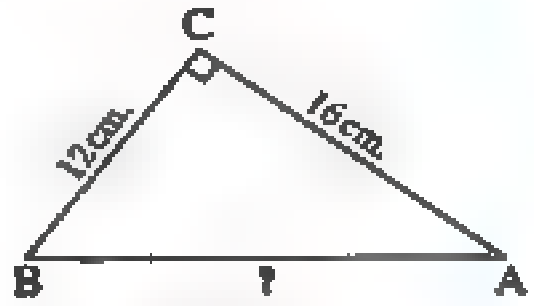


Fig. (3)

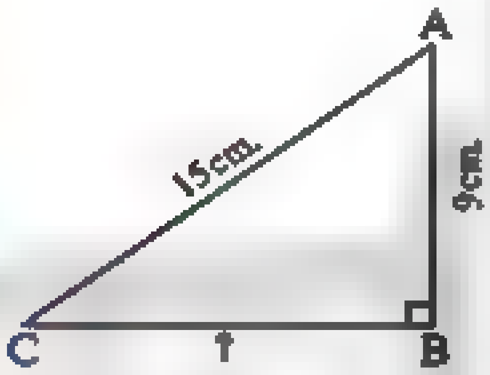


Fig. (4)

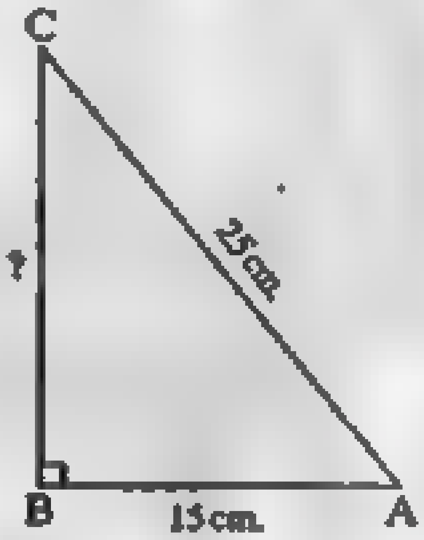


Fig. (5)

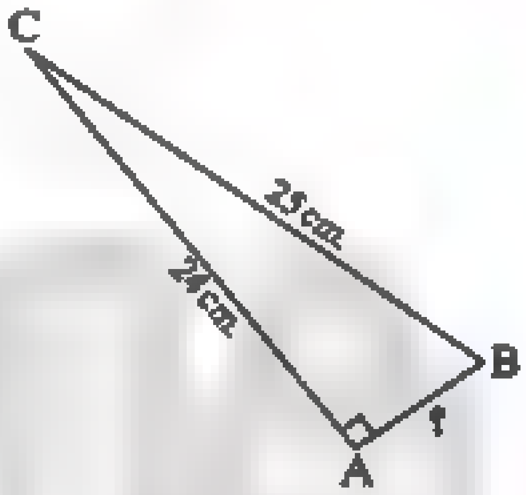
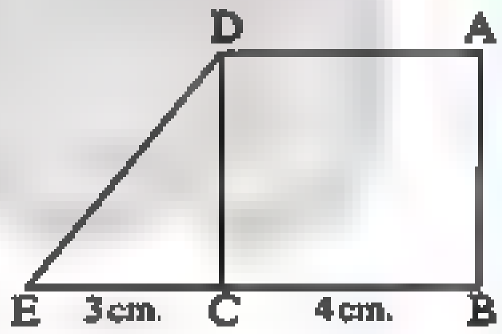


Fig. (6)

2 **In the opposite figure :**

ABCD is a square whose side length = 4 cm.

and $E \in \overrightarrow{BC}$ where CE = 3 cm.



Complete the following proof to find the length of $\overline{DE}$

$\because$ ABCD is a square

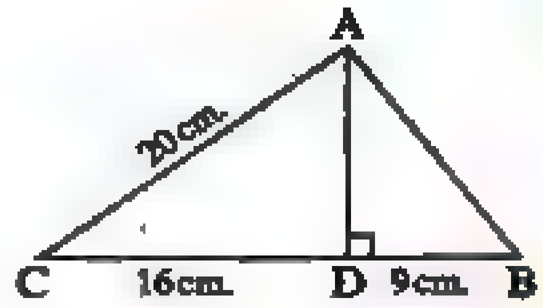
$\therefore$ DC = ... cm. , m ($\angle$ DCB) = ...°

$\therefore$ In Δ DCE : $(DE)^2 = (\text{..........})^2 + (CE)^2 = (\text{..........})^2 + (3)^2$

= .... + ... = ...

$\therefore$ DE = ... cm. (The req.)

3 **In the opposite figure :**



$\overline{AD} \perp \overline{BC}$, BD = 9 cm. ,

DC = 16 cm. and AC = 20 cm.

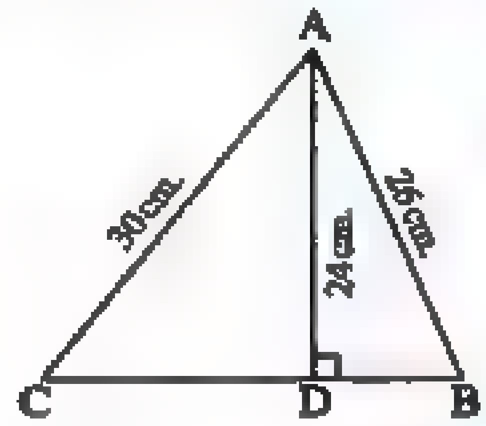
Find :

- AD
- AB
- The area of Δ ABC

« 12 cm. , 15 cm. , 150 cm^2 »

---

4 **In the opposite figure :**



ABC is a triangle and $\overline{AD} \perp \overline{BC}$

If AD = 24 cm.

, AB = 26 cm.

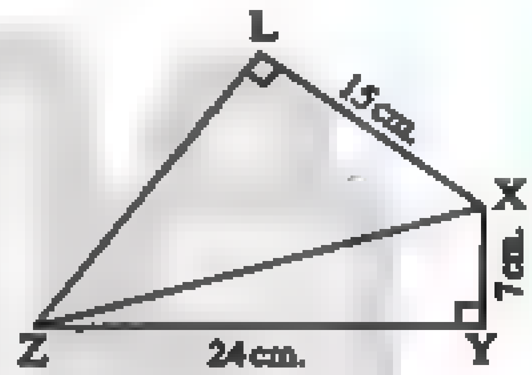
and AC = 30 cm.

find BC and calculate the area of Δ ABC

« 28 cm. , 336 cm^2 »

---

5 **In the opposite figure :**



XYZL is a quadrilateral in which :

m ($\angle$ XYZ) = m ($\angle$ XLZ) = 90° ,

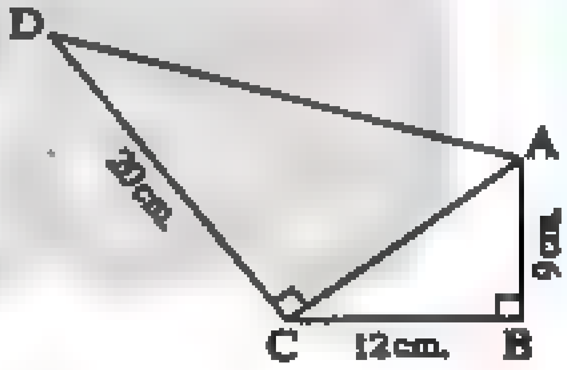
XY = 7 cm. , YZ = 24 cm. and XL = 15 cm.

Find the length of each of : $\overline{XZ}$ and $\overline{ZL}$

« 25 cm. , 20 cm. »

---

6 **In the opposite figure :**



m ($\angle$ B) = m ($\angle$ ACD) = 90°

, AB = 9 cm. , BC = 12 cm.

and DC = 20 cm.

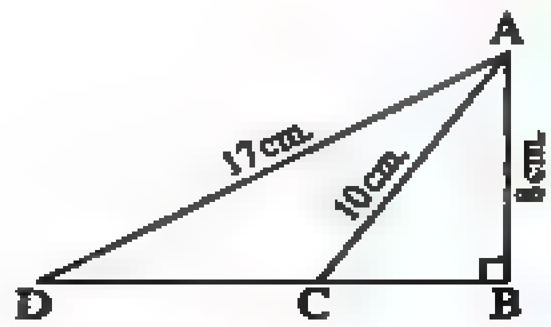
Find :

- The length of $\overline{AC}$
- The length of $\overline{AD}$
- The perimeter of the figure ABCD
- The area of the figure ABCD

« 15 cm. , 25 cm. , 66 cm. , 204 cm^2 »

---

7 **In the opposite figure :**



Δ ABD is a right-angled triangle at B

, AB = 8 cm. , AD = 17 cm.

and C $\in \overline{BD}$ such that AC = 10 cm.

Find the length of each of : $\overline{CB}$, $\overline{BD}$ and $\overline{CD}$

« 6 cm. , 15 cm. , 9 cm. »

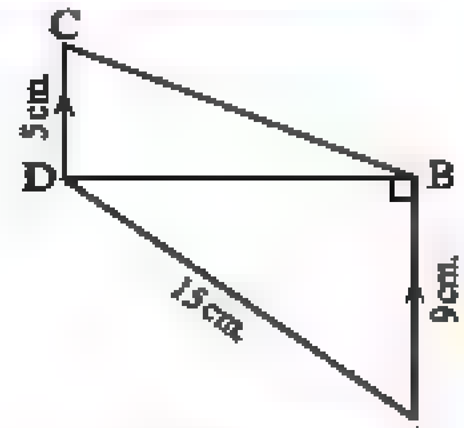
8 **In the opposite figure :**

$m(\angle ABD) = 90°$, $\overline{BA} \parallel \overline{CD}$

, AB = 9 cm. , AD = 15 cm.

and DC = 5 cm.

Calculate the length of : $\overline{BC}$



« 13 cm »

9 **In the opposite figure :**

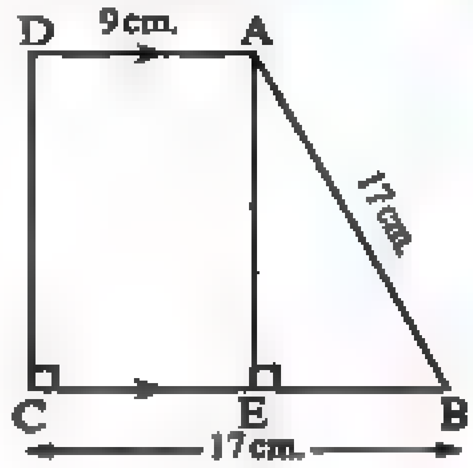
ABCD is a trapezium , $\overline{AD} \parallel \overline{BC}$

, $m(\angle DCB) = 90°$, $\overline{AE} \perp \overline{BC}$

If : AB = BC = 17 cm. and AD = 9 cm.

, find the length of $\overline{DC}$ and calculate

the area of the trapezium ABCD



« 15 cm. , 195 cm^2 »

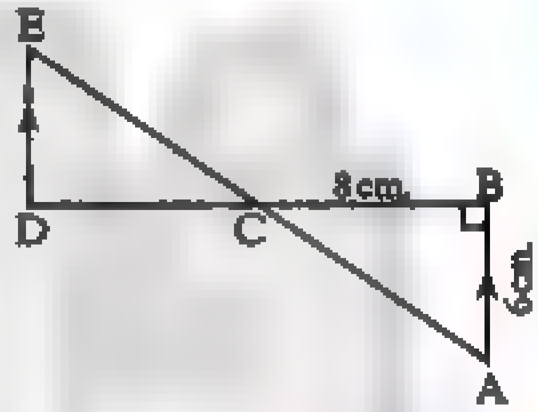
10 **In the opposite figure :**

$\overline{BD} \cap \overline{AE} = \{C\}$, $\overline{AB} \parallel \overline{DE}$

, AB = 6 cm. , BC = 8 cm.

and C is the midpoint of $\overline{BD}$

Calculate the length of : $\overline{CE}$



« 10 cm. »

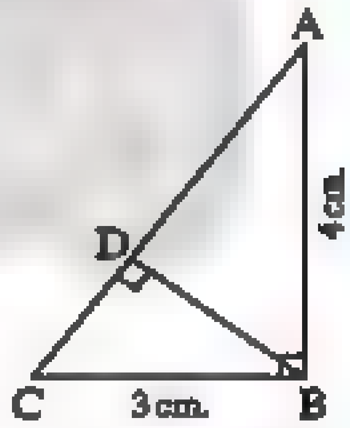
11 **In the opposite figure :**

ABC is a right-angled triangle at B

, $\overline{BD} \perp \overline{AC}$, AB = 4 cm.

and BC = 3 cm.

Calculate the length of : $\overline{BD}$



« 2.4 cm. »

12 **Complete the following :**

- In the right-angled triangle , the area of the square on the hypotenuse equals ....
- If XYZ is a right-angled triangle at X , XY = 12 cm. and XZ = 9 cm. ,
  then YZ = ... cm.
- If ABC is a right-angled triangle at B , AB = 20 cm. and AC = 25 cm. ,
  then BC = ... cm.

(4) **In the opposite figure :**

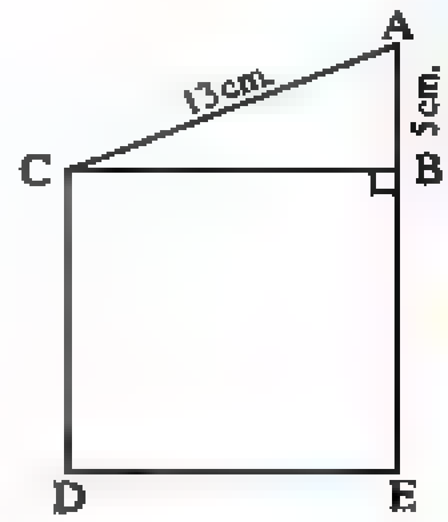
If m (∠ ABC) = 90°

, AB = 5 cm.

and AC = 13 cm.

, then the area of

the square BEDC = ... cm^2.



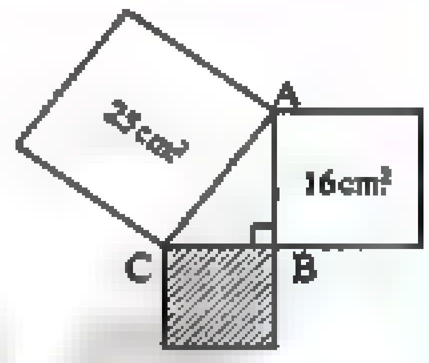
(5) A rectangle is of length 8 cm. and width 6 cm. , then the length of its diagonal equals .. .. cm.

(6) If the area of a rectangle equals 60 cm^2 and its width is 5 cm. , then the length of its diagonal = ... cm.

(7) If Δ ABC is right-angled at B ,

then the side length of the shaded

square = ... cm.

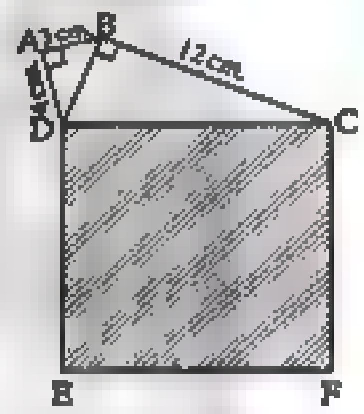


(8) If Δ ABD is right-angled at A

and Δ BCD is right-angled at B ,

then the area of the shaded

square = ... cm^2.



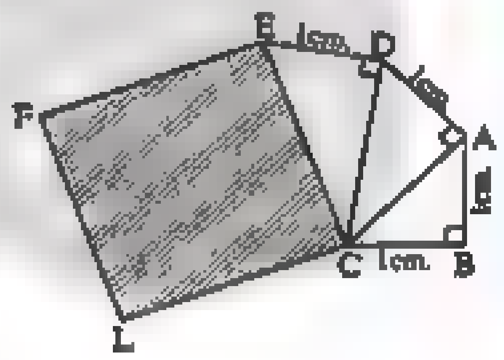
(9) If each of the triangles ABC , ACD and DCE

are right-angled at B , A and D respectively ,

AB = BC = AD = DE = 1 cm.

, then the area of the shaded

square = ... cm^2.



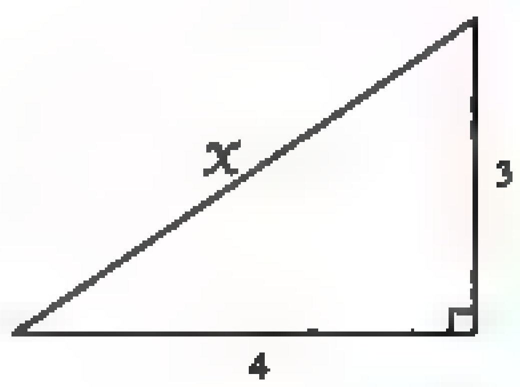
13 **Choose the correct answer from those given :**

(1) **In the opposite figure :**

Which of the following relations is true ?

(a) $x = 4^2 + 3^2$ (b) $x^2 = 4^2 - 3^3$

(c) $x^2 + 9 = 16$ (d) $x^2 = 25$

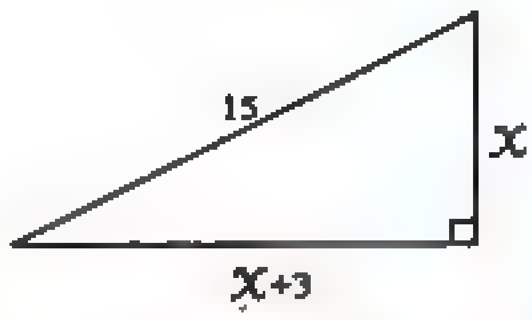


(2) **In the opposite figure :**

Which of the following relations is true ?

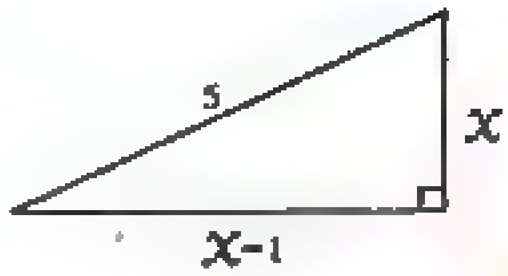
(a) $x + 3 + x = 15$ (b) $x^2 + 3x = 108$

(c) $(x + 3)^2 = 15 - x^2$ (d) $x^2 + 6x + 9 = 225$



In the opposite figure :

Which of the following relations is true ?



(a) $x^2 + (x-1)^2 = 5$ (b) $x + (x-1) = 25$

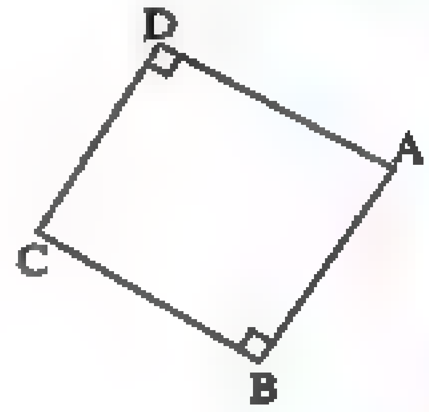
(c) $x^2 - x = 12$ (d) $(x-1)^2 - x^2 = 25$

If ABCD is a square , then $(AC)^2 =$ ....

(a) AB (b) $(AB)^2$ (c) $2\ (AB)^2$ (d) $4\ (AB)^2$

---

14 **In the opposite figure :**

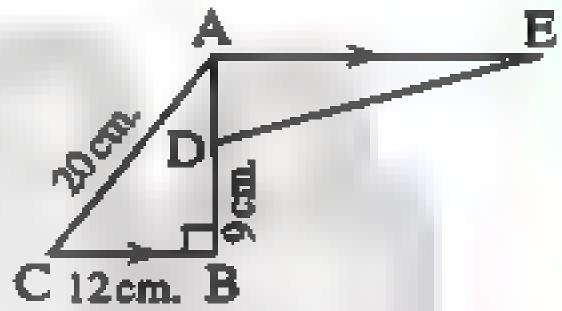


If m ($\angle$ B) = m ($\angle$ D) = 90°

Prove that : $(AB)^2 + (BC)^2 = (AD)^2 + (DC)^2$

---

15 **In the opposite figure :**



ABC is a triangle , m ($\angle$ B) = 90°

, $\overline{AE}$ // $\overline{BC}$, If BC = 12 cm. , AC = 20 cm.

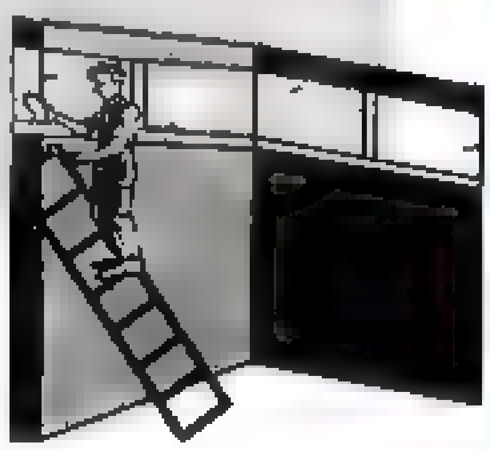
, D $\in \overline{AB}$ where BD = 9 cm. and $\vec{AE} = 2\ \vec{BC}$

, **find the length of each of :** $\overline{AD}$ and $\overline{ED}$

« 7 cm. , 25 cm. »

Life Applications

16 A window cleaner has a ladder which is 5 metres long.



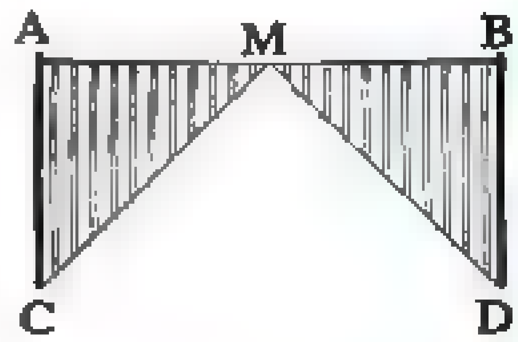
He places it so that it reaches a window sill 4 metres from the ground.

How far is the wall from the foot of the ladder ?

« 3 m. »

---

17 A wooden bridge $\overline{AB}$ of length 15 m. is built horizontally on two vertical walls $\overline{AC}$ and $\overline{BD}$ resting on two supports $\overline{MC}$ and $\overline{MD}$



If AC = 4 m. and AM = MB ,

calculate the length of the support $\overline{MC}$

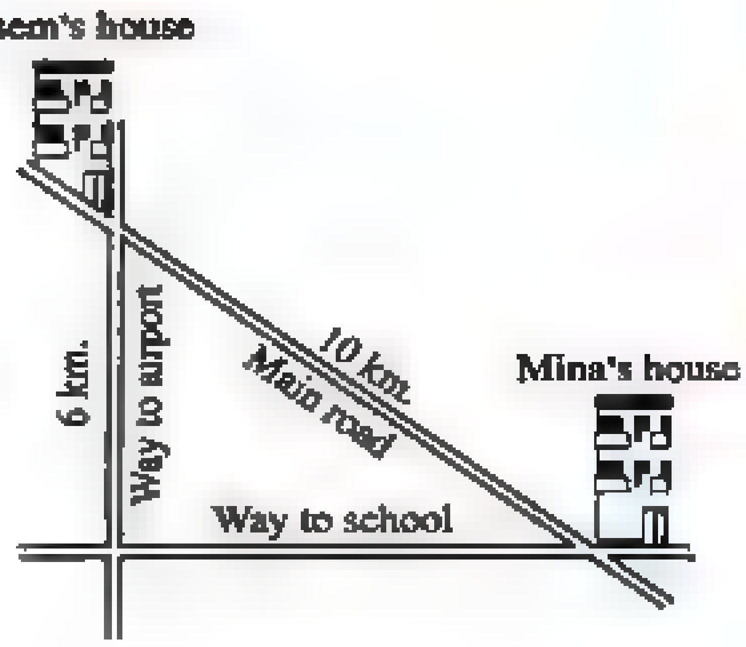
« 8.5 m. »

Unit 3

18 If Mina wants to go to the house of his friend Bassem.

What is the distance he saves if he takes the main road instead of the other two roads ?

« 4 km. »



For excellent pupils

19 If Δ ABC is right-angled at B , D is the midpoint of $\overline{BC}$,

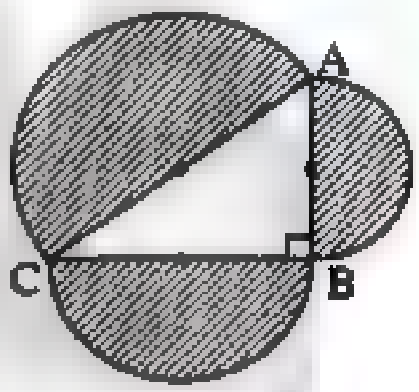
Prove that : $(AC)^2 - (AD)^2 = 3\,(BD)^2$

---

20 **In the opposite figure :**

Prove that the sum of areas of the two semicircles drawn on the two sides of the right angle in a right-angled triangle equals the area of the semicircle drawn on the hypotenuse?

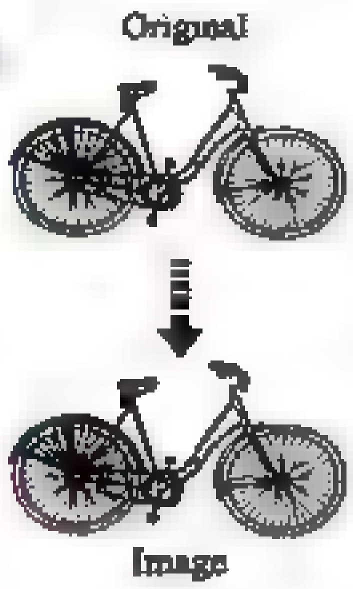
[Given the area of the circle $= \pi r^2$]

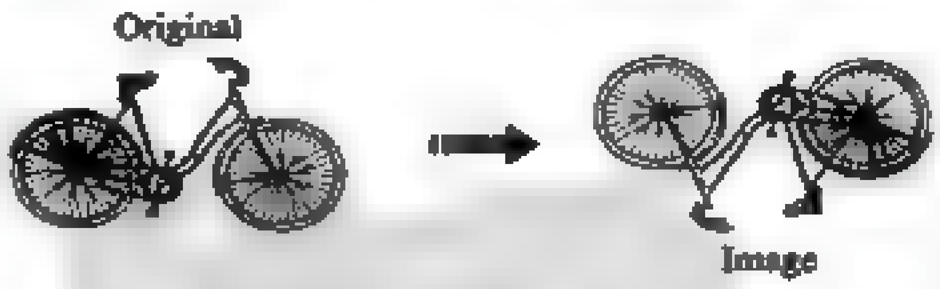


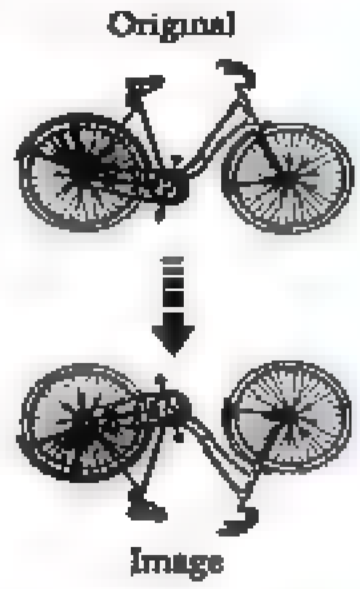
From the school book

Exercise 8 On geometric transformations

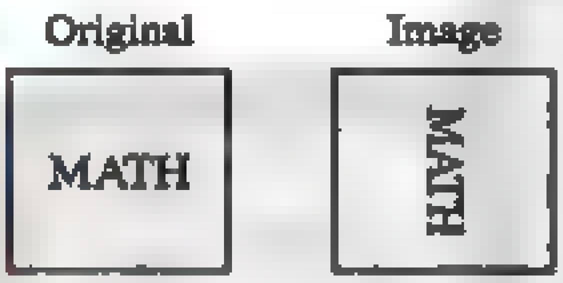
1 Describe the type of the geometric transformation (reflection , translation or rotation) in each of the following :

(1)


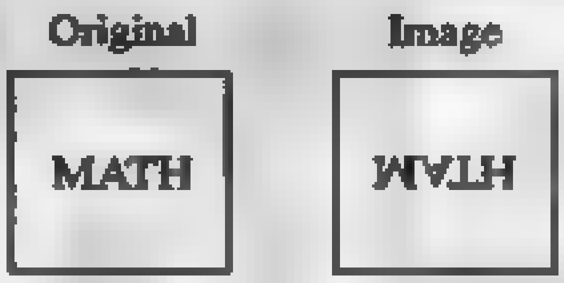
(2)


(3)


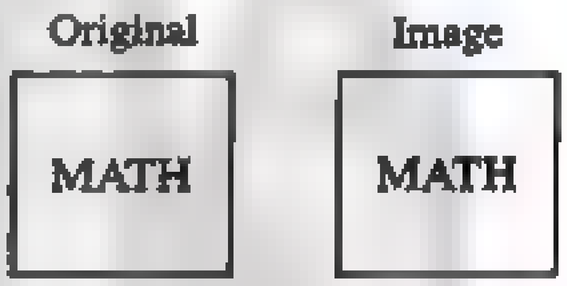
2 Write below each shape the type of the geometric transformation (reflection , translation or rotation) :

(1)


....

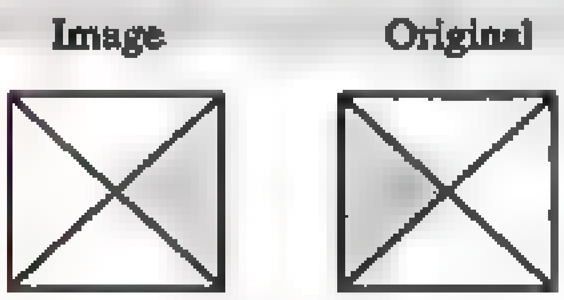
(2)


....

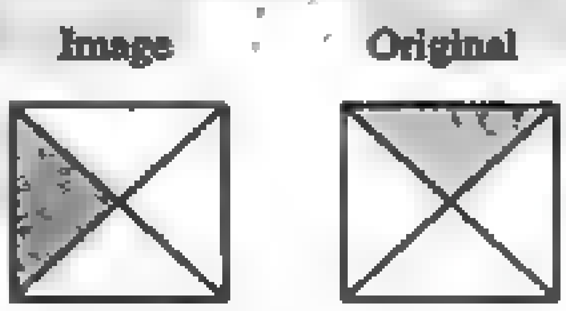
(3)


....

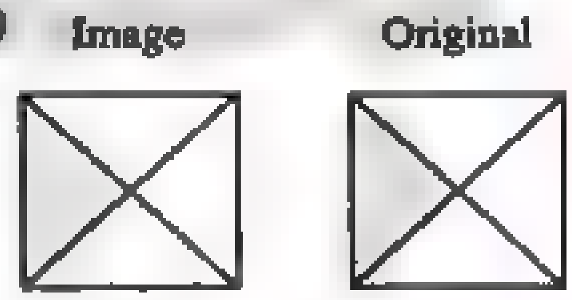
3 Write below each shape the type of the geometric transformation (reflection , translation or rotation) :

(1)


....

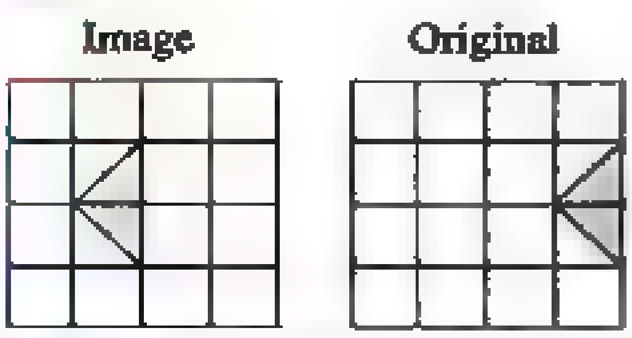
(2)


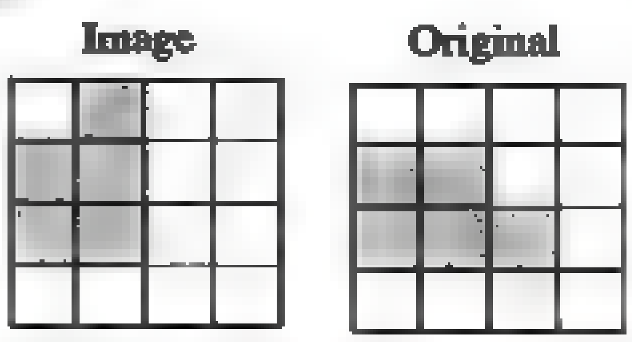
....

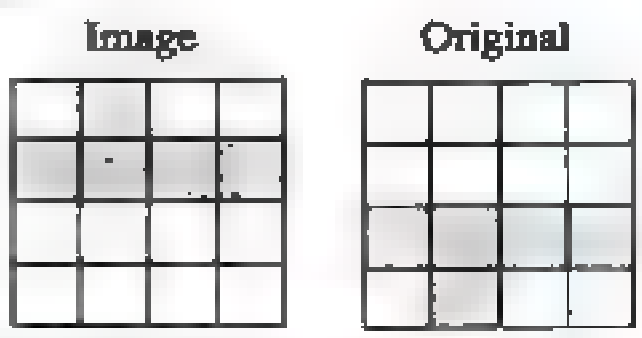
(3)


....

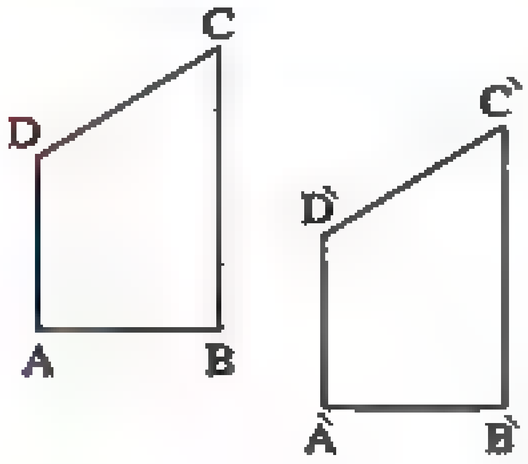
4 Write the type of the geometric transformation in each of the following shapes :

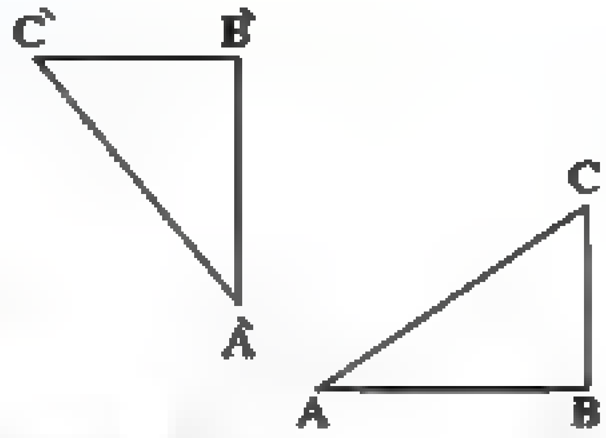
(1)


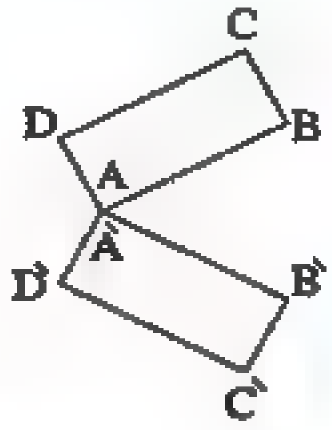
(2)


(3)


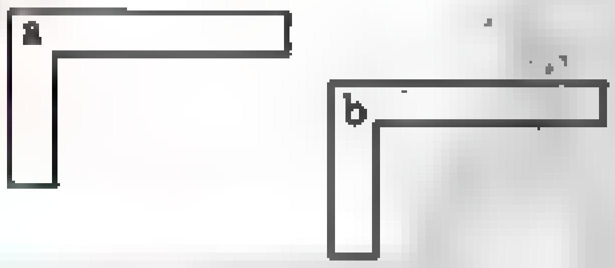
5 Describe the type of transformation in each of the following figures (reflection , translation , rotation) :

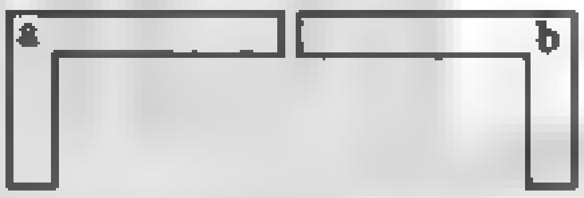

(1) ....

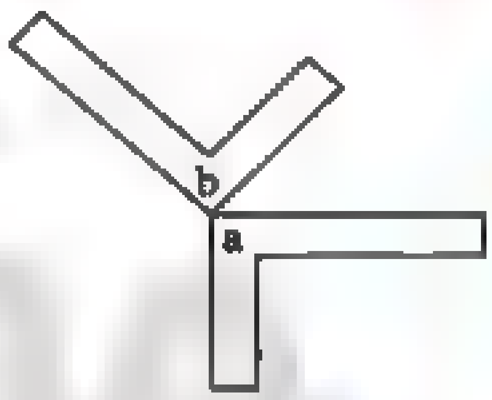

(2) ....

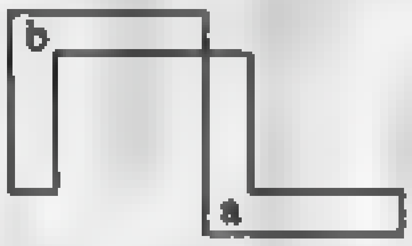

(3) ....

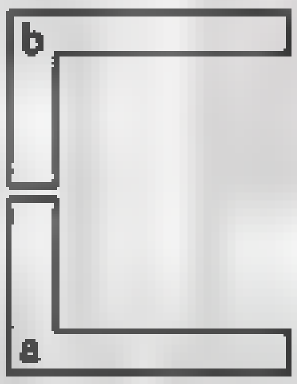
6 Figure b is the image of figure a by a geometric transformation. Identify each transformation as (a translation , a reflection or a rotation) :

(1)


(2)


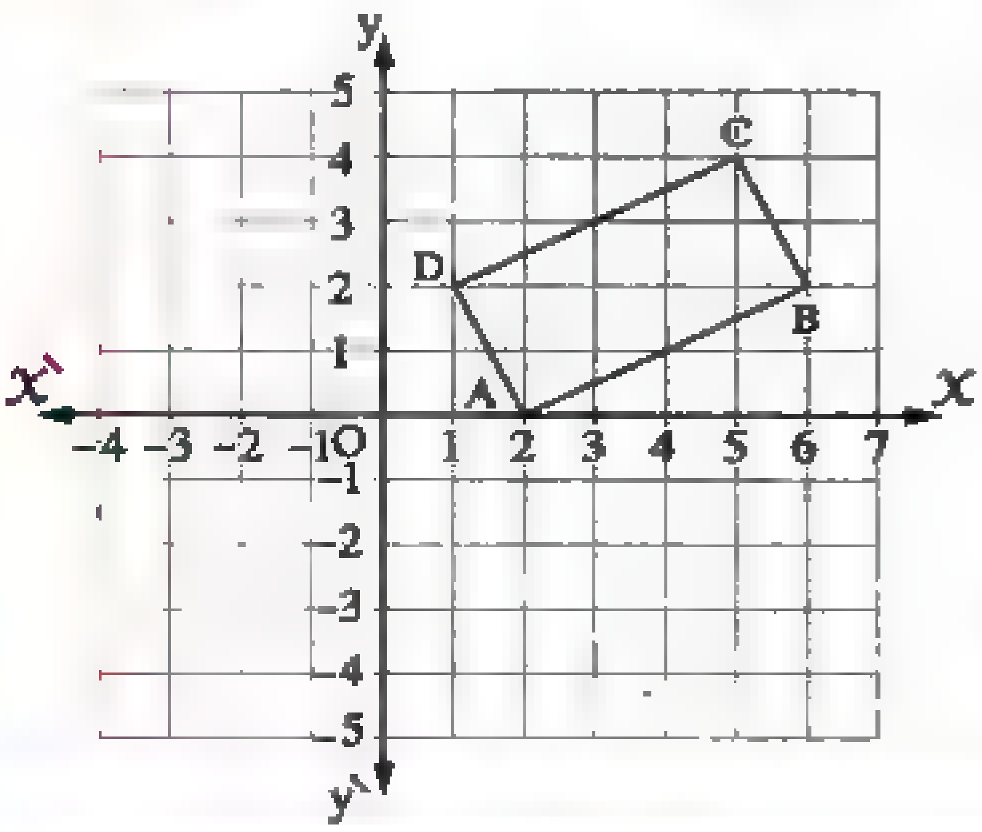
(3)


(4)


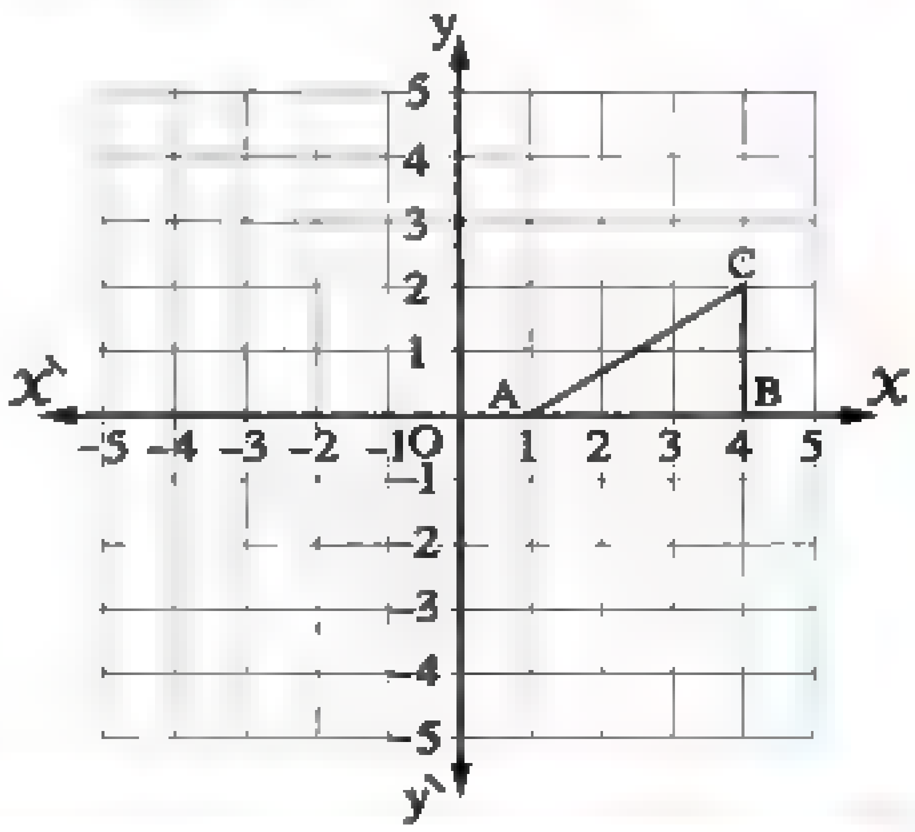
(5)


7 Draw the image of each figure according to the shown transformation , then describe each type :

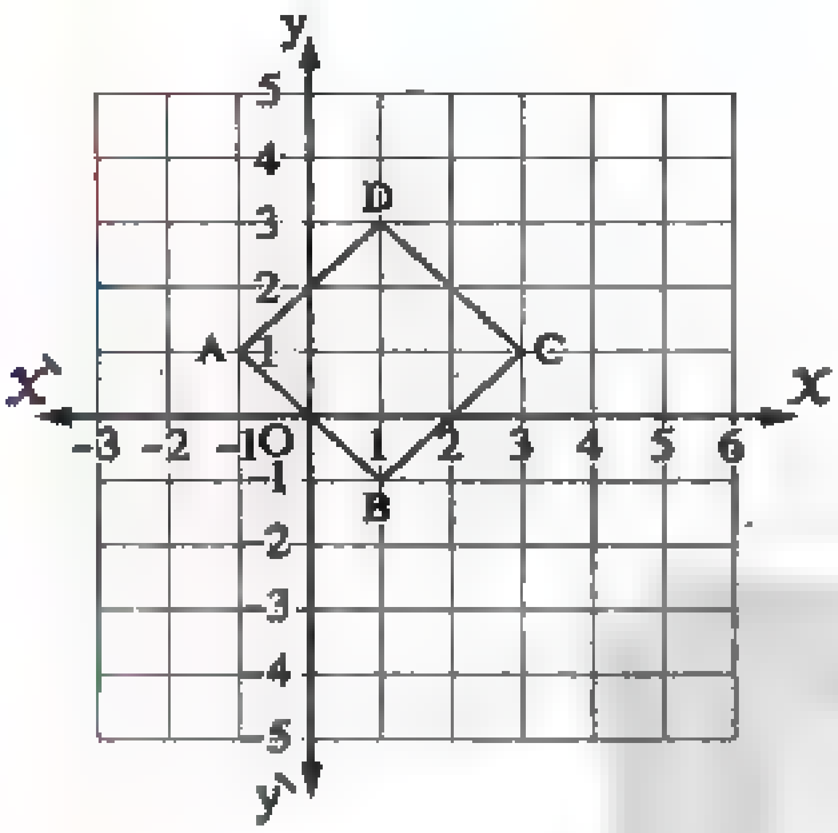
(1) $(X , y) \longrightarrow (-X , y)$



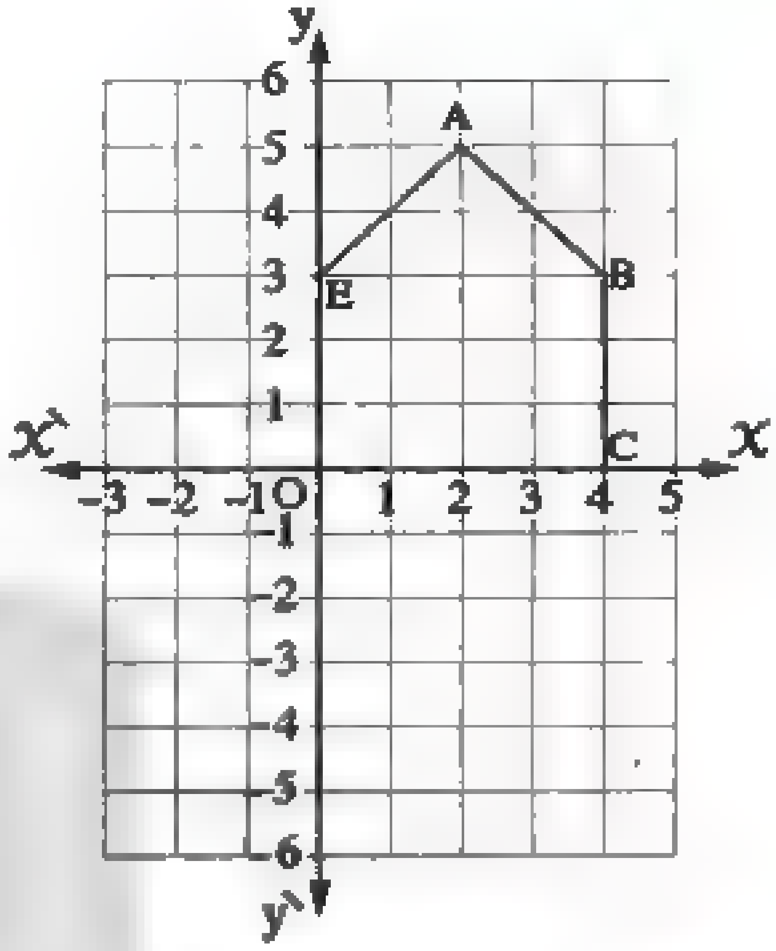
(2) $(X , y) \longrightarrow (-X , -y)$



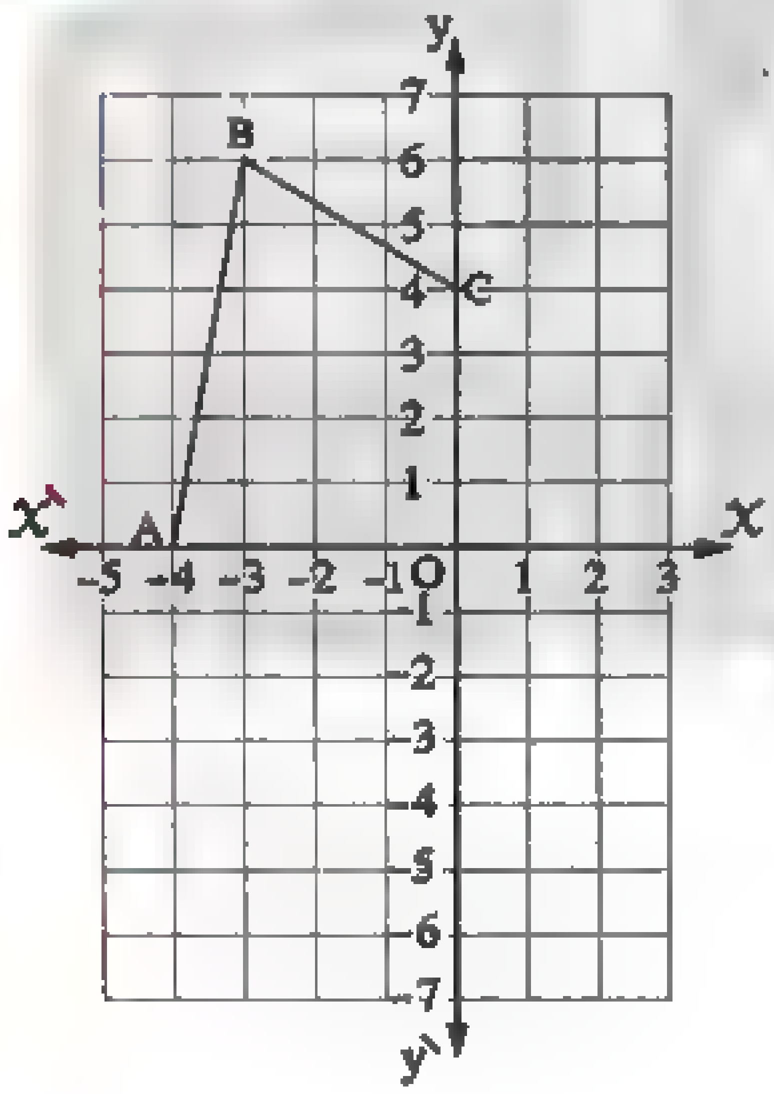
(3) $(x, y) \longrightarrow (x+2, y+3)$



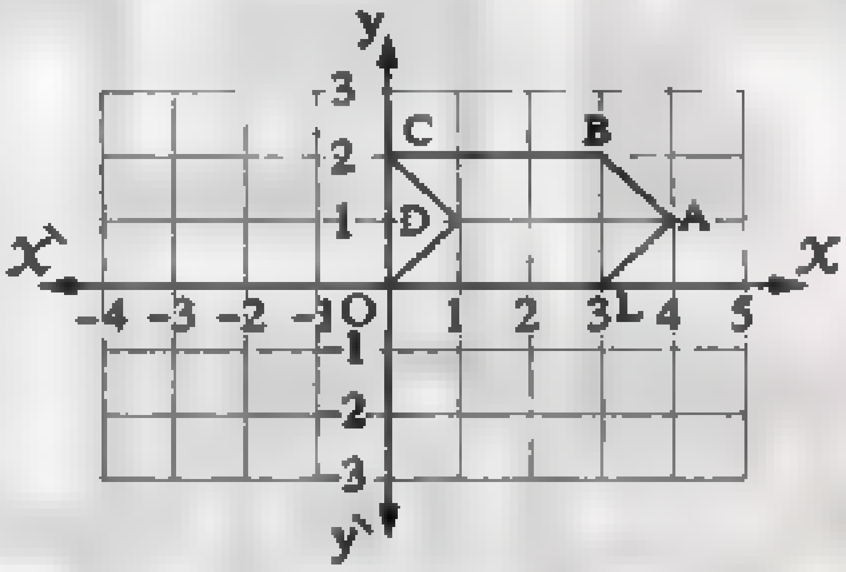
(4) $(x, y) \longrightarrow (x, y-3)$



(5) $(x, y) \longrightarrow (y, -x)$



(6) $(x, y) \longrightarrow (-x, y)$



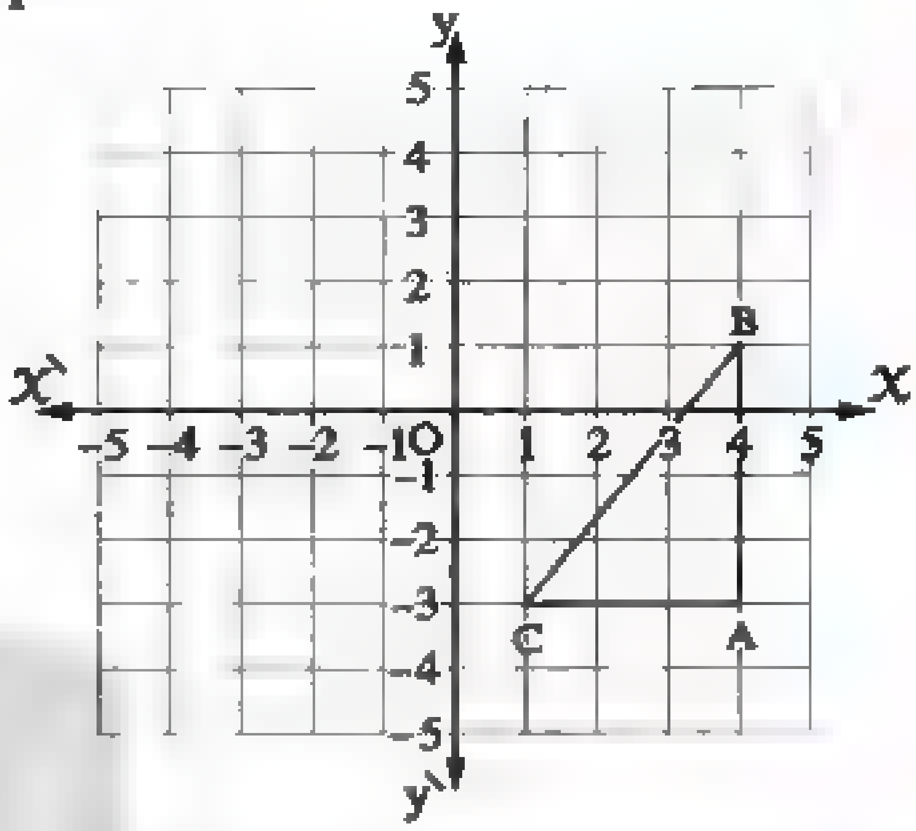
8 Map the image of ΔABC where $A(4, -3)$, $B(4, 1)$, $C(1, -3)$ according to the following transformations then describe its type :

(1) $(x, y) \longrightarrow (-x, y)$

(2) $(x, y) \longrightarrow (-x, -y)$

(3) $(x, y) \longrightarrow (x, y-2)$

(4) $(x, y) \longrightarrow (-y, x)$



9 Draw the image of the polygon ABCDEO according to each transformation and describe the type :

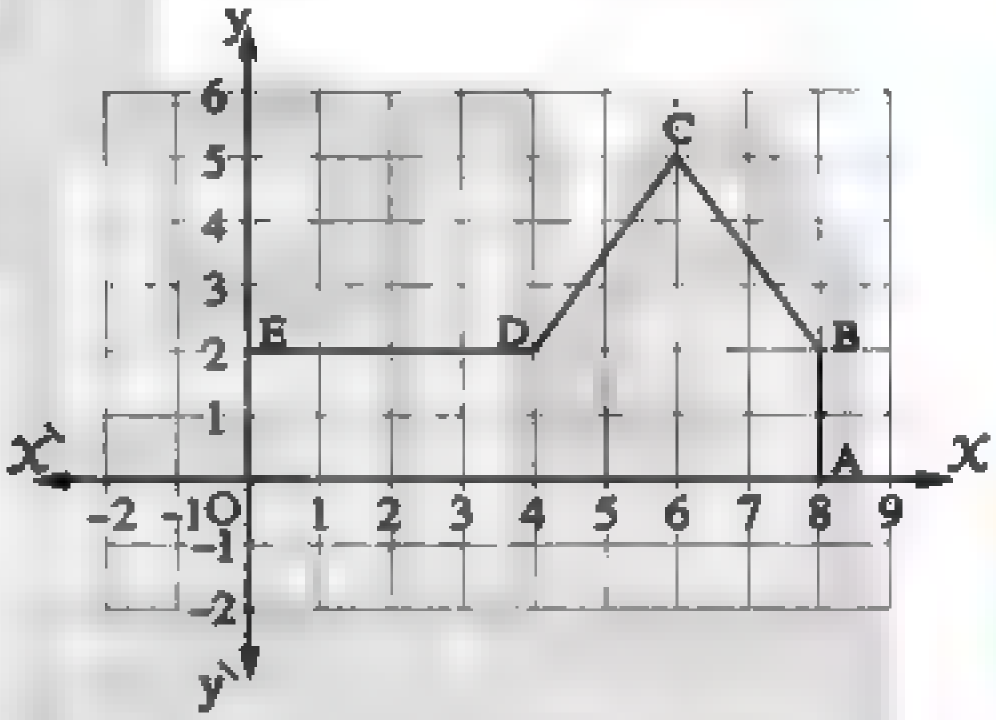
(1) $(x, y) \longrightarrow (-x, y)$

(2) $(x, y) \longrightarrow (x, y+5)$

(3) $(x, y) \longrightarrow (-x, -y)$

(4) $(x, y) \longrightarrow (x-5, y)$

(5) $(x, y) \longrightarrow (x, -y)$



10 Draw the image of ΔABC where $A(1, 2)$, $B(3, 2)$ and $C(3, 5)$ by the following transformations :

(1) $(x, y) \longrightarrow (x, -y)$

(2) $(x, y) \longrightarrow (x+1, y-3)$

(3) $(x, y) \longrightarrow (-y, x)$

11 On a square lattice, draw ΔABO where $A(3, 1)$, $B(1, 3)$ and O is the origin point, and draw its images by the following transformations then discribe its type :

(1) $(x, y) \longrightarrow (x+1, y-2)$

(2) $(x, y) \longrightarrow (x, -y)$

(3) $(x , y) \longrightarrow (-y , x)$

(4) $(x , y) \longrightarrow (-x , -y)$

12 **On a square lattice, draw the quadrilateral ABCD where A (1 , 1), B (4 , 2) , C (3 , 4) and D (1 , 4) , and draw its image by the following transformation, then discribe its type :**

(1) $(x , y) \longrightarrow (y , -x)$

(2) $(x , y) \longrightarrow (-x , y)$

(3) $(x , y) \longrightarrow (x - 1 , y + 1)$

For excellent pupils

13 Draw Δ ABC whose image Δ $\grave{A}$ $\grave{B}$ $\grave{C}$ by the transformation $(x , y) \longrightarrow (-y , x)$ where $\grave{A}$ (1 , – 1) , $\grave{B}$ (3 , 1) and $\grave{C}$ (4 , – 1) , then discribe the transformation type.

From the school book

Exercise 9 On reflection in a straight line

First : Problems on reflection in the plane :

1 Find the image of each of A , $\overline{AB}$ and Δ ABC by reflection in the straight line L :

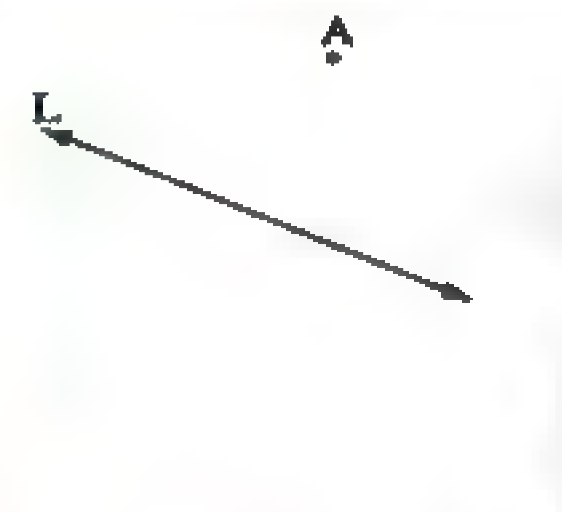


fig. (1)

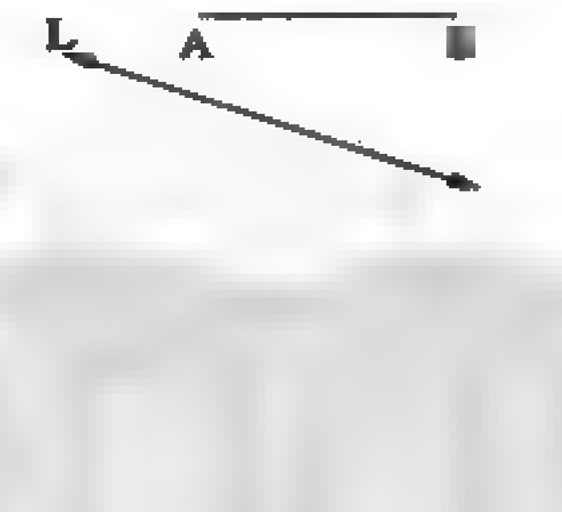


fig. (2)

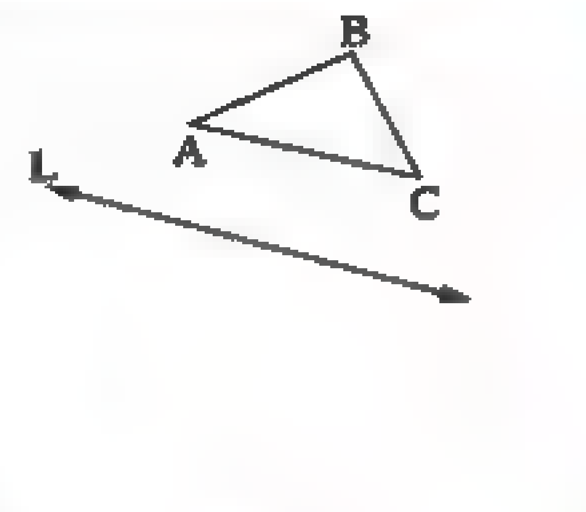
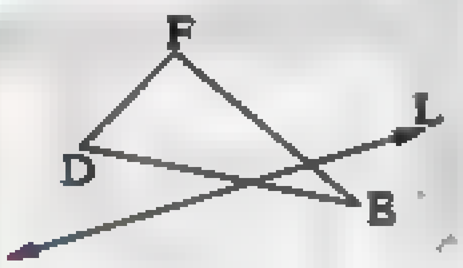


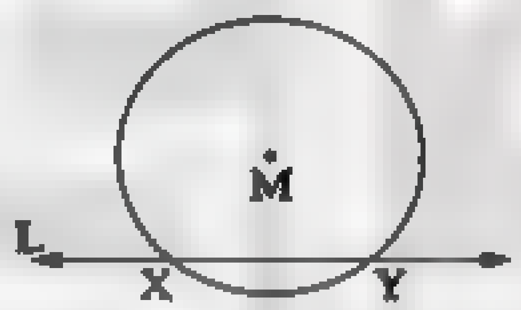
fig. (3)

2 Copy the figures below in your notebook , then draw the images of Δ DEF and the circle M by reflection in L :

(1)

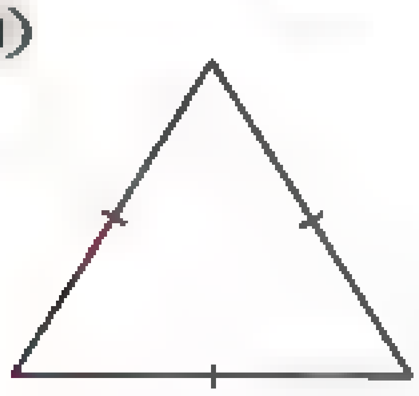


(2)

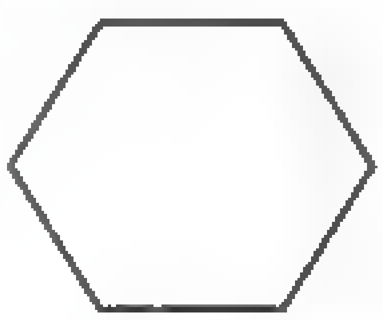


3 Draw the axes of symmetry of each of the following figures if they are existed :

(1)



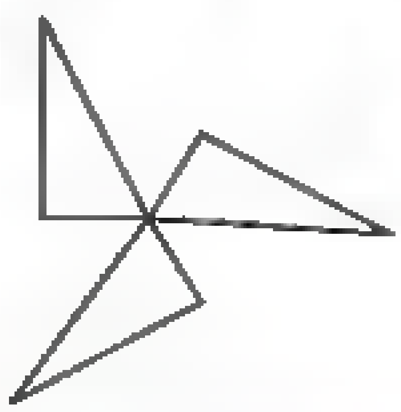
(2)



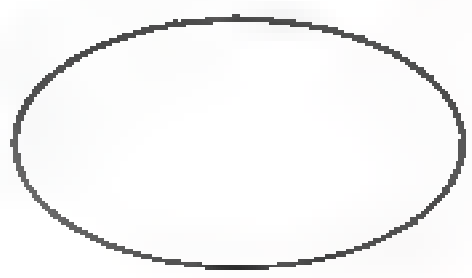
(3)

(4)

(5)



(6)



4 **Draw the triangle ABC in which :** AB = 6 cm. , m (∠ A) = 90° and m (∠ B) = 30°
Then draw its image by reflection in $\overleftrightarrow{AB}$

5 **Draw the image of Δ ABC in which :** AB = 3 cm. , BC = 4 cm. and AC = 5 cm. by reflection in the straight line containing the shortest side.

6 **Draw the image of Δ XYZ in which :** XY = 3 cm. , YZ = 5 cm. and ZX = 7 cm. by reflection in the straight line containing the longest side.

7 **Draw Δ ABC in which :** AB = 3 cm. , BC = 6 cm. and m (∠ ABC) = 90° , then find its image by reflection in the straight line L which is perpendicular to $\overline{BC}$ at C

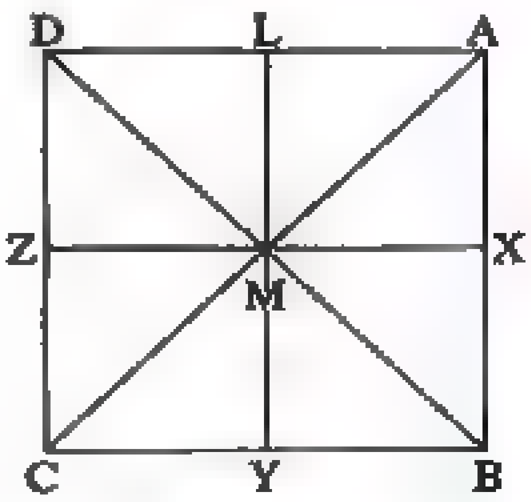
8 **Draw the rectangle ABCD in which :** AB = 6 cm. and CB = 4 cm. , then draw its image by reflection in $\overleftrightarrow{AD}$. Say the name of the resulting figure which consists of the rectangle and its image , then find its perimeter. « 32 cm. »

9 Draw the image of the circle M whose radius length is 2 cm. , by reflection in the straight line which is far from the centre by 1 cm.

10 Draw the circle N with radius length 2.5 cm. , then draw its image by reflection in the straight line which is far from its centre by 2.5 cm.

11 Draw Δ ABC where BC = 3 cm. , AB = 4 cm. and AC = 5 cm.
If the point D is the image of the point C by reflection in $\overleftrightarrow{AB}$
Find :
(1) The perimeter of Δ ACD
(2) The area of Δ ACD « 16 cm. , 12 cm.^{2} »

12 **In the opposite figure :**



ABCD is a square. M is the point of intersection of its diagonals X , Y , Z and L are the midpoints of its sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively. **Complete the following :**
(1) The image of the point A by reflection in $\overleftrightarrow{LY}$ is ....
(2) The image of the $\overline{AM}$ by reflection in $\overleftrightarrow{XM}$ is ....
(3) The image of the Δ ALM by reflection in $\overleftrightarrow{LY}$ is ....
(4) The image of the Δ ALM by reflection in $\overleftrightarrow{XZ}$ is ....
(5) The image of the Δ ALM by reflection in $\overleftrightarrow{AM}$ is ....

(6) The image of the Δ AMB by reflection in $\overleftrightarrow{LY}$ is ....

(7) The image of the Δ AMB by reflection in $\overleftrightarrow{XZ}$ is ....

(8) The image of the square AXML by reflection in $\overleftrightarrow{LY}$ is ....
and by reflection in $\overleftrightarrow{AM}$ is ....

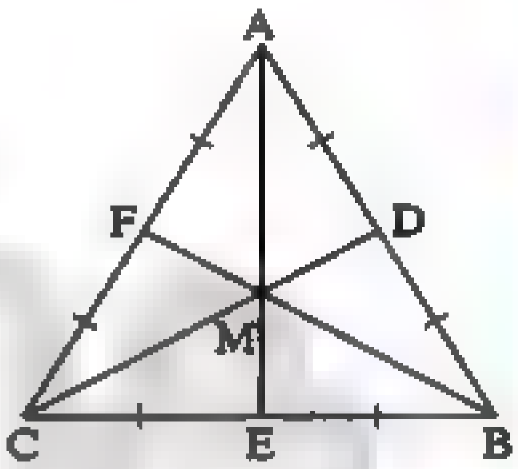
(9) The image of the square ABCD by reflection in $\overleftrightarrow{LY}$ is ....

(10) Δ MZD is the image of Δ MZC by reflection in ....

(11) Δ AXM is the image of Δ CYM by reflection in ....

13 **In the opposite figure :**

Δ ABC is an equilateral triangle , where D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively , and $\overline{AE} \cap \overline{BF} \cap \overline{CD} = \{M\}$:



Complete :

(1) Axes of symmetry of Δ ABC are ....

(2) $\overline{AB}$ is the reflected image of $\overline{AC}$ by reflection in ....

(3) The reflected image of $\overline{AF}$ by reflection in $\overleftrightarrow{BF}$ is .... ,
and the reflected image of $\overline{CF}$ in $\overleftrightarrow{AE}$ is ....

(4) The reflected image of Δ AMD by reflection in $\overleftrightarrow{AE}$ is ....
$\therefore$ m ($\angle$ AMD) = m ($\angle$ ....) , because reflection in a line reserves ....

(5) The reflected image of Δ AMB by reflection in $\overleftrightarrow{AE}$ is ....

(6) Δ BMC is the reflected image of .... by reflection in $\overleftrightarrow{CD}$, and the reflected image of .... by reflection in $\overleftrightarrow{BF}$
$\therefore$ BM = AM , and CM = AM , because the reflection reserves ....

14 **Complete the following :**

(1) The reflection in a plane reserves :
(a) .... (b) .... (c) .... (d) ....

(2) If the reflection in a straight line transforms the figure to itself then this straight line is called ....

(3) The number of axes of symmetry of :

(a) The equilateral triangle is ....
(b) The isosceles triangle is ....
(c) The scalene triangle is ....
(d) The parallelogram is ....
(e) The rectangle is ....
(f) The rhombus is ....
(g) The square is ....
(h) The trapezium which is not isosceles is ....
(i) The isosceles trapezium ....
(j) The circle ....

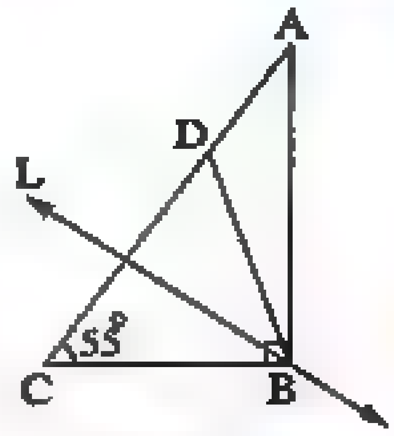
(4) In the opposite figure :

If m (∠ ABC) = 90°

and m (∠ C) = 55° , the straight line L

is the axis of symmetry of Δ DBC

, then m (∠ ABD) = ... °



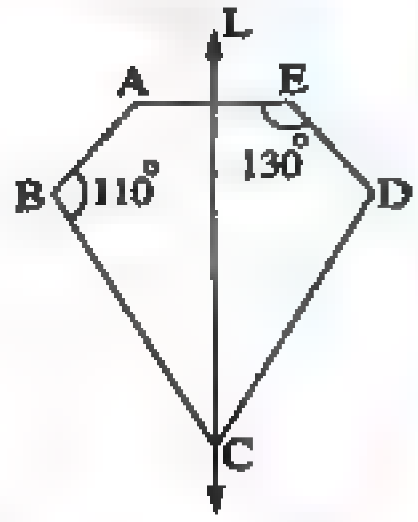
15 In the opposite figure :

If the straight line L

is the axis of symmetry

of the figure ABCDE ,

calculate : m (∠ BCD)



« 60° »

16 By using geometric instruments , draw the rectangle ABCD , where AB = 3 cm. and BC = 4 cm. locate $\grave{A}$ as the reflected image of A by reflection in $\overleftrightarrow{CD}$ and locate $\grave{C}$ as the reflected image of C by reflection in $\overleftrightarrow{AB}$

Prove that :

(1) m (∠ $\grave{C}$AC) = 2 m (∠ CAB)   (2) $\overleftrightarrow{A\grave{C}}$ // $\overleftrightarrow{\grave{A}C}$

Life Applications

17 By using the reflection we can complete drawing some of the symmetrical shapes like the butterfly in the opposite figure.

Use the reflection to complete each of the following figures :

18 In our daily life , we see many figures having one or more axes of symmetry in front of you , there are some signs of cars. Draw their axes of symmetry if they exist :

(1) (2) (3) (4)

(5) (6) (7) (8)

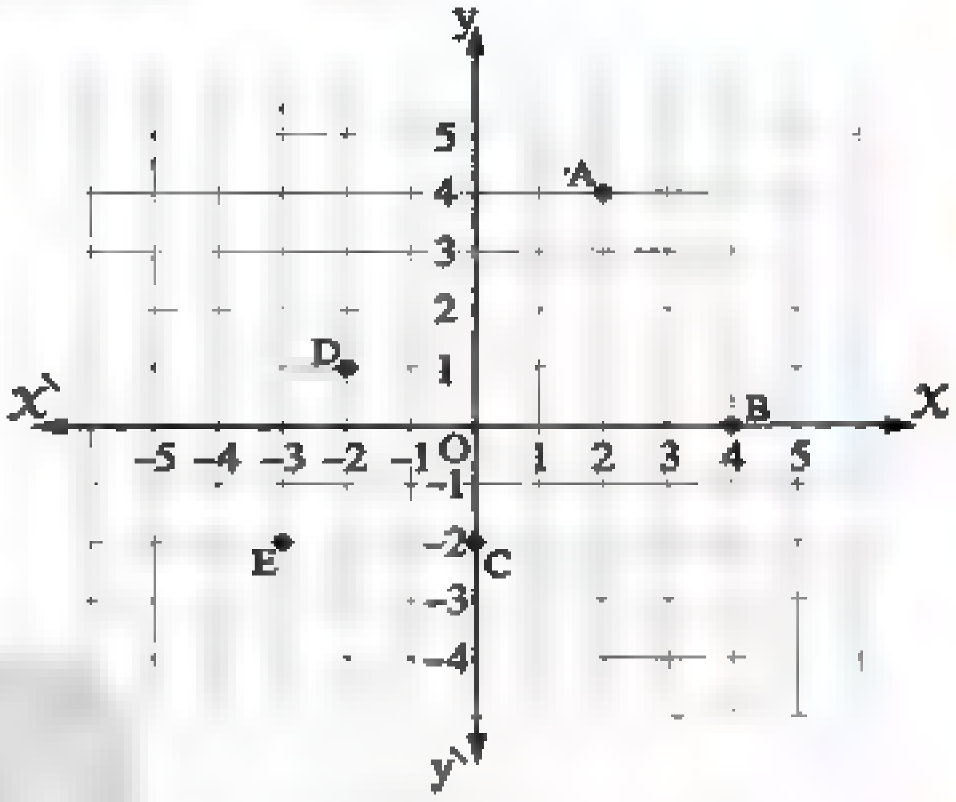
Second | Problems on reflection in the Cartesian plane :

1 In the opposite figure :

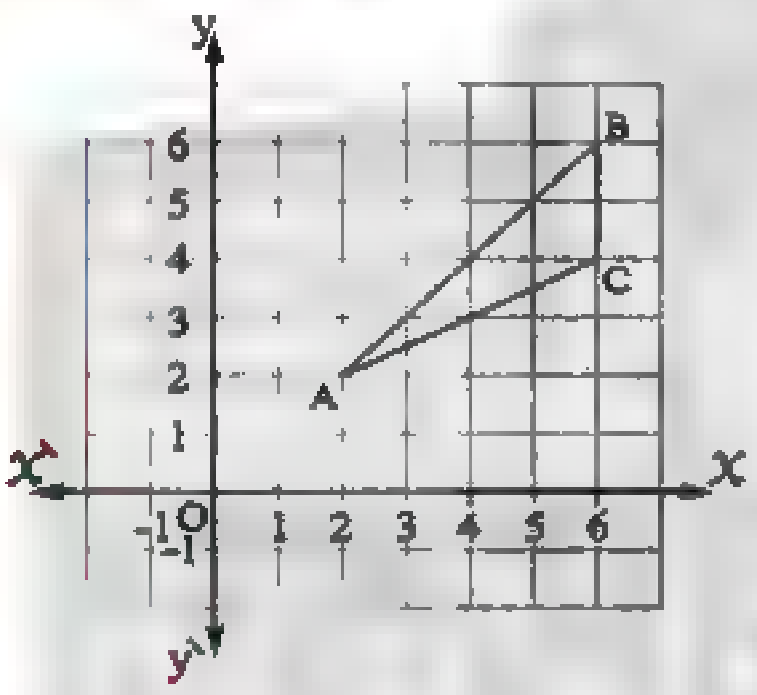
Write the coordinates of the image of each point by reflection in :

(1) The x-axis

(2) The y-axis

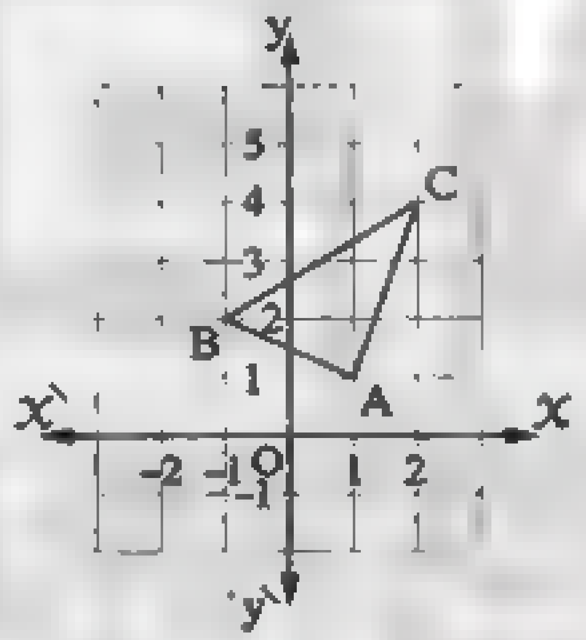


2 Copy each of the following figures on a lattice and draw the images of the figure by a geometric transformation as shown below each figure , then write the coordinates of each vertex of the figure.



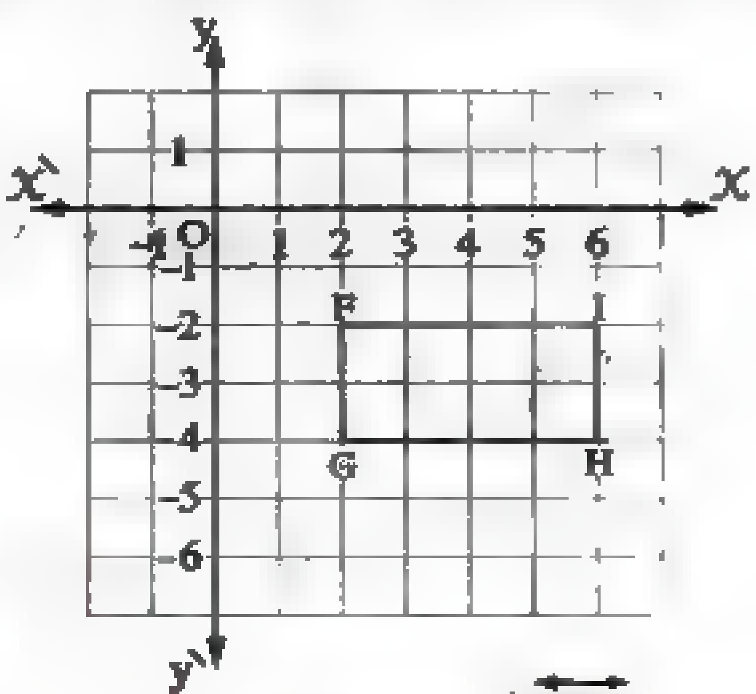
reflection in the x-axis

fig. (1)



reflection in the y-axis

fig. (2)



reflection in $\overleftrightarrow{FG}$

fig. (3)

3 Draw $\overline{AB}$ where A (4 , 3) and B (1 , – 2) , then draw its image by reflection in :

(1) The X-axis (2) The y-axis

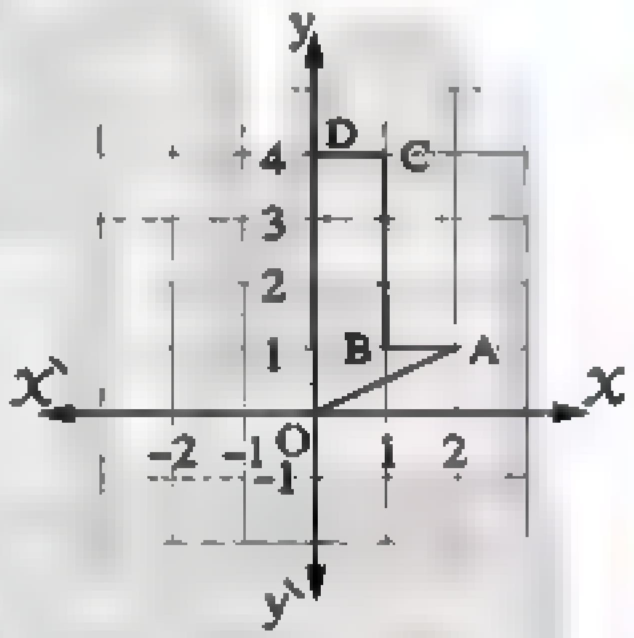
4 If A (3 , 1) and B (3 , – 2) , find $\overline{DC}$ which is the image of $\overline{AB}$ by reflection in the y-axis and name the figure ABCD and calculate its perimeter. « 18 length units »

5 📖 Find the image of Δ ABC where A (– 6 , – 1) , B (– 2 , – 1) and C (– 5 , – 6) by reflection in the X-axis

6 Draw the image of Δ ABC where A (0 , 0) , B (3 , 0) and C (– 1 , 2) by reflection in the y-axis

7 On a square lattice , draw Δ ABC where A (2 , – 2) , B (3 , 4) and C (– 3 , 2) then draw Δ ÀB̀C̀ which is the image of Δ ABC by reflection in the y-axis then draw Δ A̋B̋C̋ which is the image of Δ ÀB̀C̀ by reflection in the X-axis

8 Draw the image of the opposite figure by reflection in the X-axis and in the y-axis another time



9 On a square lattice , draw the rectangle whose vertices are A (3 , 2) , B (8 , 2) , C (8 , 6) and D (3 , 6) , then draw its image by reflection in the y-axis

10 📖 **Graph the square ABCD and its image :** by reflection in the X-axis , then compare the length of the sides and the area where A (0 , 2) , B (– 5 , 0) , C (– 3 , – 5) and D (2 , – 3)

11 ABCD is a rectangle in which : A (1 , 1) , B (1 , 3) and C (– 3 , 3) Determine the coordinates of D from the graph , then find the image of the rectangle ABCD by reflection in the X-axis.

12 📖 Draw the image of the square ABCD where A (2 , 3) and B (2 , – 1) by reflection in the y-axis. What do you notice ?

13 📖 Draw the image of the rectangle XYZL where X (2 , 2) and Y (– 3 , 2) with width 3 units by reflection in the X-axis.

14 Complete the following table :

No.	The point	Its image by reflection in the X-axis	Its image by reflection in the y-axis
1	(3 , – 2)	....	....
2	....	(1 , 2)	....
3	....	....	(– 2 , 4)
4	(0 , 5)	....	....
5	....	(3 , 0)	....
6	....	....	(0 , 0)

15 Complete the following :

(1) The image of the point (1 , 3) by reflection in the X-axis is ....

(2) The image of the point (– 2 , 5) by reflection in the y-axis ....

(3) The image of the point (2 , – 3) by reflection in the .... is (2 , 3)

(4) The image of the point (– 1 , – 4) by reflection in the .... is (1 , – 4)

(5) The image of the point (0 , 3) by reflection in the .... is itself.

(6) The image of the point (– 5 , 0) by reflection in the .... is itself.

(7) The image of the point (2 , 1) by reflection in the X-axis followed by reflection in the y-axis is ....

(8) The image of the point (2 , – 3) by reflection in the y-axis followed by reflection in the X-axis is ....

(9) If $\grave{A}$ (– 2 , 3) is the image of the point A (2 , 3) by reflection in y-axis , then the image of the point $\grave{A}$ by reflection in the y-axis is ....

For excellent pupils

16 Determine on a square lattice the points A (5 , 4) , B (5 , 1) , C (2 , 1) , $\grave{A}$ (4 , 5) , $\grave{B}$ (1 , 5) and $\grave{C}$ (1 , 2)

(1) If $\Delta\ \grave{A}\grave{B}\grave{C}$ is the image of Δ ABC by reflection in the straight line L , draw this straight line.

(2) If the figure $AB\grave{B}\grave{A}$ is the image of the figure $CB\grave{B}\grave{C}$ by reflection in the straight line M , draw this straight line.

17 If the geometric transformation $(X , y) \longrightarrow (y , X)$ is a reflection in a straight line L , draw on a square lattice the straight line L

From the school book

Exercise 10 On reflection in a point

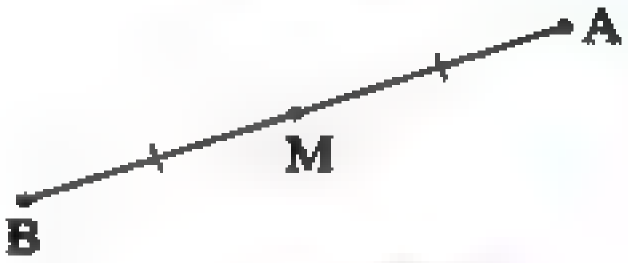
First Problems on reflection in the plane :

1 Choose the correct answer from the given ones :

(1) If $\overline{A'B'}$ is the image of $\overline{AB}$ by reflection in M , then A'B' ……… AB

(a) > (b) < (c) = (d) ≠

(2) **In the opposite figure :**

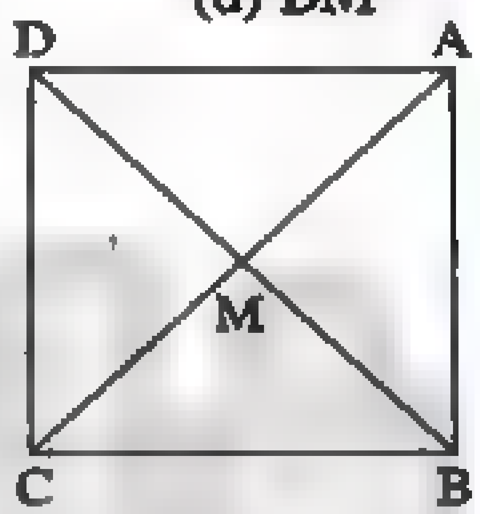


The image of $\overline{AB}$

by reflection in the point M is ………

(a) $\overline{AM}$ (b) $\overline{AB}$ (c) $\overline{BA}$ (d) $\overline{BM}$

(3) **In the opposite figure :**



ABCD is a square whose diagonals intersect at M The image of Δ ABM by reflection in M is Δ ………

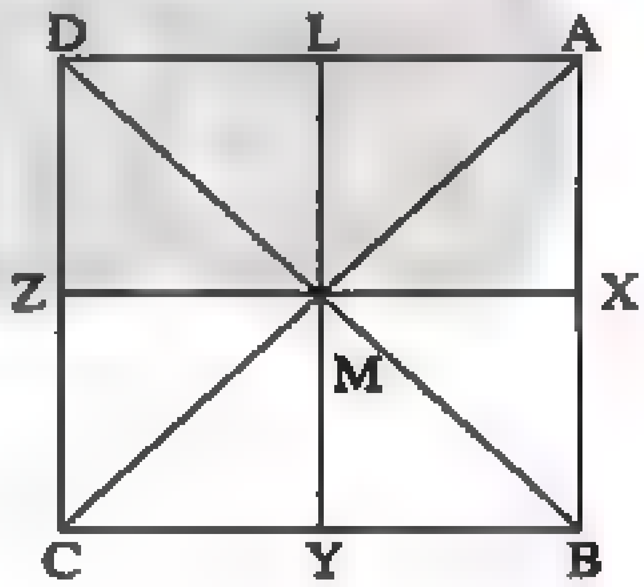
(a) ADM (b) BCM

(c) DCM (d) CDM

(4) If A' is the image of A by reflection in M and if MA = 5 cm. , then AA' = ………

(a) 5 cm. (b) 7 cm. (c) 10 cm. (d) 15 cm.

2 **In the opposite figure :**



ABCD is a square whose diagonals intersect at M

X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Complete the following :

(1) The image of the point A by reflection in M is ………

(2) The image of the point X by reflection in M is ………

(3) The image of $\overline{AL}$ by reflection in M is ………

(4) The image of $\overline{MZ}$ by reflection in M is ………

(5) The image of $\overline{BM}$ by reflection in M is ………

(6) The image of $\overline{AX}$ by reflection in X is ………

(7) The image of Δ ALM by reflection in M is ………

(8) The image of Δ BXM by reflection in M is ………

(9) The image of Δ AMB by reflection in M is ………

(10) The image of the square AXML by reflection in M is ………

3 Using the geometric tools , draw the image of each of the following by reflection in A (Answer in the same page of the book) :

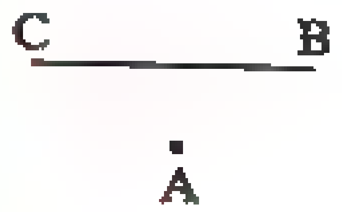


Fig. (1)

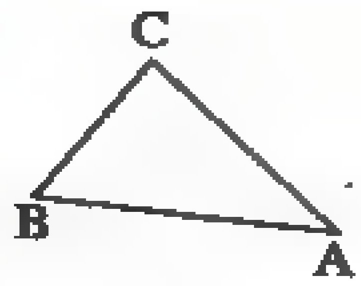


Fig. (2)

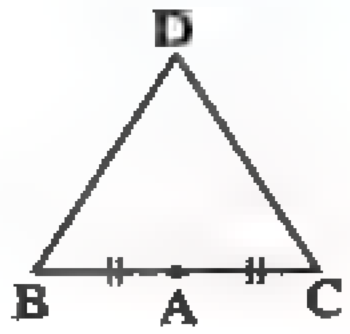


Fig. (3)

4 Draw Δ ABC in which AB = BC = 4 cm. and AC = 5 cm. , then find its image by reflection in the point B

5 In each of the following figures, draw $\Delta\, \grave{A}\grave{B}\grave{C}$ as the image of Δ ABC by reflection in the point B and mention the name of the figure $A\,\grave{C}\grave{A}C$ giving reason.

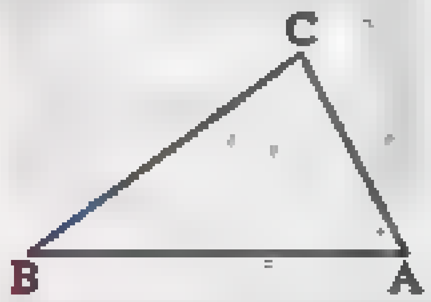


Fig. (1)

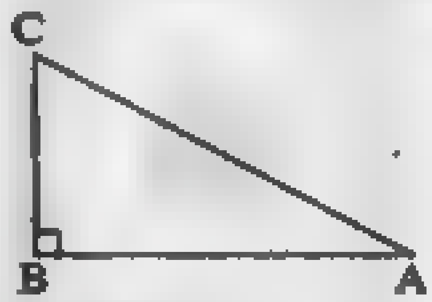


Fig. (2)

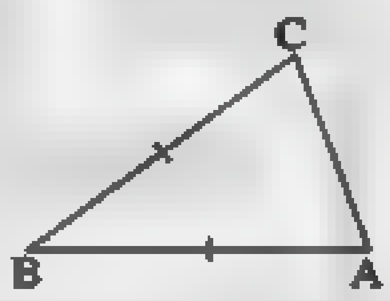


Fig. (3)

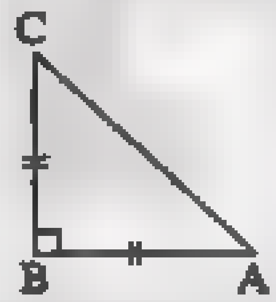


Fig. (4)

6 Draw Δ ABC in which BC = 3 cm. , AB = 4 cm. and m ($\angle$ B) = 90° , then draw $\Delta\, \grave{A}\grave{B}C$ as the image of Δ ABC by reflection in C Prove that the quadrilateral $ABA\grave{}\grave{B}$ is a parallelogram.

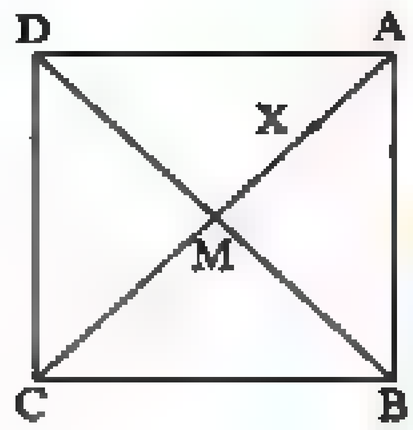
7 Draw the square ABCD whose side length is 5 cm. then draw its image by reflection in the point M where M is the point of intersection of its diagonals. What do you observe ?

8 ABC is a triangle , F is the midpoint of $\overline{AC}$ Draw D as the image of B by reflection in F What is the type of the figure ABCD and what is the type of the triangle ABC required to transfer the figure ABCD to ....

(1) rectangle. (2) rhombus.

9 **In the opposite figure :**

ABCD is a square , M is the point of intersection of its diagonals and $X \in \overline{AM}$ Find Y as the image of X by reflection in M then,

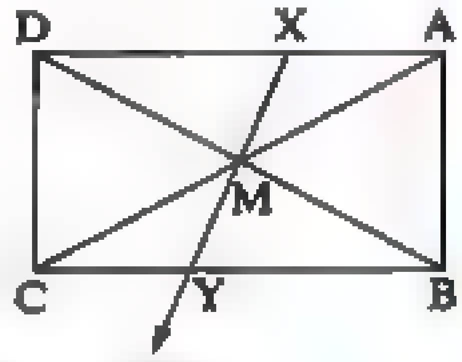


Prove that : (1) Δ DAX $\equiv \Delta$ BCY

(2) The figure DXBY is a parallelogram.

10 **In the opposite figure :**

ABCD is a rectangle , M is the point of intersection of its diagonals , $X \in \overline{AD}$ and $\overrightarrow{XM} \cap \overline{BC} = \{Y\}$



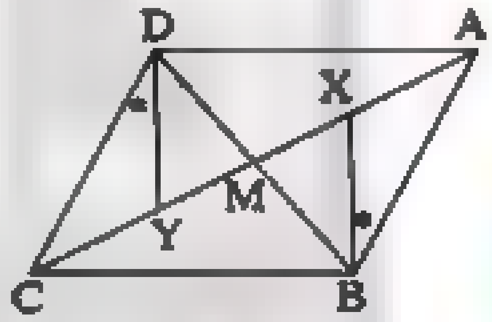
Prove that :

(1) Y is the reflected image of X in M

(2) The figure AXCY is a parallelogram.

11 **In the opposite figure :**

ABCD is a parallelogram , M is the point of intersection of its diagonals and $X \in \overline{AC}$, $Y \in \overline{AC}$ such that m ($\angle$ ABX) = m ($\angle$ CDY)



Prove that :

(1) Δ ABX is the image of Δ CDY by reflection in M

(2) The figure XBYD is a parallelogram.

Life Application

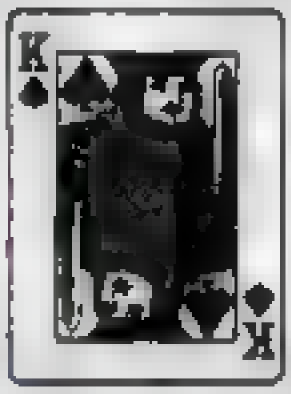
12 In front of you , an English letter. Check that if we find the image of this letter by reflection in the midpoint of the inclined line of the letter , then we get the same letter. And we notice that we will see the same figure if we look at it from above or down , right or left. Which of the following figures has the same property ?

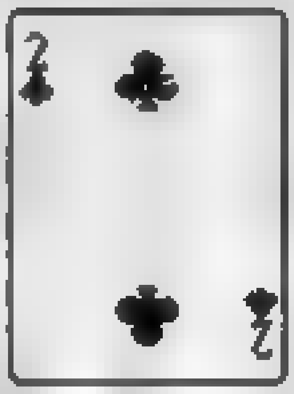
N

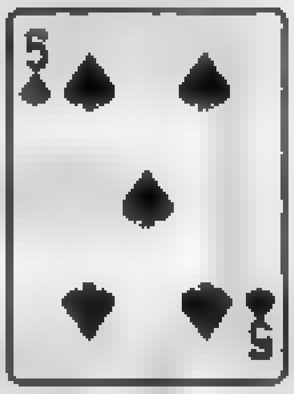
Enrich information

This point is called "point of symmetry"

M	I	H	E	A
(1)	(2)	(3)	(4)	(5)
Z	X	W	T	S
(6)	(7)	(8)	(9)	(10)


(11)


(12)


(13)

Second Problems on reflection in the Cartesian plane :

1 Choose the correct answer from those given :

(1) The image of the point (– 3 , 2) by reflection in the origin point is ....

(a) (3 , 2) (b) (– 3 , – 2) (c) (3 , – 2) (d) (– 3 , 2)

(2) The point (5 , – 2) is the image of the point .... by reflection in the origin point.

(a) (5 , – 2) (b) (– 5 , – 2) (c) (– 5 , 2) (d) (5 , 2)

(3) The point whose image by reflection in the origin point is itself is ....

(a) (0 , 1) (b) (1 , 0) (c) (0 , 0) (d) (– 1 , 0)

(4) The image of the point (3 , – 2) by reflection in the origin point followed by reflection in X-axis is ....

(a) (3 , – 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (3 , 2)

2 On a square lattice , draw ΔABC where $A(3, 1)$, $B(1, 4)$ and $C(0, 0)$, then find its image by reflection in the point C

---

3 In Xy - coordinate plane , draw ΔABC, where : $A(-2, 4)$, $B(5, 0)$ and $C(3, -3)$, then find the reflected image of ΔABC in the origin point.

---

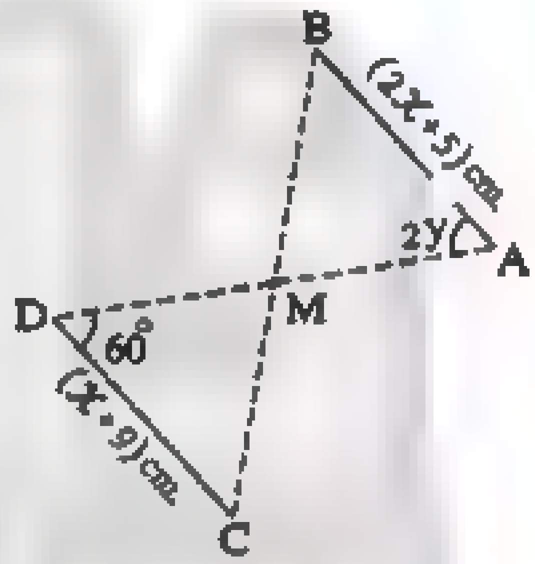
4 On a square lattice , draw ΔABC where $A(2, -2)$, $B(3, 4)$ and $C(-3, 2)$, then map $\Delta A'B'C'$ as the image of ΔABC by reflection in y-axis then map $\Delta A''B''C''$ as the image of $\Delta A'B'C'$ by reflection in X-axis. What is the image of ΔABC by reflection in the origin point ? What do you deduce ?

---

5 ABCD is a rectangle where $A(2, 5)$, $B(6, 5)$, $C(6, 8)$ and $D(2, 8)$, then find the image of the rectangle ABCD by reflection in the origin point.

For excellent pupils

6 In the opposite figure :



If $\overline{CD}$ is the image of $\overline{BA}$ by reflection in the point M and $BA = (2X + 5)$ cm. , $CD = (X + 9)$ cm. , $m(\angle A) = 2y$ and $m(\angle D) = 60°$

Find : (1) The length of $\overline{CD}$

(2) The value of y « 13 cm. , 30° »

From the school book

Exercise 11 On translation

First Problems on translation in the plane :

1 Using the geometric tools , draw the image of each of the following :

By translation MN in the direction of $\overrightarrow{MN}$ as shown in each case.

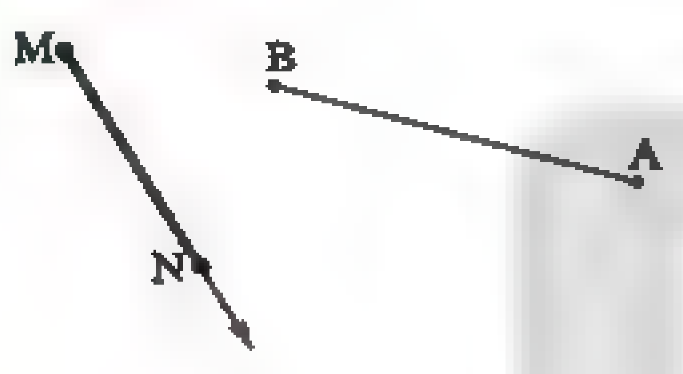


fig. (1)

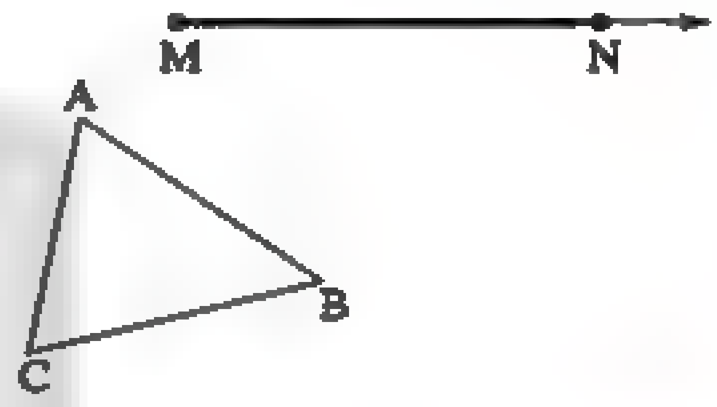
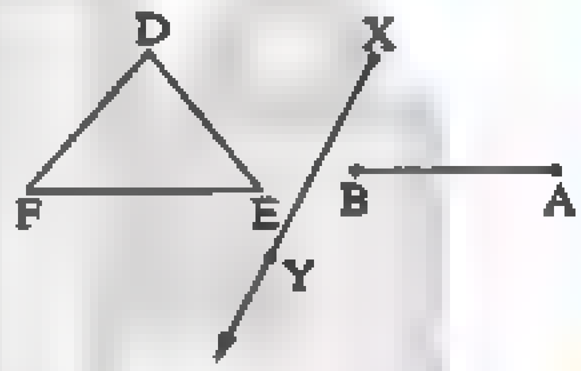


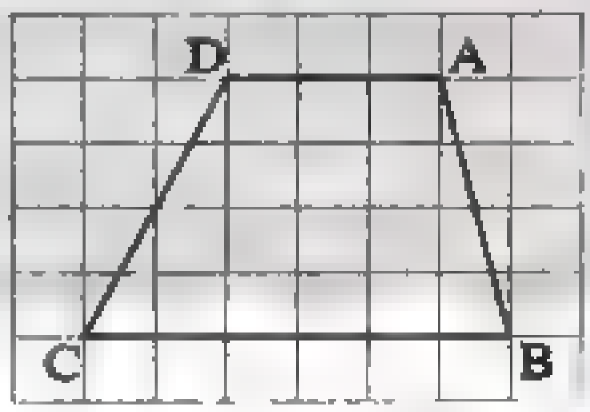
fig. (2)

2 In the opposite figure :

Using the geometric tools , find the image of the following figures by the translation of displacement XY in the direction of $\overrightarrow{XY}$



3 Using the grid , draw the image of the figure ABCD by the translation of 4 units in the direction of $\overrightarrow{BC}$

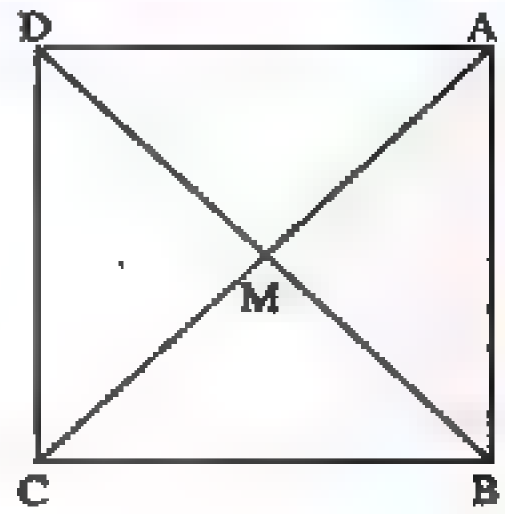


4 Draw a line segment $\overline{AB}$ where AB = 5 cm. , then draw the image of $\overline{AB}$ by a translation of magnitude of 8 cm. in the direction of $\overrightarrow{AB}$

5 Using the geometric instruments , draw the square ABCD whose side length is 4 cm. , then draw its image by translation of magnitude of 4 cm. in the direction of $\overrightarrow{AB}$

6 Draw Δ ABC in which AB = 4 cm. , BC = 6 cm. and CA = 5 cm. , then draw the image of Δ ABC by a translation of magnitude of 3 cm. in the direction of $\overrightarrow{CB}$

7 📖 **In the opposite figure :**



ABCD is a square whose side length is 4 cm.

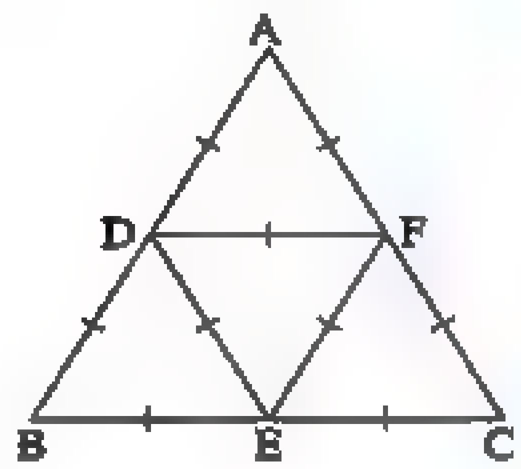
M is the point of intersection of its diagonals. Draw :

(1) The image of Δ MAB by the translation of distance 2 cm. in the direction of $\overrightarrow{AD}$

(2) The image of Δ AMB by the translation AM in the direction of $\overrightarrow{AM}$

---

8 📖 **In the opposite figure :**



The triangles ADF , BDE , DEF and EFC are congruent.

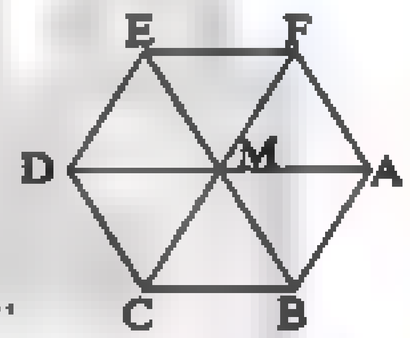
Complete :

(1) The image of Δ ADF by a translation of magnitude of AD in the direction of $\overrightarrow{AD}$ is ....

(2) Δ FEC is the image of Δ DBE by a translation of magnitude .... in the direction of ....

---

9 **In the opposite figure :**



ABCDEF is a regular hexagon. **Complete the following :**

(1) The image of the point D by translation DM in the direction of $\overrightarrow{DM}$ is ....

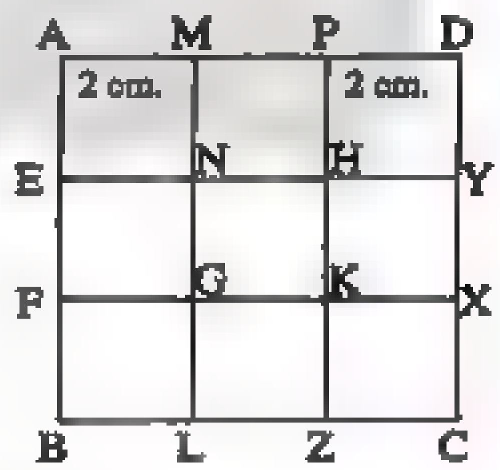
(2) The image of $\overline{AF}$ by translation ED in the direction of $\overrightarrow{ED}$ is ....

(3) The image of Δ MCD by translation EF in the direction of $\overrightarrow{EF}$ is ....

(4) The translation which makes Δ DME the image of Δ MAF is ....

---

10 📖 **In the opposite figure :**



ABCD is a square and all the interior squares are congruent.

Complete :

(1) The image of $\overline{AE}$ by a translation of magnitude of 2 cm. in the direction of $\overrightarrow{GK}$ is ....

(2) The image of the square AENM by a translation of magnitude of 4 cm. in the direction of $\overrightarrow{PK}$ is ....

(3) The square MNHP is the image of the square GLZK by a translation of magnitude .... in the direction of ....

---

11 📖 Δ ABC is right-angled at B where AB = 3 cm. and BC = 4 cm. If $\Delta\, \grave{A}\grave{B}\grave{C}$ is the image of Δ ABC by translation of a distance 3 cm. in the direction of $\overrightarrow{CB}$

Prove that : The figure $A\grave{A}\grave{C}C$ is a parallelogram.

12 Draw Δ ABC which is right-angled at B , in which AB = BC = 3 cm.
then find the image of Δ ABC by translation of a distance 3 cm. in the direction of $\overrightarrow{AB}$, then

Prove that : The figure BB̀C̀C is a square.

---

13 ABCD is a rectangle , where E ∈ $\overline{AD}$ Find the translated image of Δ ABE by translation of a magnitude DA in the direction of $\overrightarrow{AD}$ If È is the image of E by the same translation.

Prove that : The figure BCÈE is a parallelogram.

---

14 ABCD is a parallelogram , $\overline{BE} \perp \overline{AD}$ cutting it at E Find Δ ÀB̀D as the image of Δ ABE by translation of a distance ED in the direction of $\overrightarrow{AD}$, then prove that the figure EBB̀D is a rectangle.

Second Problems on translation in the Cartesian plane :

1 Complete the following :

(1) The image of the point (2 , 5) by translation $(X , y) \longrightarrow (X + 2 , y + 1)$ is ....

(2) The image of the point (3 , 2) by translation $(X , y) \longrightarrow (X + 3 , y - 2)$ is ....

(3) The image of the point (– 5 , 4) by translation $(X , y) \longrightarrow (X + 4 , y - 5)$ is ....

(4) The image of the point (– 2 , – 5) by translation $(X , y) \longrightarrow (X - 2 , y)$ is ....

(5) The image of the point (3 , – 2) by translation $(X , y) \longrightarrow (X , y + 3)$ is ....

---

2 Choose the correct answer from those given :

(1) The image of the point (– 1 , 2) by translation of magnitude of 3 units in the positive direction of the X-axis is ....

(a) (– 1 , 5) (b) (2 , 2) (c) (– 2 , 2) (d) (– 1 , 3)

(2) The image of the point (– 3 , 4) by translation of magnitude of 4 units in the negative direction of the y-axis is ....

(a) (– 3 , 0) (b) (– 7 , 4) (c) (– 3 , 8) (d) (– 1 , 4)

(3) If À (3 , – 3) is the image of A by translation $(X , y) \longrightarrow (X - 1 , y - 4)$, then the point A is ....

(a) (2 , – 7) (b) (4 , 1) (c) (– 4 , – 1) (d) (2 , 1)

(4) The image of the point (– 1 , 4) by the translation (3 , – 2) followed by reflection in the X-axis is ....

(a) (2 , 2) (b) (– 2 , 2) (c) (– 2 , – 2) (d) (2 , – 2)

(5) If the point (a , – 1) is the image of (2 , 4) by the translation $(X , y) \longrightarrow (X + 1 , y - b)$, then (a , b) is ....

(a) (3 , 3) (b) (1 , 3) (c) (3 , 5) (d) (1 , – 5)

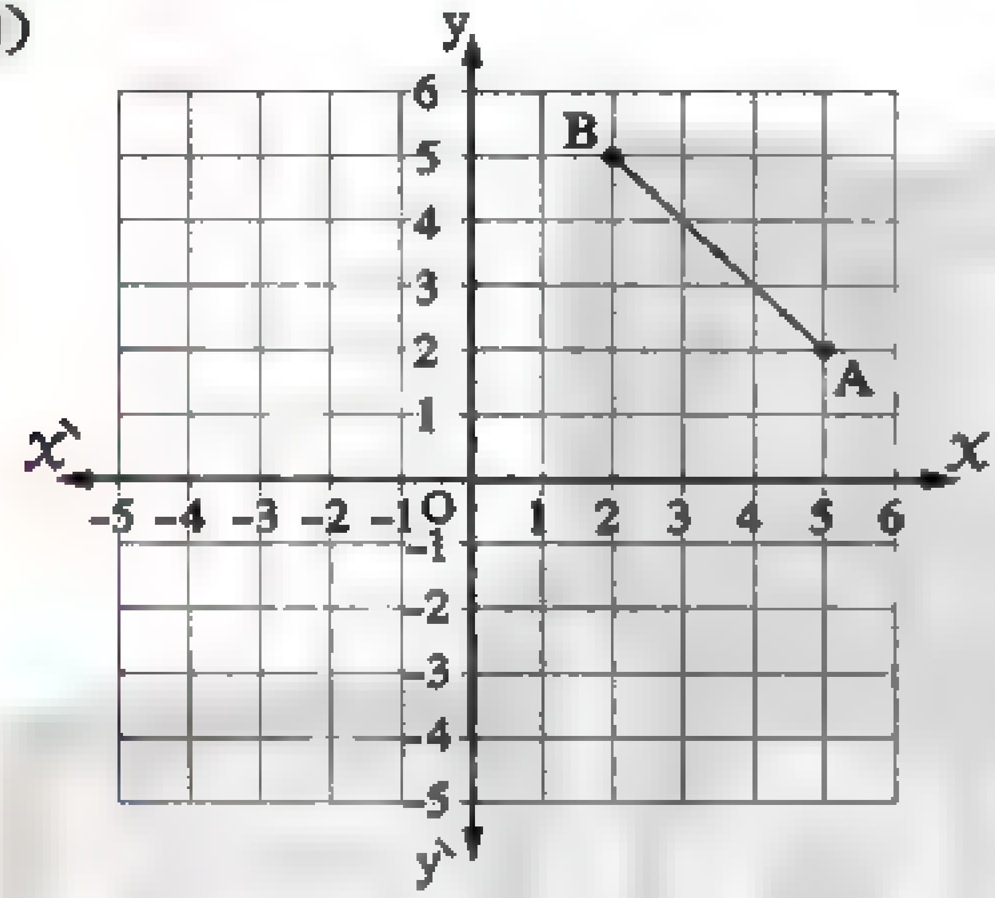
(6) If : $\acute{A}$ is the image of the point A (2 , 3) by reflection in the y-axis , then A is the image of $\acute{A}$ by the translation ....

(a) $(x , y) \longrightarrow (x + 4 , y)$ (b) $(x , y) \longrightarrow (x , y + 6)$

(c) $(x , y) \longrightarrow (x - 4 , y)$ (d) $(x , y) \longrightarrow (x , y - 6)$

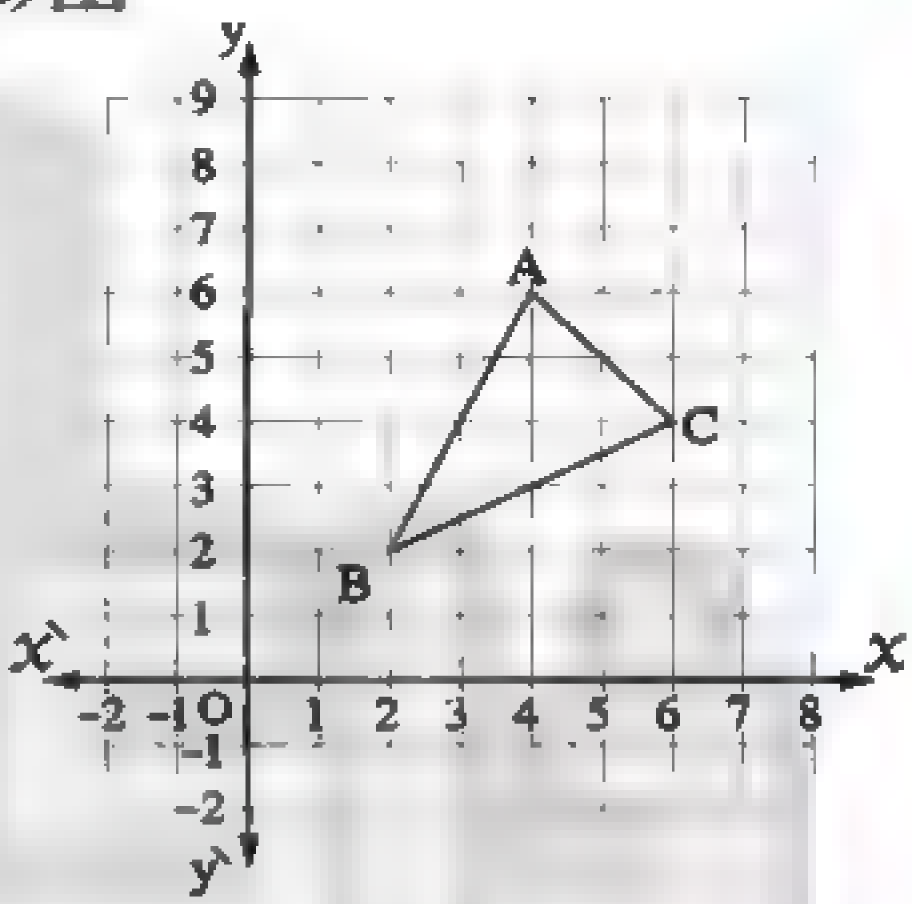
3 **Find the image of each of the following figures by the translation shown under each figure :**

(1)



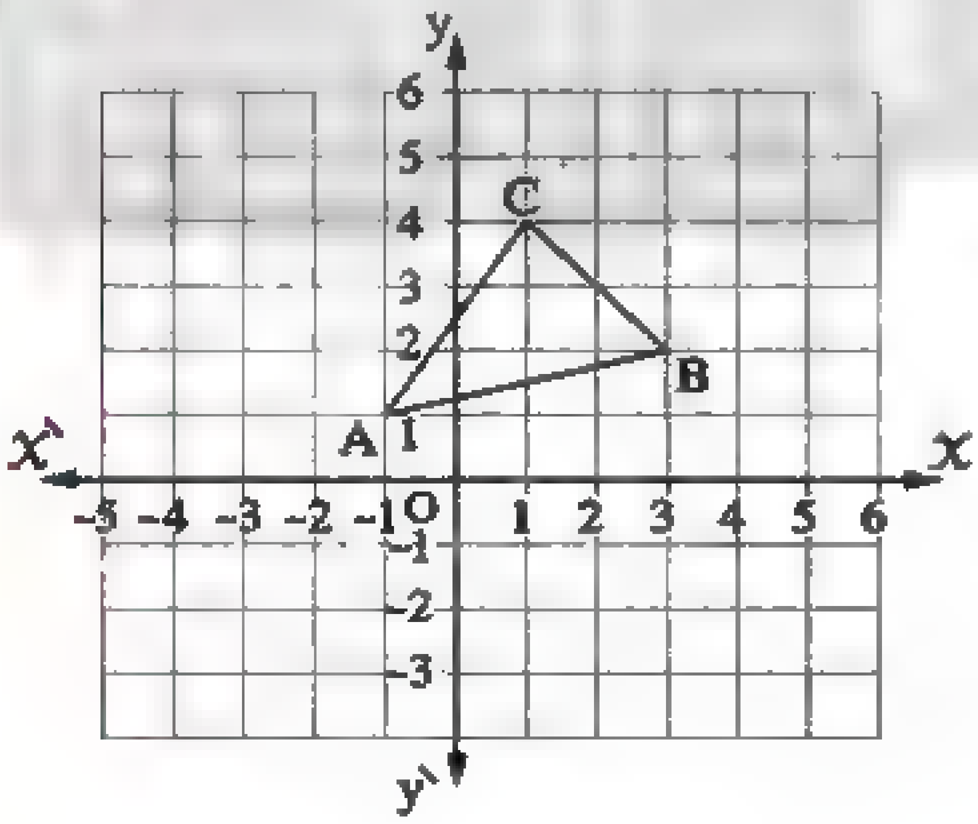
$(x , y) \longrightarrow (x - 3 , y - 4)$

(2) 📖



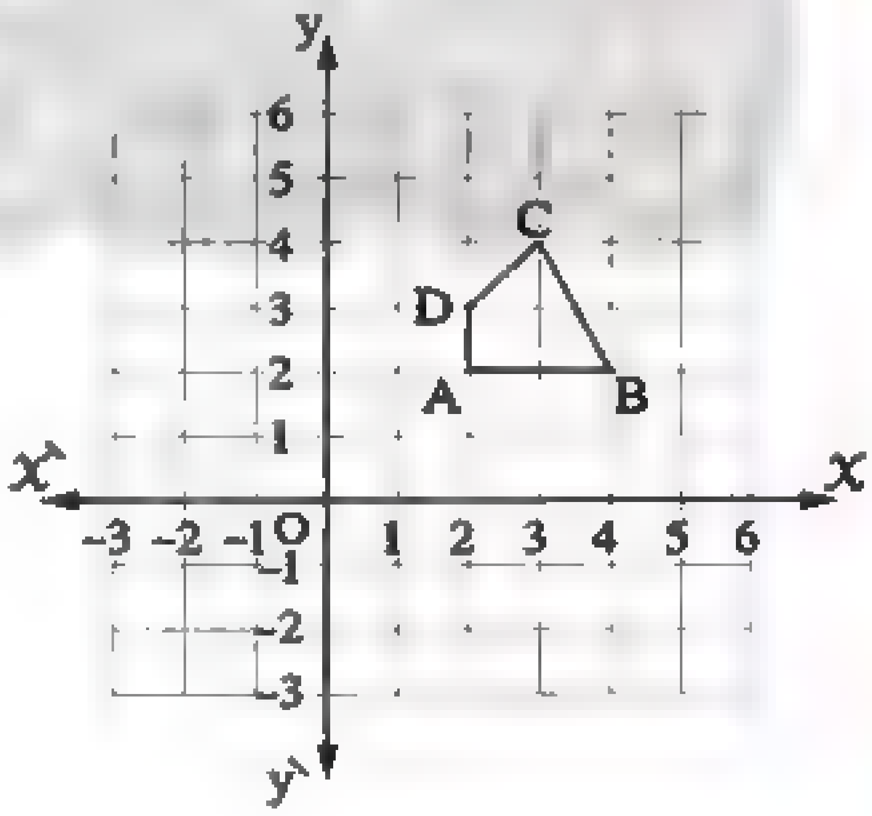
$(x , y) \longrightarrow (x + 2 , y + 3)$

(3)



$(x , y) \longrightarrow (x + 2 , y)$

(4)



$(x , y) \longrightarrow (x + 3 , y - 2)$

10 On a square lattice , draw ΔABC where $A(2, 1)$, $B(1, -1)$ and $C(0, 1)$, then draw its image by a translation of 2 AB in the direction of $\overrightarrow{AB}$

---

11 📖 A square has vertices $A(1, 1)$, $B(4, 2)$, $C(3, 5)$ and $D(0, 4)$

(1) Graph the square and its image under the translation which maps vertex A onto vertex B

(2) Write the mapping rule for the translation.

---

12 📖 Use the translation : $(x, y) \longrightarrow (x+2, y+3)$ to locate the point whose image is $(2, 3)$

---

13 📖 If the image of the point $A(1, 1)$ by translation in the plane is $A'(2, 2)$, find the images of the points $O(0, 0)$, $B(-1, 3)$ and $C(-3, 5)$ by the same translation.

---

14 If $A(-3, 1)$ and $B(1, -2)$, write the mapping rule of the translation that makes B the image of A

---

15 If $A(3, 2)$, $B(5, 1)$, find :

(1) C' which is the image of $C(1, -1)$ under translation of AB in the direction of $\overrightarrow{AB}$

(2) D whose image is $D'(2, 1)$ under translation of AB in the direction of $\overrightarrow{AB}$

---

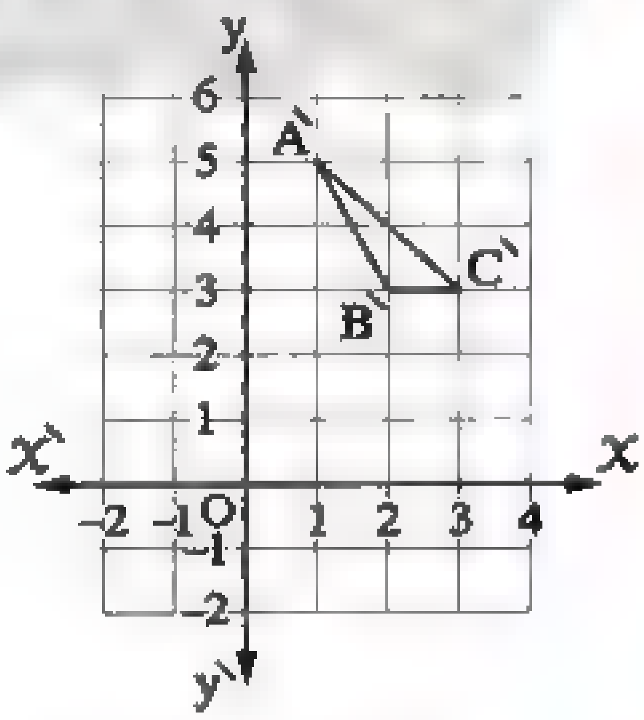
16 📖 The point $A'(3, -3)$ is the image of the point A by the translation $(x, y) \longrightarrow (x-1, y-4)$ Locate A then by the same translation , draw the image of ΔABC where $B(5, 0)$ and $C(-1, -2)$

---

17 📖 **In the opposite figure :**

Copy the graph , then draw the triangle ABC whose image is $\Delta A'B'C'$ by the translation

$(x, y) \longrightarrow (x+2, y+3)$



---

18 If $A(-2, 1)$ and $B(1, 3)$, find on a square lattice the image of $\overline{AB}$ by reflection in x-axis followed by the translation $(x, y) \longrightarrow (x+4, y)$

19 State whether the graph shows a reflection or a translation :

(1) Name the line of reflection.

(2) Describe the translation.

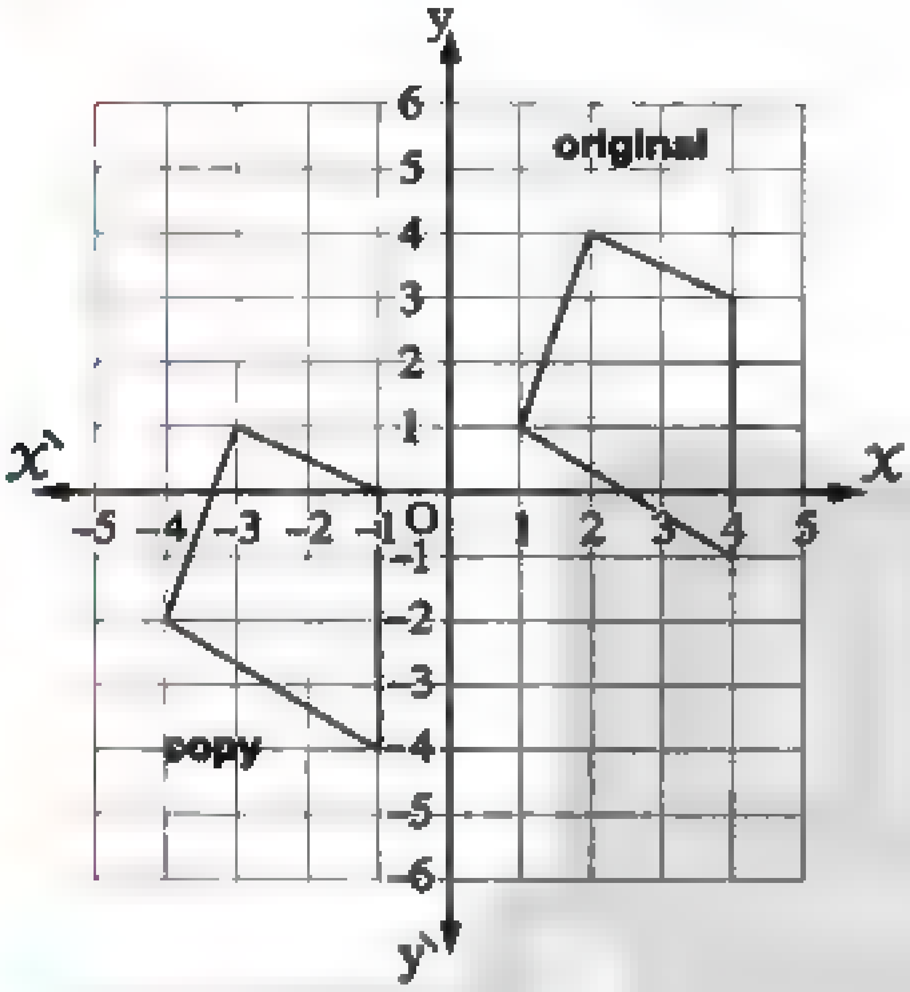


fig. (1)

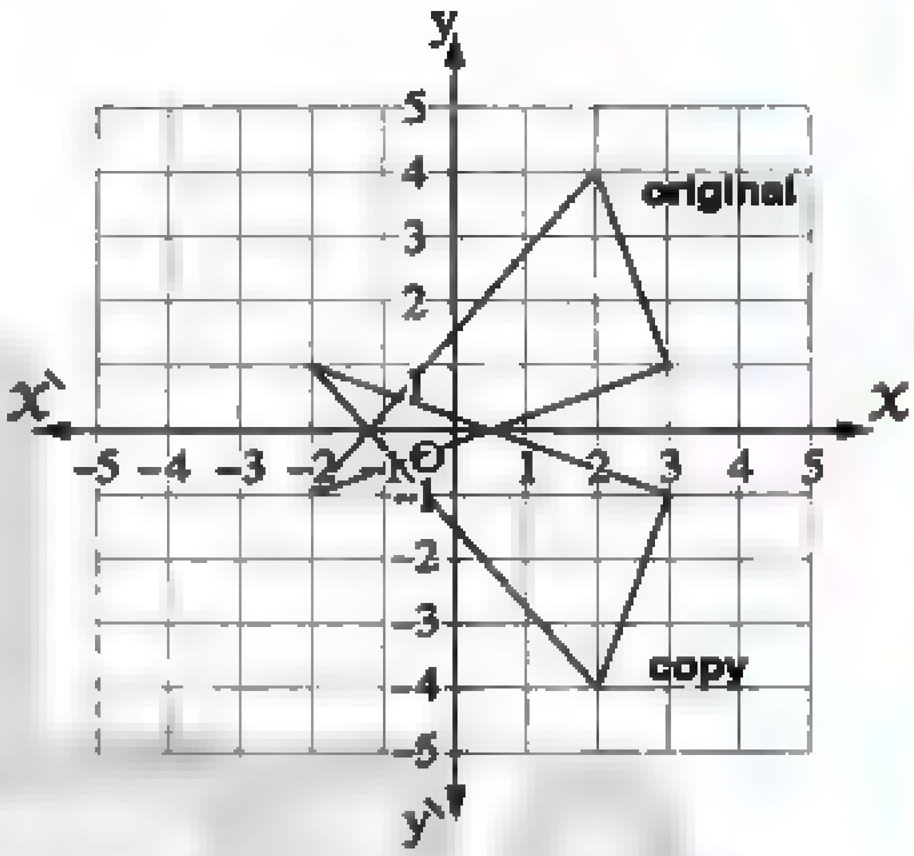


fig. (2)

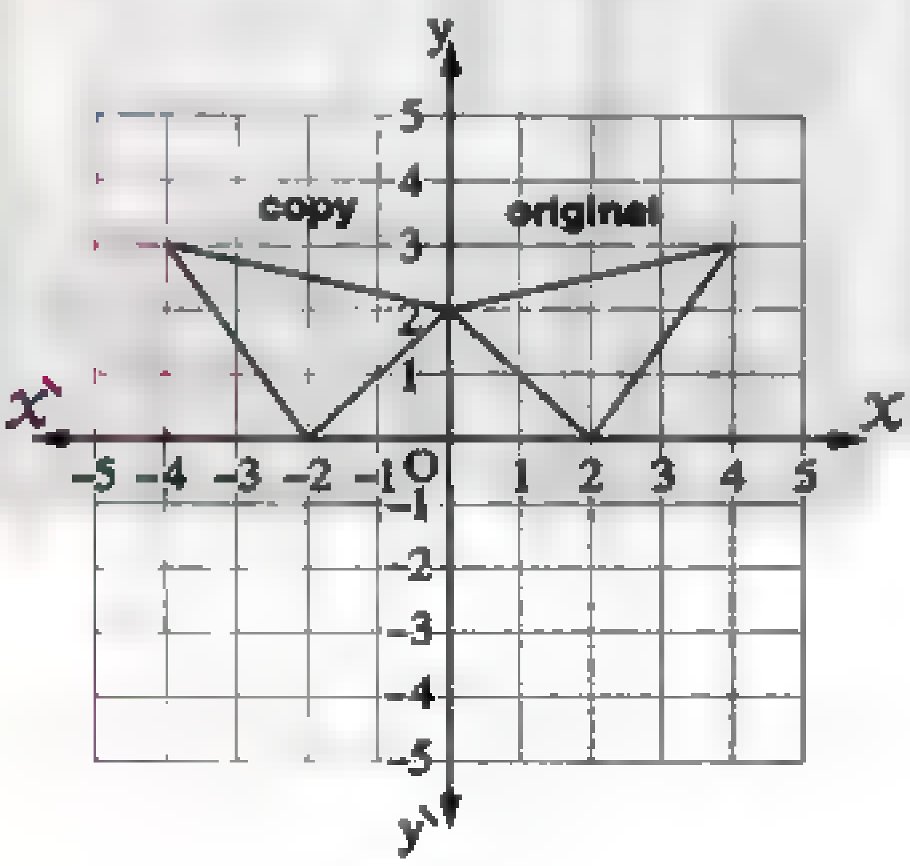


fig. (3)

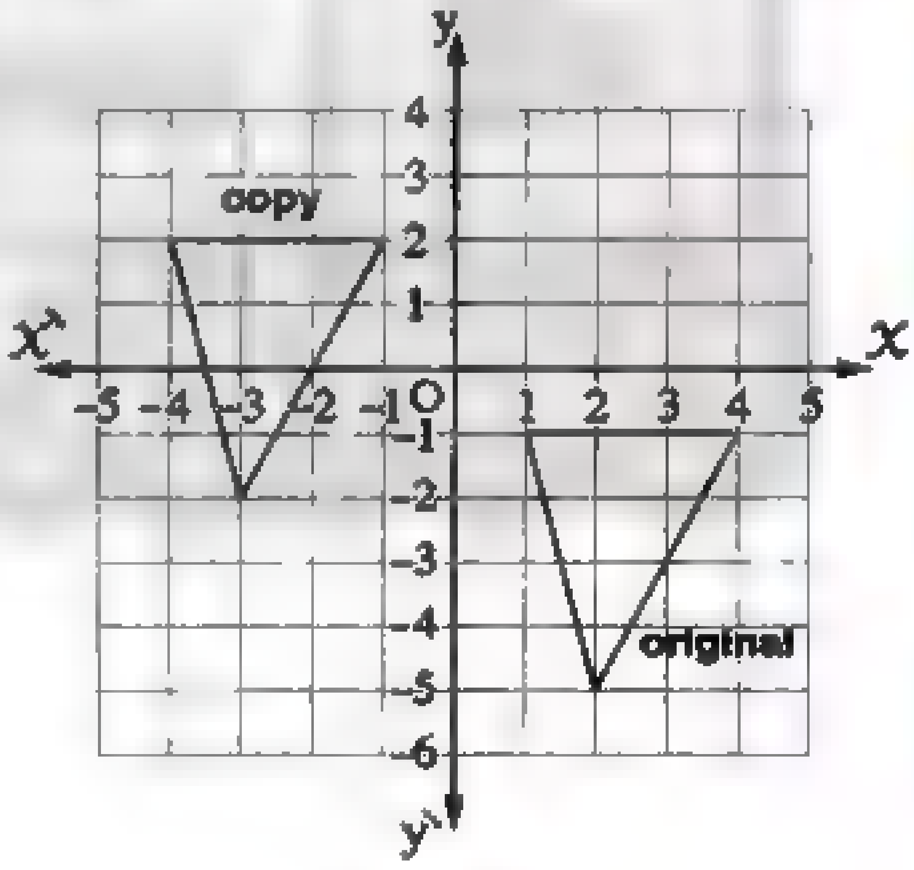


fig. (4)

Unit 3

From the school book

Exercise 12 On rotation

First Problems on rotation in the plane :

1 Draw the image of the point A , $\overline{AB}$ and Δ ABC by the required rotation :

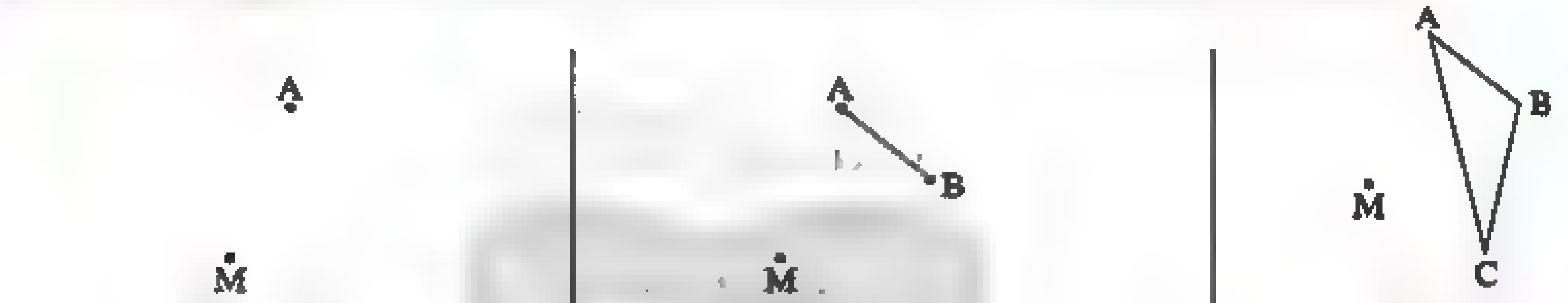


Fig. (1) R (M , 30°)

Fig. (2) R (M , – 60°)

Fig. (3) R (M , 60°)

2 Use the geometric tools to draw $\overline{AB}$ with length 3 cm. , then draw its image by rotation R (B , 135°)

3 Draw the equilateral triangle ABC with side length 6 cm. Draw the image of the triangle ABC by rotation R (A , 60°)

4 📖 Draw the triangle ABC in which AB = 5 cm. , BC = 6 cm. and CA = 7 cm. , then draw the image of Δ ABC by rotation

(1) R (A , 180°) (2) R (A , 360°)

5 Draw the triangle XYZ in which XY = XZ = 3 cm. and YZ = 4 cm. , then draw the image of Δ XYZ in each of the two cases :

(1) By rotation about X with an angle of measure 90°

(2) 📖 By rotation about X with an angle of measure (270°)

6 📖 Draw Δ ABC in which AB = 5 cm. , AC = 3 cm. , m ($\angle$ A) = 40° , then draw $\grave{C}$ the image of C by rotation R (A , 40°) , $\grave{B}$ the image of B by rotation R (A , – 40°)

7 Draw the square ABCD with side length 5 cm. Draw the image of the square ABCD :

(1) By rotation R (B , 90°)

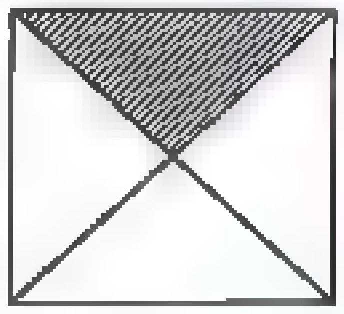
(2) By rotation R (A , 180°)

8 Using the geometric tools , draw the square ABCD with side length 4 cm. , then draw its image by rotation about its centre (The point of diagonals intersection) with an angle of measure 90°

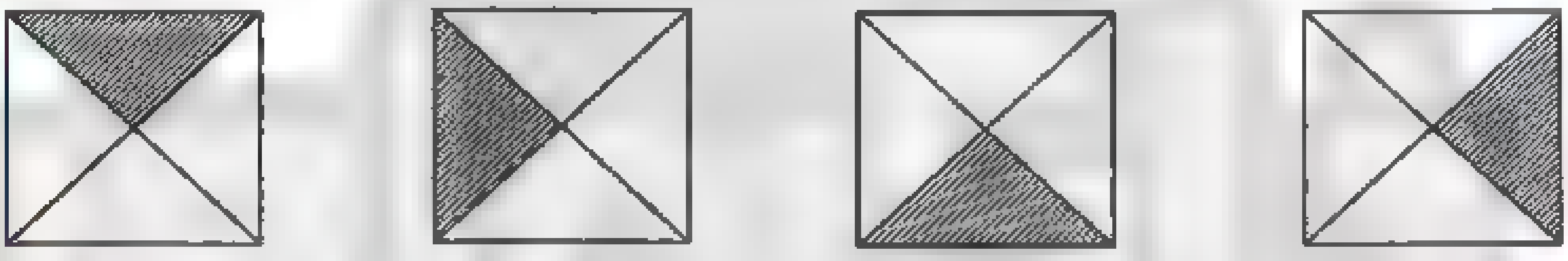
9 Draw the rectangle ABCD in which BC = 6 cm., AB = 4 cm. Draw the image of the rectangle ABCD :

1. By rotation R (A , 90°)
2. By rotation R (M , 180°) where M is the point of intersection of its diagonals.

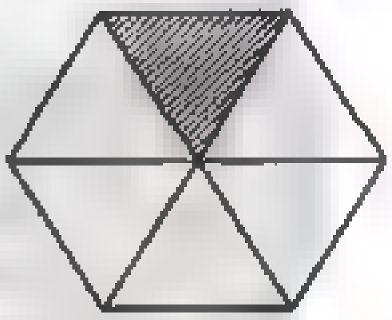
10 **Choose the correct answer from those given :**



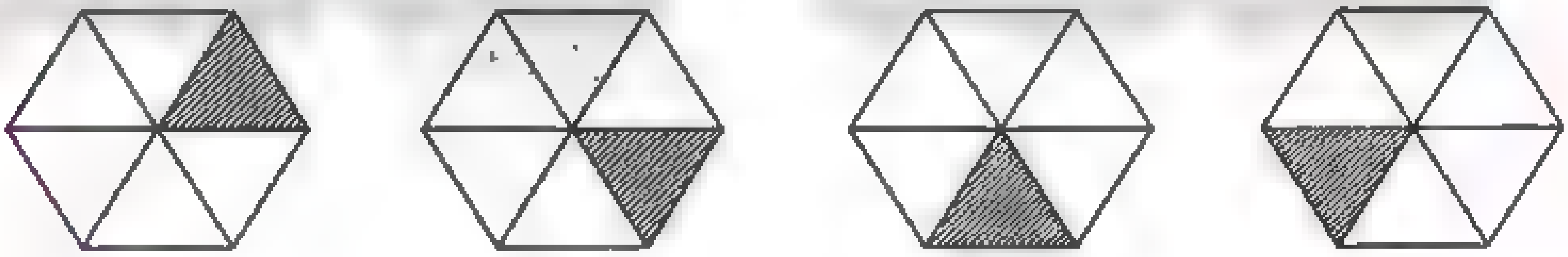
1. Which of these figures represents the rotation of the opposite square about its centre with an angle of measure 270° ?



(a) (b) (c) (d)



2. Which of these figures represents the rotation of the opposite regular hexagon about its centre with an angle of measure (– 120°) ?



(a) (b) (c) (d)

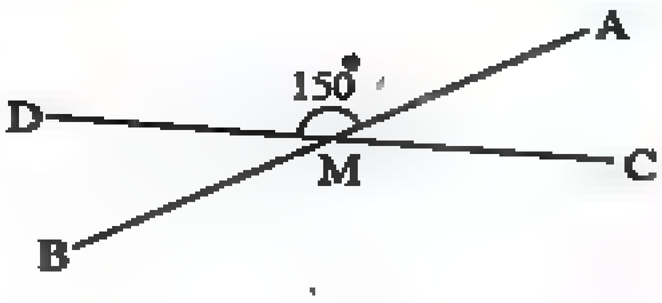
3. **In the opposite figure :**

If B is the midpoint of $\overline{AC}$, then the image of $\overline{AC}$ by rotation about B with an angle of 180° is ....

(a) $\overline{AC}$ (b) $\overline{AB}$ (c) $\overline{CA}$ (d) $\overline{CB}$

- In the opposite figure :

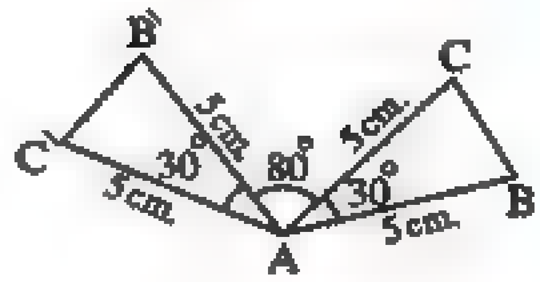
$\overline{CD}$ is the image of $\overline{AB}$ under a rotation about M and the measure of its angle is ....



(a) 75° (b) 30° (c) – 30° (d) – 150°

- In the opposite figure :

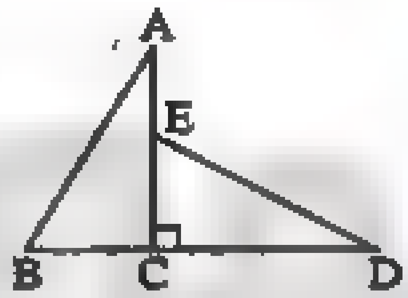
$\Delta A\grave{B}\grave{C}$ is the image of ΔABC by a rotation about A with an angle of measure ....



(a) – 110° (b) 80° (c) 110° (d) 140°

- In the opposite figure :

ΔABC is the image of ΔDEC which is right-angled at C by rotation about C with an angle of measure ....



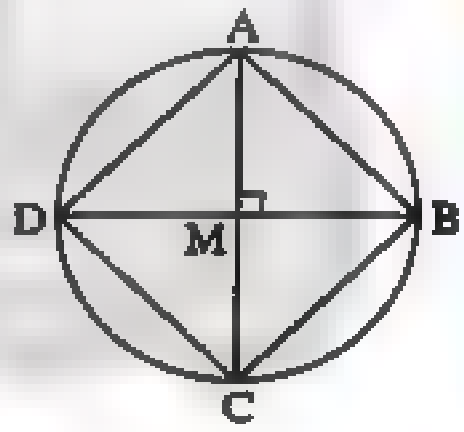
(a) 90° (b) – 90° (c) 180° (d) 360°

---

11 In the opposite figure :

The radius length of circle M is 3 cm. ،

$\overline{AC}$ and $\overline{BD}$ are two perpendicular diameters in it.



Complete :

- By the rotation R (M ، 90°) ، then the image of the point A is ....

and the image of the point B is ....

∴ The image of $\overline{AB}$ is .... and the image of $\overrightarrow{AB}$ is ....

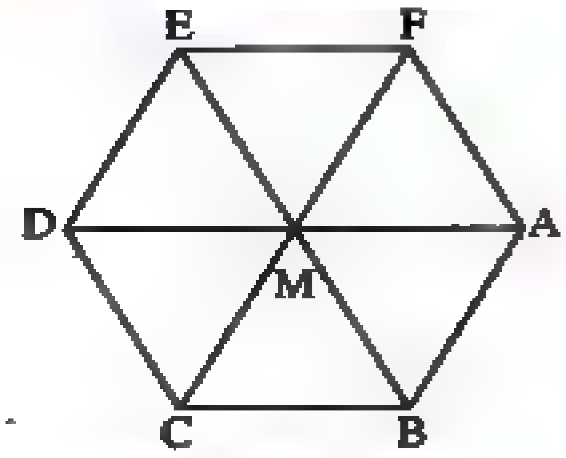
- By rotation R (M ، – 90°) ، the image of $\overline{AB}$ is .... and the image of $\overrightarrow{AB}$ is .... and the image of $\overleftrightarrow{AB}$ is ....

- By rotation R (M ، 180°) ، the image of the point A is .... ، the image of the point B is ....

∴ The image of $\overline{AB}$ is ....

- By rotation R (M ، – 180°) ، the image of $\overline{AB}$ is ....

12 **In the opposite figure :**



ABCDEF is a regular hexagon whose centre is M. Complete the following :

- The image of the point E by rotation about M with an angle of measure 120° is ·········
- The image of $\overline{AF}$ by rotation about M with an angle of measure 180° is ·········
- The image of $\overline{DE}$ by rotation about M with an angle of measure – 60° is ·········
- The image of Δ MCD by rotation about M with an angle of measure 300° is ·········
- Δ ABM is the image of Δ CDM by rotation about ········· with an angle of measure ·········
- Δ BMC is the image of ········· by rotation about M with an angle of measure (– 120°)

13 **Referring to the opposite figure , choose the correct answer from those given :**

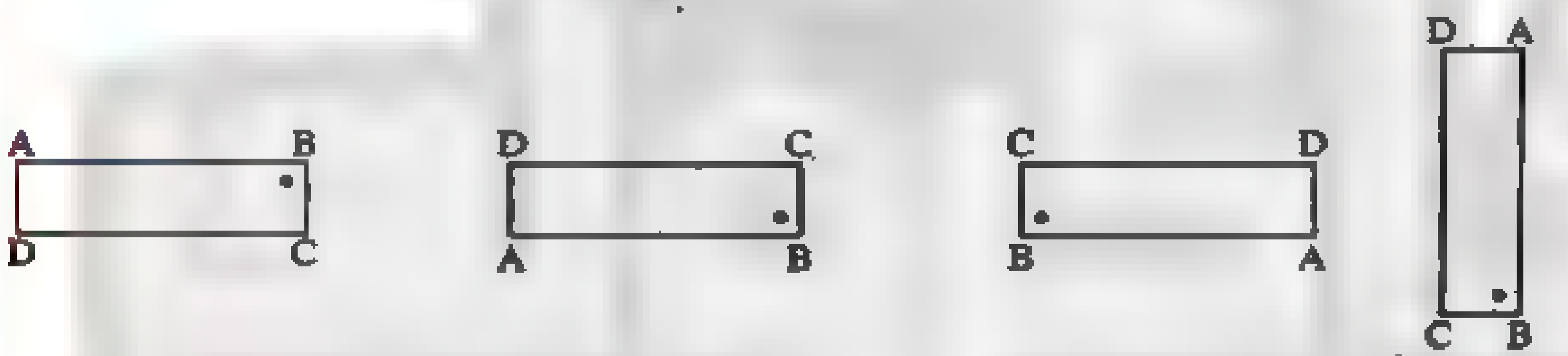


fig. (1) fig. (2) fig. (3) fig. (4)

- The image of the figure by reflection in $\overleftrightarrow{AD}$ is ·········

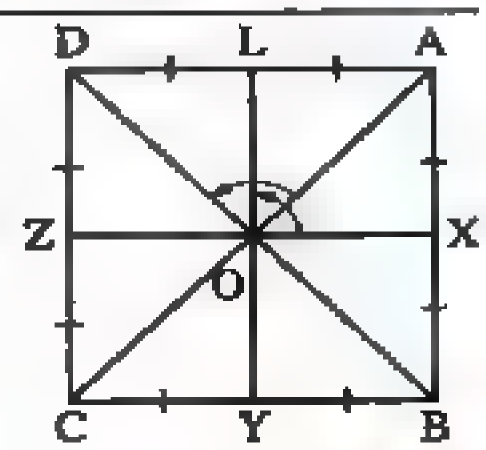
  (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)
- The image of the figure by rotation about A with an angle of measure 90° is ·········

  (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)
- The image of the figure by translation to the right is ·········

  (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)
- The image of the figure by rotation about A with an angle of measure 180° is ·········

  (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)

14 **In the opposite figure :**



ABCD is a square , O is the point of intersection of its diagonals X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Find :

(1) The image of Δ AXO by reflection in $\overleftrightarrow{AO}$ followed by another reflection in $\overleftrightarrow{LO}$

(2) The image of Δ AXO by rotation R (O , 90°)

---

15 ABC is a right-angled triangle with AB = 5 cm. and BC = 12 cm. **Find :**

(1) X as the image of B by translation 9 cm. in the direction of $\overrightarrow{BA}$

(2) Y as the image of B by rotation R (A , – 90°)

(3) The length of $\overline{XY}$

---

16 **In the opposite figure :**

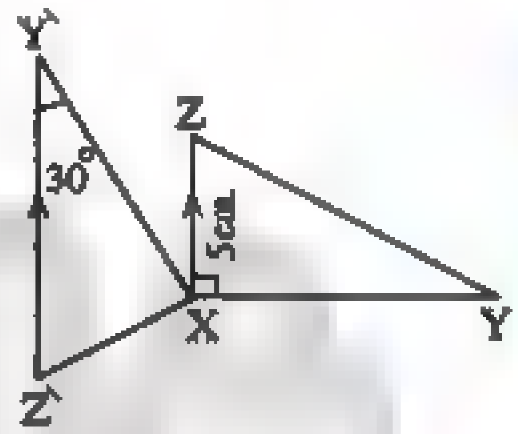
If the point X is the centre of rotation such that $\grave{Y}$ is the image of Y and $\grave{Z}$ is the image of Z

If $\overline{XZ}$ // $\overline{\grave{Y}Z}$, **find :**

(1) The measure of the angle of rotation.

(2) The length of $\overline{XZ}$

« 120° , 5cm. »

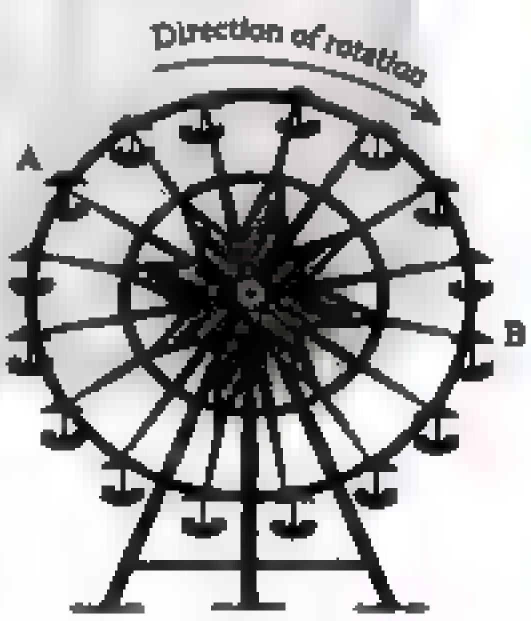


Life Applications

17 **In the opposite figure :**

The wheel is one of the rides in the funfair ,
if the wheel moves from position A to position B
What is the measure of the rotation angle in this case ?

« 157.5° »



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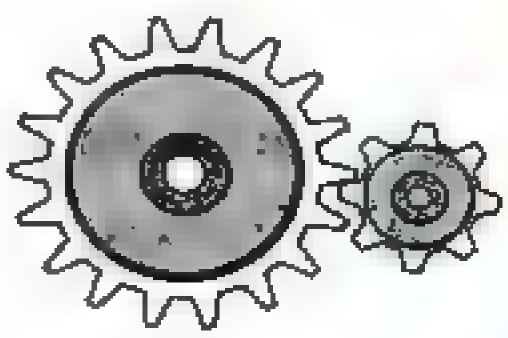
18 Gears are used to change the speed and direction of rotating parts of some machines
In the opposite figure , if the smaller gear makes one complete rotation clockwise direction , then what is the measure of the angle of rotation of the greater gear , and what is the direction of its rotation ?

« 160° »



Second Problem on rotation in the Cartesian plane :

1 Complete the following :

1. The image of the point $(2 , -3)$ by rotation about the origin point with an angle of measure $90°$ is .... and with an angle of measure $180°$ is ....
2. The image of the point $(-1 , 0)$ by rotation about the origin point with an angle of measure $90°$ is .... and with an angle of measure $360°$ is ....
3. The point $(3 , -2)$ is the image of the point $(2 , 3)$ by rotation about the origin point with an angle of measure ....
4. The image of the point .... by rotation about the origin point with an angle of measure $90°$ is $(-1 , 4)$
5. The image of the point .... by rotation about the origin point with an angle of measure $(-180°)$ is $(5 , -2)$
6. The image of the point $(-3 , 7)$ by rotation $90°$ about the origin point followed by reflection in y-axis is ....
7. The image of the point $(-2 , 0)$ by translation $(x , y) \longrightarrow (x + 3 , y - 1)$ followed by rotation about the origin point with an angle of measure $90°$ is ....
8. The rotation with an angle of measure $90°$ about the origin point maps the point $(x , -y)$ onto the point ....
9. The image of (a , b) is the same point by rotation about the origin point with an angle of measure ....
10. If the image of the point (x , y) by rotation about the origin point with an angle of measure $90°$ is (a , b) , then $a + y =$ ....

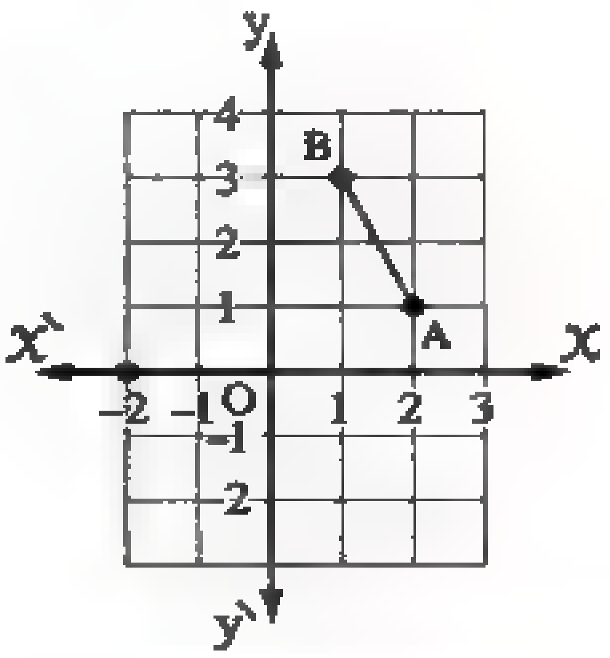
2 In the opposite figure :

The point A $(2 , 1)$ and B $(1 , 3)$

Find the image of $\overline{AB}$

by rotation about the origin point

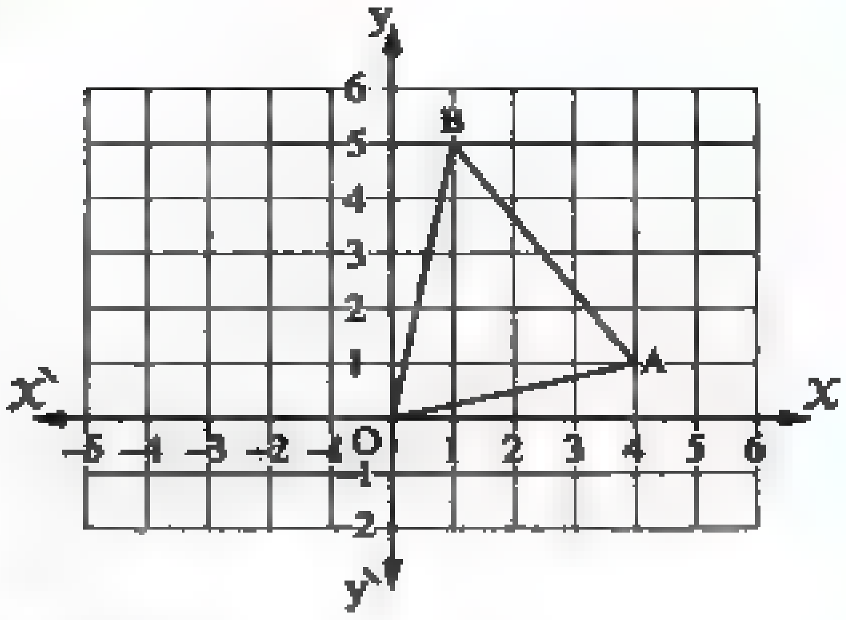
with an angle of measure $90°$



Unit 3

3 On the lattice , draw the image of Δ OAB by rotation about the origin with an angle of measure :

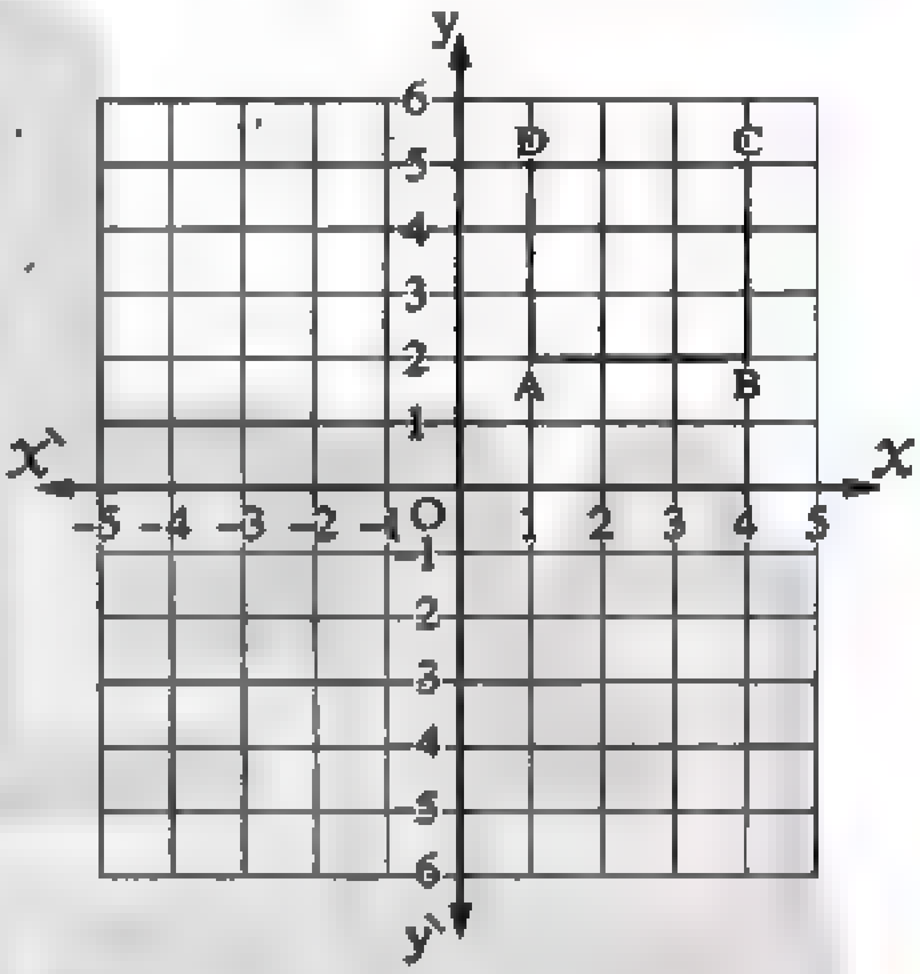
- 90°
- 180°



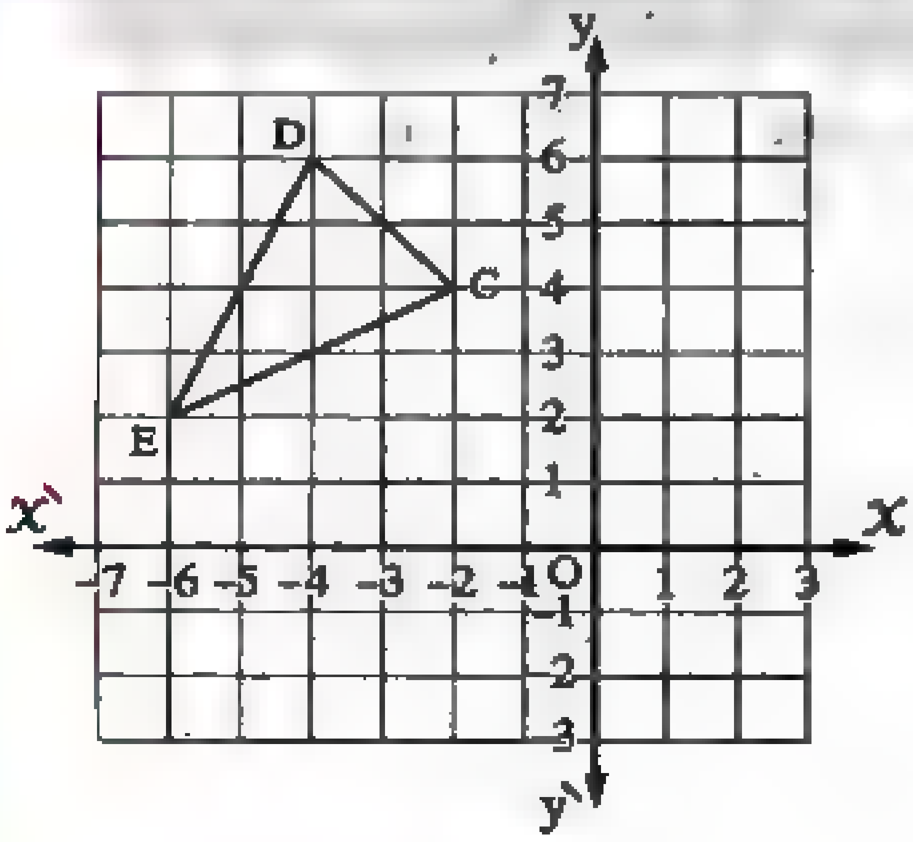
4 **In the opposite figure :**

Draw the image of the square ABCD by a rotation about the origin with an angle of measure :

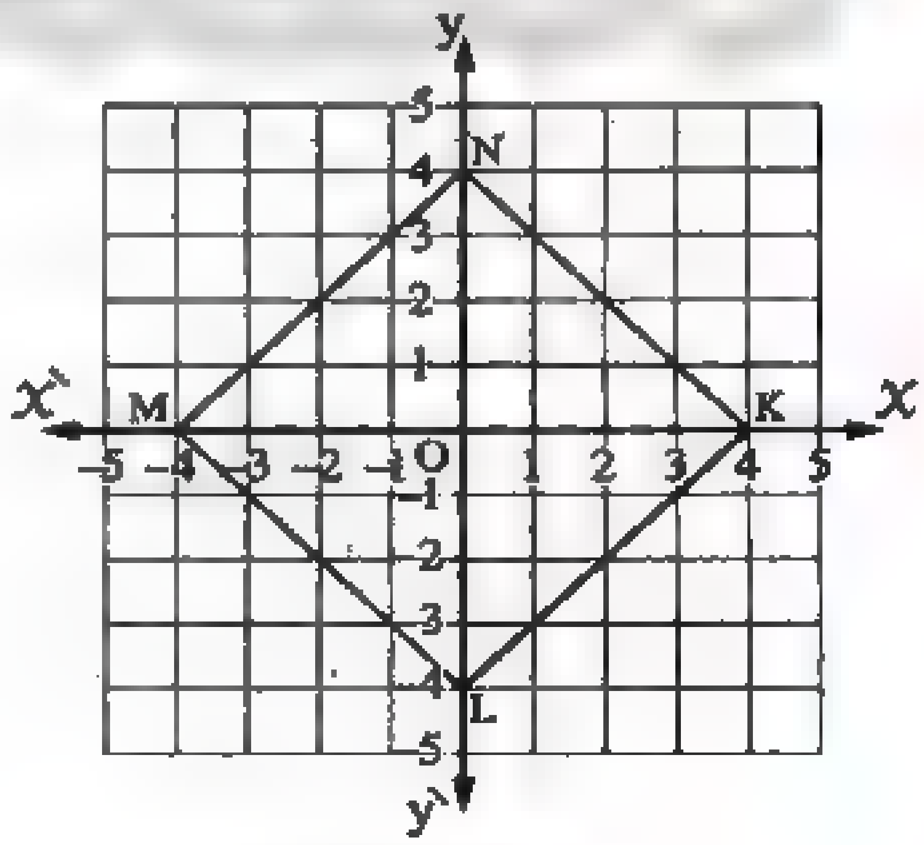
- 90°
- 180°



5 Copy each figure on a graph paper. Draw their images under the transformation indicated. Give the coordinates of the images vertices in each case.



Rotation of 90°
Clockwise about O



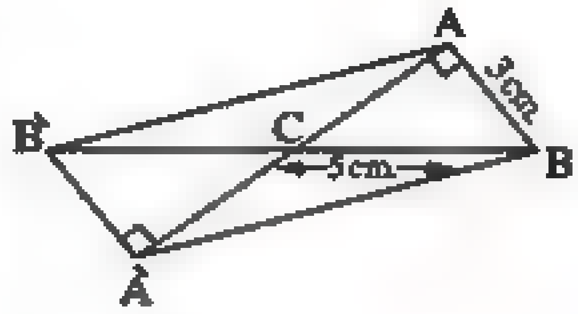
Rotation of 90°
anticlockwise about O

(c) Draw three images formed by rotating the rectangle about its centre through an angle of measure

① 90°   ② 180°   ③ 270°

For excellent pupils

14 **In the opposite figure :**



ABC is a right-angled triangle at A

AB = 3 cm. and BC = 5 cm.

If : Δ CÀB̀ is the image of Δ CAB

by rotation about C and with an angle of measure 180°

Find the area of Δ AÀB̀ « 12 cm^2 »

General Exercises on Unit Three "Geometry and Measurement"

First : Completion questions :

1 Complete the following :

1. If two straight lines intersect , then each two vertically opposite angles are ……….
2. The sum of the measures of the accumulative angles at a point equals ……….°
3. The sum of measures of the interior angles of the heptagon = ……….°
4. The measure of the interior angle of the regular octagon = ……….°
5. The number of diagonals of the pentagon = ……….
6. The quadrilateral in which two sides are parallel is called ……….
7. In a parallelogram , every two opposite sides are ………. , ……….
8. ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = ……….°
9. The parallelogram whose two diagonals are equal in length is called ……….
10. The parallelogram whose two diagonals are equal in length and perpendicular is called ……….
11. The quadrilateral whose sides are equal in length is called ……….
12. The rectangle is a ………. with a right angle.
13. The square is a ………. with a right angle.
14. The square is a ………. in which its diagonals are perpendicular.
15. If XYZL is a rhombus , then ………. ⊥ ……….
16. The rhombus whose perimeter is 82 cm. , its side length = ………. cm.
17. The sum of measures of the interior angles of a triangle = ………. °
18. The measure of the exterior angle of a triangle is equal to the sum of ……….
19. In Δ ABC : If m (∠ A) + m (∠ C) = m (∠ B) , then m (∠ B) = ………. °
20. The ray drawn from the midpoint of a side of a triangle parallel to another side ……….
21. The length of the line segment joining the midpoints of two sides of a triangle equals ……….
22. In the right-angled triangle , the area of the square on the hypotenuse equals ……….
23. A rhombus whose diagonals lengths are 16 cm. and 12 cm. , then its side length = ………. cm.

2 Complete the following :

1. The image of the point (– 3 , 2) by reflection in y-axis is ……….
2. The reflection in a plane reserves : ………. , ………. , ………. , ……….

- The measure of the exterior angle of the equilateral triangle at any one of its vertices equals ....
  (a) 60° (b) 120° (c) 150° (d) 30°
- In Δ ABC , if D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , BC = 8 cm. , then DE = ... cm.
  (a) 16 (b) 8 (c) 4 (d) 2
- A rectangle is of length 4 cm. and width 3 cm. , then the length of its diagonal = ... cm.
  (a) 25 (b) 3 (c) 4 (d) 5
- In Δ ABC , if m ($\angle$ A) = 90° , BC = 25 cm. and AC = 20 cm. , then AB = ... cm.
  (a) 20 (b) 25 (c) 10 (d) 15

2 Choose the correct answer from those given :

- The image of the point (– 3 , – 5) by reflection in the X-axis is ....
  (a) (3 , – 5) (b) (3 , 5) (c) (– 3 , 5) (d) (– 3 , – 5)
- The number of axes of symmetry of the equilateral triangle is ....
  (a) zero (b) 1 (c) 2 (d) 3
- The image of the point (2 , – 7) by reflection in the origin point is ....
  (a) (2 , 7) (b) (– 2 , 7) (c) (– 2 , – 7) (d) (2 , – 7)
- If $\grave{A}$ is the image of A by reflection in M and MA = 6 cm. , then A$\grave{A}$ = ...
  (a) 6 cm. (b) 3 cm. (c) 12 cm. (d) 9 cm.
- The image of the point (– 2 , 3) by translation of magnitude of 4 units in the negative direction of the y-axis is ....
  (a) (2 , 3) (b) (– 2 , 7) (c) (– 6 , 3) (d) (– 2 , – 1)
- If $\grave{A}$ (4 , – 5) is the image of A by translation (X , y) $\longrightarrow$ (X – 2 , y + 1) , then the point A is ....
  (a) (6 , – 4) (b) (4 , – 4) (c) (2 , – 4) (d) (6 , – 6)
- The image of the point (– 4 , 2) by rotation around the origin point with an angle of measure 90° is ....
  (a) (– 2 , 4) (b) (– 2 , – 4) (c) (4 , – 2) (d) (2 , – 4)
- The image of the square by rotation around the origin point with an angle of measure – 90° is ....
  (a) a rectangle (b) a square (c) a rhombus (d) a trapezium
- The image of the triangle by rotation around the origin point with an angle of measure 180° is ....
  (a) a triangle (b) a line segment (c) a point (d) a straight line

(10) If the image of the point (5 , – 2) by rotation around the origin point is the same point , then the measure of the rotation angle = ...

(a) 90° (b) 180° (c) 270° (d) 360°

Third : Essay questions

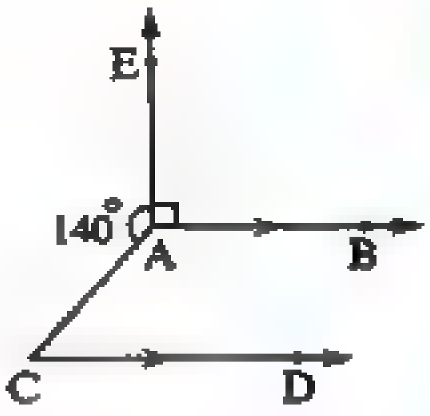
1 **In the opposite figure :**

$\overrightarrow{AB} // \overrightarrow{CD}$, m (∠ EAC) = 140°

and m (∠ EAB) = 90°

Find :

(1) m (∠ BAC) (2) m (∠ C)



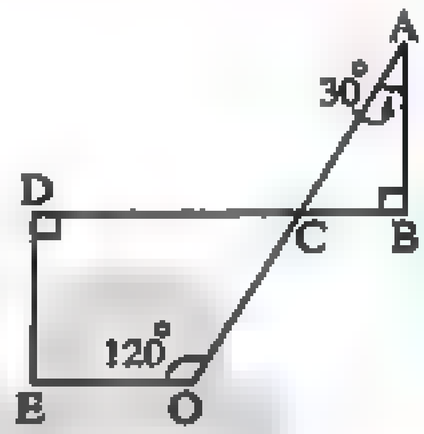
2 **In the opposite figure :**

$\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{DB}$

, $\overline{BD} \cap \overline{AO} = \{C\}$, m (∠ A) = 30°

and m (∠ EOC) = 120°

Find : m (∠ E)

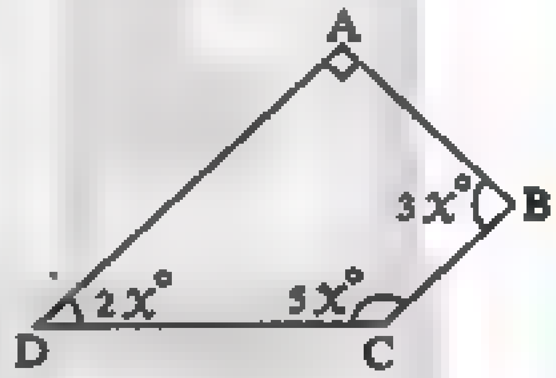


3 **In the opposite figure :**

ABCD is a quadrilateral

in which m (∠ A) = 90°

Find the value of : x



4 If the measure of the exterior angle of a regular polygon is 45° , how many sides does it have ? What is the sum of the measures of its interior angles ?

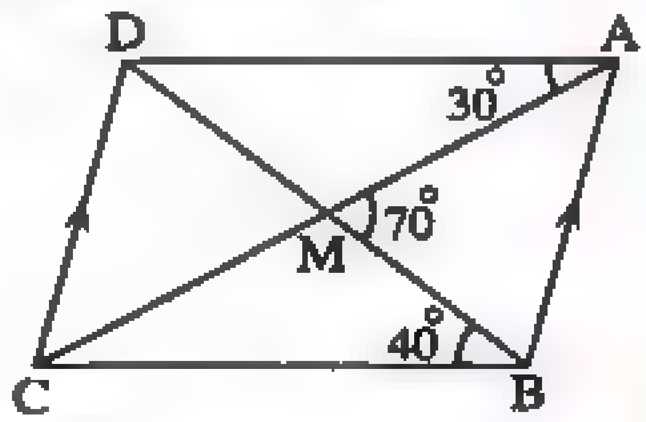
5 **In the opposite figure :**

ABCD is a quadrilateral where $\overline{AC} \cap \overline{BD} = \{M\}$,

$\overline{AB} // \overline{DC}$, m (∠ AMB) = 70° , m (∠ MBC) = 40°

and m (∠ MAD) = 30°

Prove that : ABCD is a parallelogram.



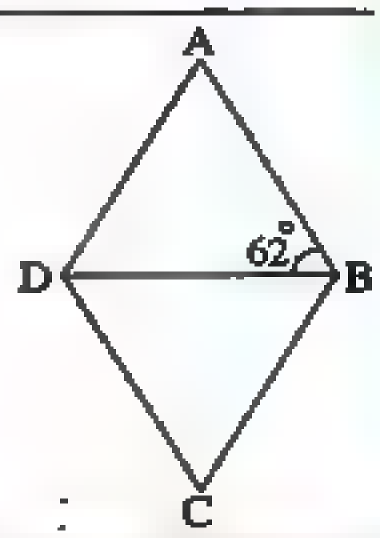
6 **In the opposite figure :**

ABCD is a rhombus ,

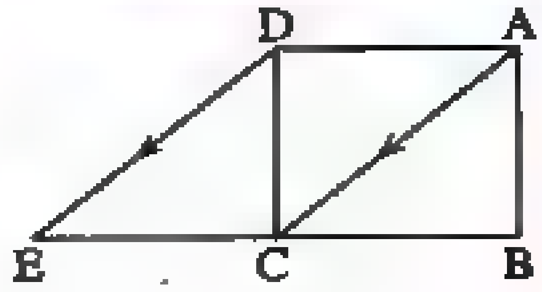
$\overline{BD}$ is a diagonal in it ,

m (∠ ABD) = 62° .

Find with proof : m (∠ A)



7 **In the opposite figure :**

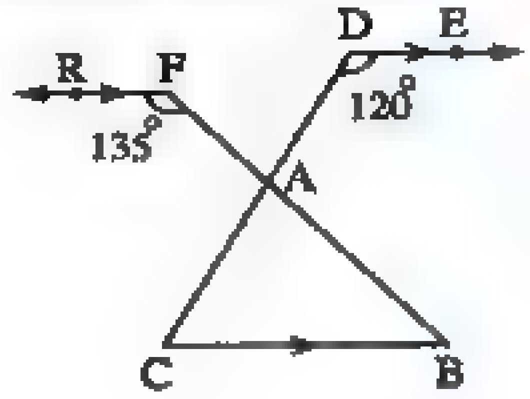


ABCD is a square, $E \in \overrightarrow{BC}$, $\overline{AC} \parallel \overline{DE}$

(1) Prove that : ACED is a parallelogram.

(2) Find : m (∠ ACE)

8 **In the opposite figure :**

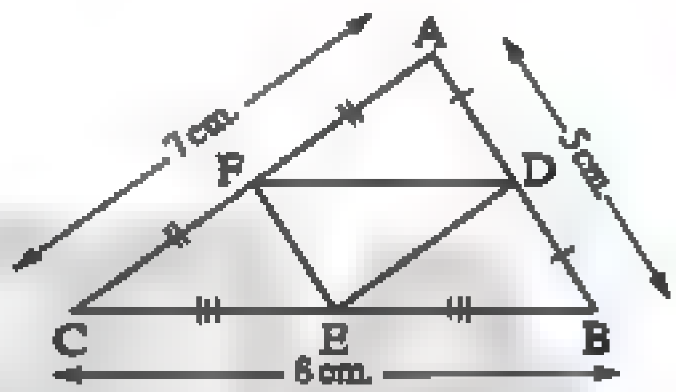


$\overrightarrow{DE} \parallel \overrightarrow{FR} \parallel \overrightarrow{BC}$,

m (∠ CDE) = 120° and m (∠ RFB) = 135°

Calculate the measures of the angles of : Δ ABC

9 **In the opposite figure :**



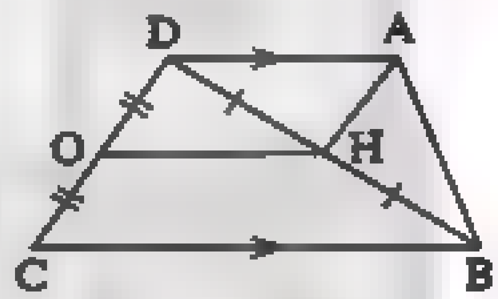
AB = 5 cm., BC = 8 cm.,

AC = 7 cm., D , E and F are the midpoints of

$\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

Calculate the perimeter of : Δ DEF

10 **In the opposite figure :**



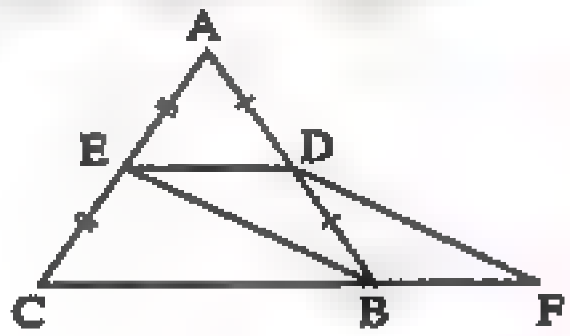
$\overline{AD} \parallel \overline{BC}$, AD = $\frac{1}{2}$ BC ,

H is the midpoint of $\overline{BD}$,

O is the midpoint of $\overline{CD}$.

Prove that : AHOD is a parallelogram.

11 **In the opposite figure :**

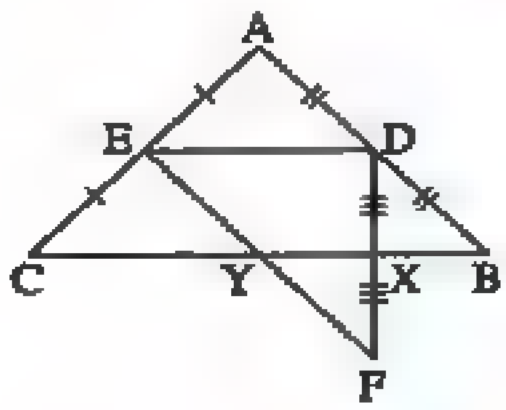


D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively,

$F \in \overrightarrow{CB}$ where BF = $\frac{1}{2}$ BC

Prove that : BEDF is a parallelogram.

12 **In the opposite figure :**



D is the midpoint of $\overline{AB}$,

E is the midpoint of $\overline{AC}$,

$\overline{DF} \cap \overline{BC} = \{X\}$, DX = XF ,

BC = 12 cm.

Find the length of : $\overline{XY}$

Unit 3

13 **In the opposite figure :**

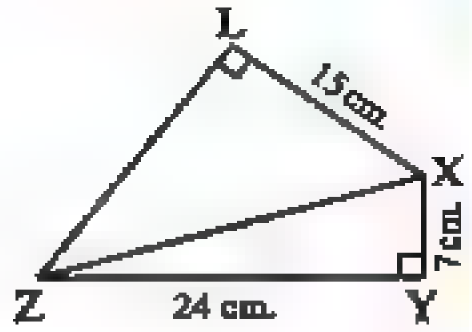
XYZL is quadrilateral in which :

$m(\angle XYZ) = m(\angle XLZ) = 90°$

, $XY = 7$ cm. , $YZ = 24$ cm.

and $XL = 15$ cm.

Find the length of each of : $\overline{XZ}$ and $\overline{LZ}$



---

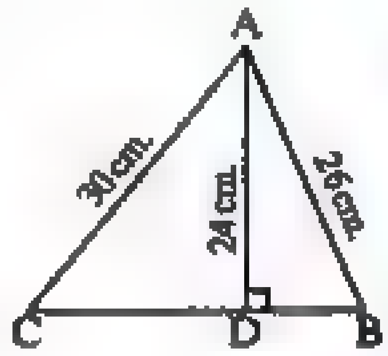
14 **In the opposite figure :**

ABC is a triangle in which : $\overline{AD} \perp \overline{BC}$

If $AD = 24$ cm. , $AB = 26$ cm.

and $AC = 30$ cm.

- **Find :** BC
- **Calculate the area of :** ΔABC



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15 Draw the image of ΔABC in which : $AB = 3$ cm. , $BC = 4$ cm. and $AC = 5$ cm. by reflection in the straight line containing the greatest side.

---

16 Draw the image of ΔABC where $A(0, 0)$, $B(4, 1)$ and $C(-1, 3)$ by reflection in the y-axis.

---

17 Draw the triangle ABC in which : $A(1, -1)$, $B(2, 3)$ and $C(0, 4)$, then find its image by reflection in the origin point.

---

18 ΔABC is right-angled at B where $AB = 3$ cm. and $BC = 4$ cm. If $\Delta \grave{A}\grave{B}\grave{C}$ is the image of ΔABC by translation of a distance 3 cm. in the direction of $\overrightarrow{CB}$

Prove that : The figure $A\grave{A}\grave{C}C$ is a parallelogram.

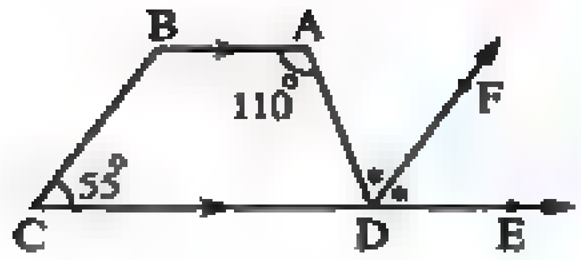
---

19 Draw ΔABC in which : $A(1, 1)$, $B(4, -2)$ and $C(6, 3)$, then find its image by rotation $R(O, -90°)$.

Worksheet 1 on lesson 1 unit 3

Answer the following questions :

1 **In the opposite figure :**



$\overline{AB} \parallel \overrightarrow{CD}$, $E \in \overrightarrow{CD}$, $m(\angle A) = 110°$

$m(\angle C) = 55°$,

$\overrightarrow{DF}$ bisects $\angle ADE$

Complete the following proof to prove that $\overrightarrow{DF} \parallel \overline{BC}$

Proof :

$\because$ $\overline{AB} \parallel$ ...①... , $\overleftrightarrow{AD}$ is a transversal to them

$\therefore$ $m(\angle EDA) = m(\angle$ ...②...$) =$ ...③...° (they are ...④...)

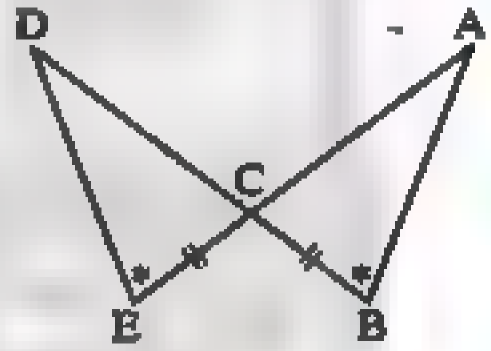
$\because$ $\overrightarrow{DF}$ bisects $\angle$ ...⑤...

$\therefore$ $m(\angle EDF) = m(\angle$ ...⑥...$) =$ ...⑦... °

$\therefore$ $m(\angle$ ...⑧...$) = m(\angle C)$ but they are ...⑨...

$\therefore$ $\overrightarrow{DF} \parallel$ ...⑩... (Q.E.D)

2 **In the opposite figure :**



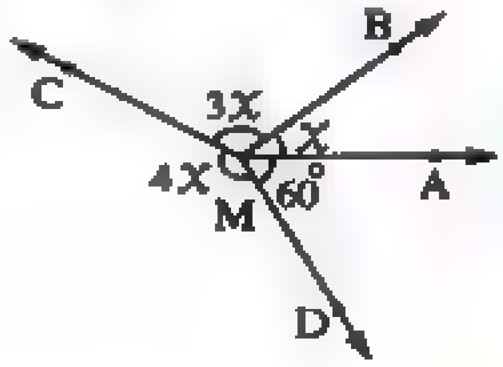
$\overline{AE} \cap \overline{BD} = \{C\}$, $BC = CE$

$m(\angle B) = m(\angle E)$ **prove that :**

(1) $AB = DE$

(2) $AE = BD$

3 **In the opposite figure :**



Find by proof the value of X

The answer of worksheet 1

Total mark 15

1 (1) .... (2) ....

(3) .... (4) ....

(5) .... (6) ....

(7) .... (8) ....

(9) .... (10) ....

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Worksheet 2 till lesson 2 unit 3

Answer the following questions :

1 Choose the correct answer from those given :

(1) The sum of measures of the interior angles of a polygon with n sides = ...

(a) $\frac{(n-2) \times 180°}{n}$ (b) $n \times 180°$ (c) $(n-2) \times 180°$ (d) $\frac{(n-2) \times 180°}{2n}$

(2) The measure of the interior angle of the regular octagon equals ....

(a) 1080° (b) 180° (c) 135° (d) 108°

(3) The sum of measures of the accumulative adjacent angles at a point = ...

(a) 180° (b) 90° (c) 360° (d) 270°

(4) The sum of measures of the exterior angles of a triangle = ...

(a) 180° (b) 360° (c) 90° (d) 100°

(5) If the measure of an interior angle of a regular polygon = 120° then the number of its sides = ...

(a) 3 (b) 4 (c) 5 (d) 6

(6) The two vertically opposite angles are ....

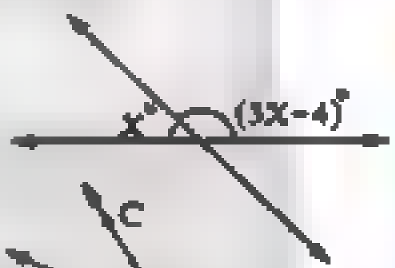
(a) complementary. (b) supplementary. (c) adjacent. (d) equal in measure.

2 Complete the following :

(1) The sum of measures of the interior angles of heptagon = ...

(2) In the opposite figure :

X = ...°

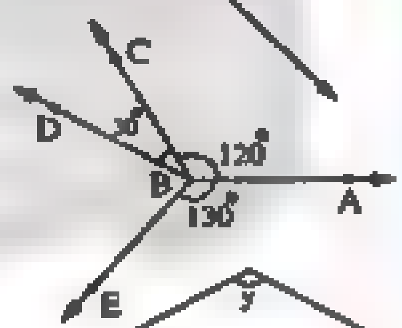


(3) In the opposite figure :

m (∠ ABC) = 120° , m (∠ CBD) = 30°

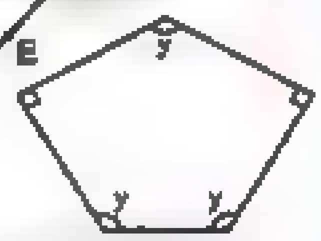
m (∠ ABE) = 130° ,

then m (∠ EBD) = ... °



(4) In the opposite figure :

y = ... °

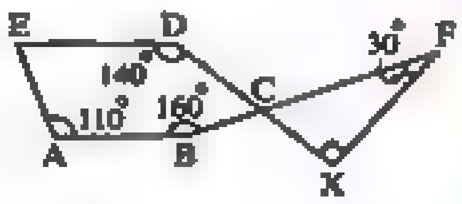


3 In the opposite figure :

$\overline{BF} \cap \overline{DX} = \{C\}$, $\overline{FX} \perp \overline{DX}$, m (∠ F) = 30°

, m (∠ B) = 160° , m (∠ D) = 140° , m (∠ A) = 110°

Prove that : $\overline{AB} \,//\, \overline{ED}$

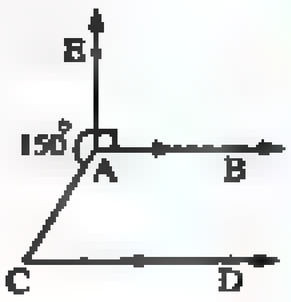


4 In the opposite figure :

$\overrightarrow{AB} \,//\, \overrightarrow{CD}$, m (∠ EAC) = 150°

$\overrightarrow{AB} \perp \overrightarrow{AE}$ **Find :**

(1) m (∠ BAC) (2) m (∠ C)



The answer of worksheet 2

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

(2) ⓐ ⓑ ⓒ ⓓ

(3) ⓐ ⓑ ⓒ ⓓ

(4) ⓐ ⓑ ⓒ ⓓ

(5) ⓐ ⓑ ⓒ ⓓ

(6) ⓐ ⓑ ⓒ ⓓ

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2 (1) ....

(2) ....

(3) ....

(4) ....

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Worksheet 3 till lesson 3 unit 3

Answer the following questions :

1 Choose the correct answer from those given :

(1) ABCD is a parallelogram in which : m (∠ A) = 120° , then m (∠ B) = ...

(a) 120° (b) 180° (c) 60° (d) 100°

(2) The measure of the interior angle of the regular hexagon = ...

(a) 108° (b) 120° (c) 135° (d) 90°

(3) The sum of the measures of the exterior angles of a polygon with n sides = ...

(a) 90° (b) 180° (c) $(n - 2) \times 180°$ (d) 360°

(4) If ABCD is a parallelogram whose perimeter = 24 cm. and AB = 8 cm. , then BC = ... cm.

(a) 8 (b) 4 (c) 6 (d) 10

(5) The sum of the measures of four accumulative angles at a point .... the sum of the measures of five accumulative angles at a point.

(a) = (b) < (c) > (d) ≠

(6) The sum of the measures of the interior angles of the quadrilateral equals the measure of ....

(a) two right angles (b) three right angles

(c) four right angles (d) right angle

2 Complete the following :

(1) If two straight lines intersect , then each two vertically opposite angles are ....

(2) If ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 110° , then m (∠ D) = ... °

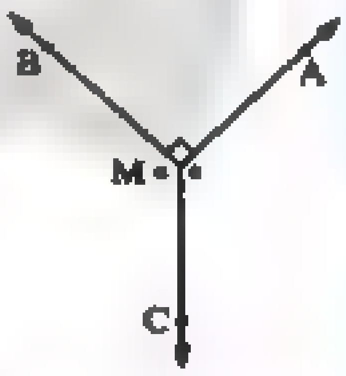
(3) The quadrilateral in which there are two parallel sides is called ....

(4) **In the opposite figure :**

$\overrightarrow{MB} \perp \overrightarrow{MA}$,

$\overrightarrow{MC}$ bisects ∠ AMB (reflex angle)

, then m (∠ AMC) = ... °

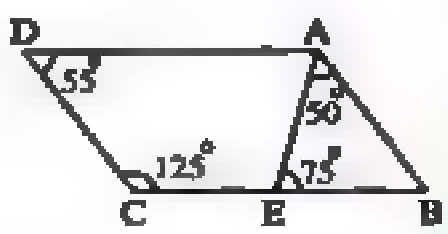


3 In the opposite figure :

E ∈ $\overline{BC}$, m (∠ BAE) = 50°

m (∠ AEB) = 75° , m (∠ D) = 55° , m (∠ C) = 125°

Prove that : the figure ABCD is a parallelogram.

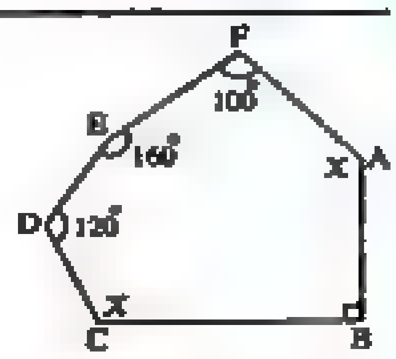


4 In the opposite figure :

ABCDEF is a hexagon and

m (∠ A) = m (∠ C) = x

Find the value of x



The answer of worksheet 3

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

(2) ⓐ ⓑ ⓒ ⓓ

(3) ⓐ ⓑ ⓒ ⓓ

(4) ⓐ ⓑ ⓒ ⓓ

(5) ⓐ ⓑ ⓒ ⓓ

(6) ⓐ ⓑ ⓒ ⓓ

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2 (1) ....°

(2) ....°

(3) ....°

(4) ....°

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Worksheet 4 till lesson 4 unit 3

Answer the following questions :

1 Choose the correct answer from those given :

(1) If ABCD is a parallelogram , m (∠ A) = 80° , then m (∠ C) = ...

(a) 80° (b) 120° (c) 100° (d) 60°

(2) A rhombus of side length 8 cm. , its perimeter = ...

(a) 16 cm. (b) 24 cm. (c) 32 cm. (d) 64 cm.

(3) If two adjacent sides in a rectangle are equal in length , then it will be ....

(a) a square. (b) a rhombus. (c) a rectangle. (d) a trapezium.

(4) If ABCD is a square , then m (∠ CAB) = ... °

(a) 30° (b) 45° (c) 60° (d) 90°

(5) .... is a parallelogram in which one of its angles is a right angle.

(a) The rectangle (b) The square (c) The rhombus (d) The trapezium

(6) The measure of the interior angle of a regular octagon = ...

(a) 120° (b) 108° (c) 135° (d) 140°

2 Complete the following :

(1) The quadrilateral in which the two diagonals bisect each other is called ....

(2) The two diagonals in .... and .... are equal in length.

(3) The sum of measures of the exterior angles of a pentagon = ... °

(4) The polygon which has an interior reflex angle is called ....

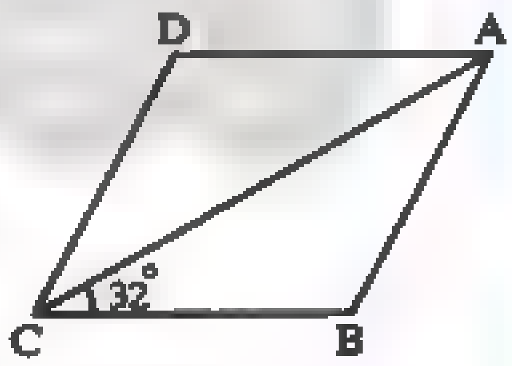
3 In the opposite figure :

ABCD is a rhombus ,

$\overline{AC}$ is a diagonal in it

m (∠ ACB) = 32°

Find : m (∠ D)



4 [a] If the measure of the exterior angle of a regular polygon equals 30° , what is the number of sides of this polygon and what is the sum of measures of its interior angles ?

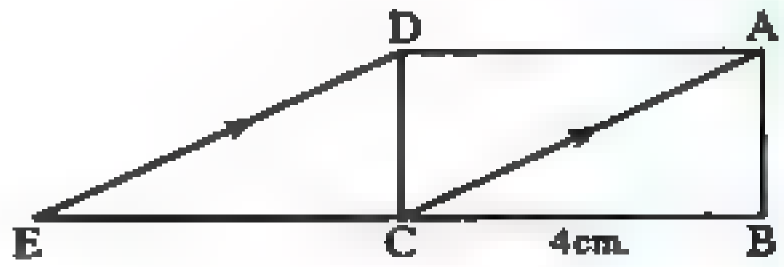
[b] In the opposite figure :

ABCD is a rectangle :

$\overline{AC} // \overline{DE}$, $E \in \overrightarrow{BC}$

(1) **Prove that :** ACED is a parallelogram

(2) **Find :** the length of $\overline{CE}$



The answer of worksheet 4

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) (a) (b) (c) (d)

(2) (a) (b) (c) (d)

(3) (a) (b) (c) (d)

(4) (a) (b) (c) (d)

(5) (a) (b) (c) (d)

(6) (a) (b) (c) (d)

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2 (1) ....

(2) ....

(3) ....

(4) ....

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4 [a] ....

....

....

[b] ....

....

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5

Worksheet 5 till lesson 5 unit 3

Answer the following questions :

1 Choose the correct answer from those given :

(1) The sum of measures of the interior angles of the triangle = ...
(a) 108° (b) 180° (c) 630° (d) 360°

(2) If the measures of two angles in a triangle are 40° and 45° , then the triangle is ....
(a) acute-angled (b) right-angled (c) obtuse-angled (d) equilateral

(3) In the parallelogram ABCD , if ∠ A is acute , then ∠ C is ....
(a) acute. (b) obtuse.
(c) right. (d) reflex.

(4) The parallelogram in which one of its angles is right is ....
(a) a trapezium. (b) a square. (c) a rhombus. (d) a rectangle.

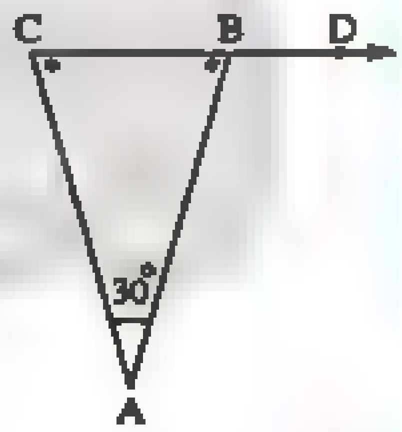
(5) The measure of the exterior angle of the equilateral triangle = ...
(a) 30° (b) 60° (c) 90° (d) 120°

(6) The number of sides of the regular polygon in which the measure of an interior angle = 120° is ....
(a) 5 (b) 6 (c) 7 (d) 8

2 Complete the following :

(1) The measure of the exterior angle of the triangle equals ....

(2) **In the opposite figure :**
$D \in \overrightarrow{CB}$, m (∠ A) = 30° ,
then m (∠ ABD) = ... °

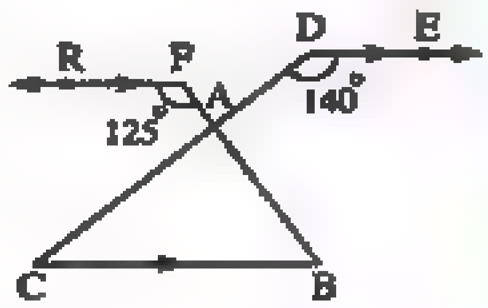


(3) In the parallelogram ABCD ,
if m (∠ A) = $\frac{1}{2}$ m (∠ B) , then m (∠ B) = ...

(4) The two diagonals of the rhombus are .... and ....

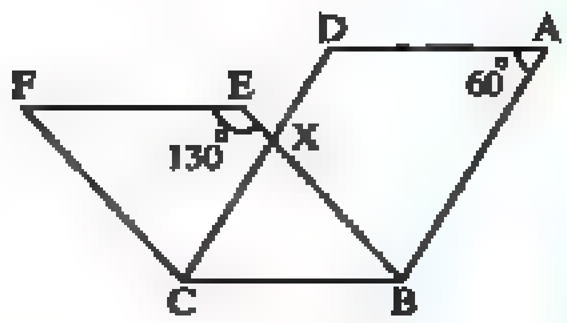
3 In the opposite figure :

$\overrightarrow{DE}$ // $\overrightarrow{FR}$ // $\overrightarrow{BC}$
m (∠ D) = 140° , m (∠ F) = 125°
Calculate the measures of the angles of Δ ABC



4 In the opposite figure :

ABCD and EBCF are two parallelograms , m (∠ A) = 60°
m (∠ E) = 130°
Find : by proof m (∠ BXC)



The answer of worksheet 5

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

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Worksheet 6 till lesson 6 unit 3

Answer the following questions :

1 Choose the correct answer from those given :

(1) The sum of measures of the interior angles of the heptagon = ………

(a) 540° (b) 720° (c) 900° (d) 180°

(2) The side length of a rhombus is 5 cm. , then its perimeter = ……… cm.

(a) 10 (b) 15 (c) 20 (d) 25

(3) The concave polygon should has a ……… angle.

(a) acute (b) right (c) obtuse (d) reflex

(4) ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 160° , then m (∠ B) = ………

(a) 20° (b) 80° (c) 100° (d) 110°

(5) The sum of measures of the interior angles of the triangle = the measure of ……… angle.

(a) a right. (b) a straight. (c) acute. (d) reflex.

(6) The length of the line segment joining the midpoints of two sides of a triangle equals ……… the length of the third side.

(a) $\frac{1}{4}$ (b) twice (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

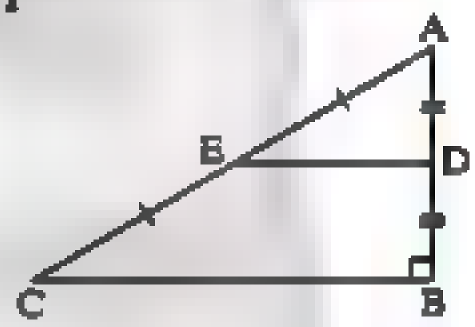
2 Complete the following :

(1) The measure of each interior angle in a regular polygon of n sides = ………

(2) The ray which is drawn from the midpoint of a side of a triangle parallel to one of the other sides ………

(3) **In the opposite figure :**

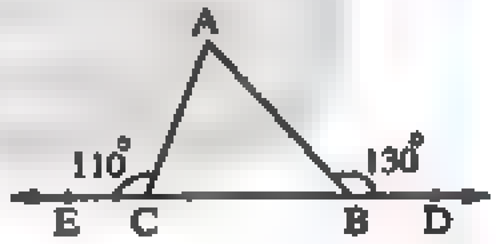
If m (∠ B) = 90° , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively then m (∠ ADE) = ……… °



(4) **In the opposite figure :**

If D ∈ $\overrightarrow{CB}$ and E ∈ $\overrightarrow{BC}$,

then m (∠ A) = ……… °



3 In the opposite figure :

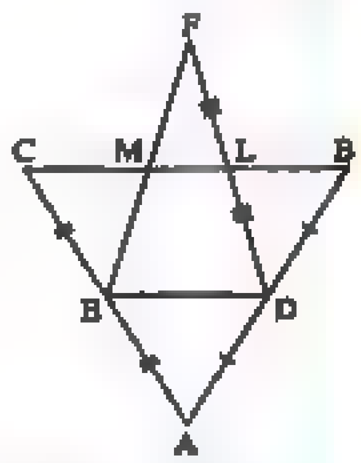
D is the midpoint of $\overline{AB}$,

E is the midpoint of $\overline{AC}$,

L is the midpoint of $\overline{FD}$

BC = 20 cm.

Find : the length of $\overline{LM}$

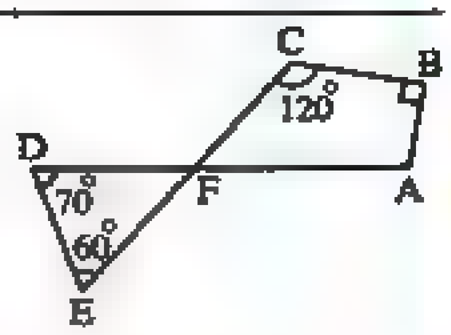


4 In the opposite figure :

$\overline{AD} \cap \overline{CE}$ = {F} , m (∠ B) = 90°

m (∠ C) = 120° , m (∠ E) = 60° , m (∠ D) = 70°

Find : m (∠ A)



The answer of worksheet 6

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

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(4) ⓐ ⓑ ⓒ ⓓ

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Worksheet 7 till lesson 7 unit 3

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) The measure of the interior angle of a regular hexagon = ……

(a) 120° (b) 102° (c) 180° (d) 360°

(2) In Δ ABC , if m ($\angle$ A) = m ($\angle$ B) = 50° , then m ($\angle$ C) = ……

(a) 130° (b) 100° (c) 80° (d) 50°

(3) The parallelogram whose one of its angles is aright angle is called ……

(a) trapezium. (b) square. (c) rhombus. (d) rectangle.

(4) A rectangle whose length = 4 cm. and width = 3 cm. , then the length of its diagonal = …… cm.

(a) 3 (b) 4 (c) 5 (d) 25

(5) The number of sides of the regular polygon which the measure of one of its interior angles is 144° = ……

(a) 4 sides. (b) 6 sides. (c) 8 sides. (d) 10 sides.

(6) In Δ ABC , if m ($\angle$ B) = m ($\angle$ A) + m ($\angle$ C) , then $\angle$ B is ……

(a) acute. (b) right. (c) obtuse. (d) reflex.

2 **Complete the following :**

(1) In the right-angled triangle , the area of the square on the hypotenuse equals ……

(2) The square is …… with a right angle.

(3) The line segment joining the midpoints of two sides of a triangle is …… the third side and its length equals ……

(4) If ABCD is a parallelogram in which m ($\angle$ A) = 40° , then m ($\angle$ B) = ……°

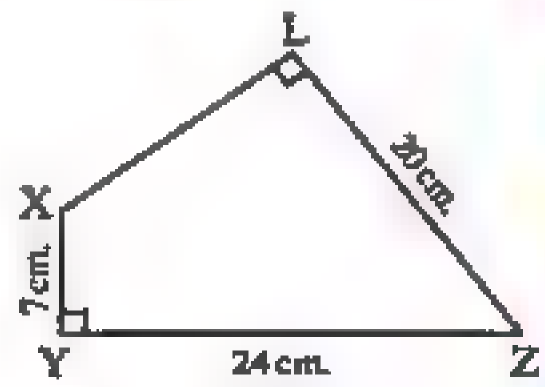
3 **In the opposite figure :**

m ($\angle$ Y) = m ($\angle$ L) = 90°

, XY = 7 cm. , YZ = 24 cm.

and L Z = 20 cm.

Find : the length of $\overline{XL}$



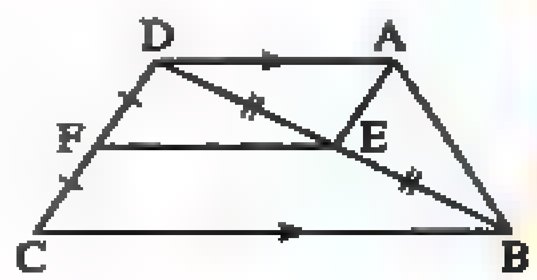
4 **In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, AD = $\frac{1}{2}$ BC

, E is the midpoint of $\overline{BD}$

and F is the midpoint of $\overline{CD}$

Prove that : AEFD is a parallelogram.



The answer of worksheet 7

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

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Worksheet 8 till lesson 8 unit 3

Answer the following questions :

1 Choose the correct answer from those given :

(1) The sum of measures of the interior angles of the hexagon = ...

(a) 4 right angles. (b) 5 right angles. (c) 6 right angles. (d) 8 right angles.

(2) If ABCD is a square , then $(AC)^2$ = ...

(a) AB (b) $(AB)^2$ (c) $2\,(AB)^2$ (d) $4\,(AB)^2$

(3) In Δ ABC , if $m(\angle B) = \frac{1}{2}\, m(\angle A) = 30°$, then the triangle will be .... triangle.

(a) acute-angled (b) right-angled (c) equilateral (d) isosceles

(4) The image of the point (1 , 2) by the transformation $(x , y) \longrightarrow (x , -y)$ is ....

(a) (– 1 , 2) (b) (– 1 , – 2) (c) (1 , – 2) (d) (2 , – 1)

(5) The sum of measures of the exterior angles of a polygon of n sides equals ....

(a) 180° (b) 360° (c) $(n - 2) \times 180°$ (d) $\frac{(n-2) \times 180}{n}$

(6) In Δ ABC , if D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively and BC = 8 cm. , then DE = ... cm.

(a) 16 (b) 12 (c) 4 (d) 2

2 Complete the following :

(1) If two straight lines intersect , then each two vertically opposite angles are ....

(2) If Δ XYZ is a right - angled triangle at Z , then $(XZ)^2$ = ...

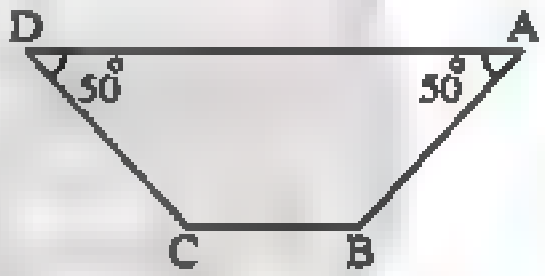
(3) **In the opposite figure :**

$m(\angle A) = m(\angle D) = 50°$

$\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ intersect at E

not shown in the figure.

, then $m(\angle E)$ = ...



(4) The quadrilateral in which two opposite sides are parallel is ....

3 [a] If the ratio among the measures of the interior angles of a quadrilateral is 2 : 2 : 3 : 5 , find the measure of the greatest angle in the quadrilateral.

[b] In the opposite figure :

ABC is a triangle in which

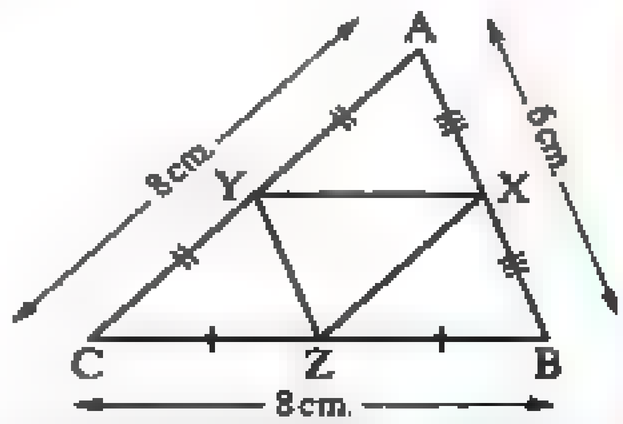
X , Y and Z are the midpoints

of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ respectively

If AB = 6 cm. , BC = 8 cm. , AC = 8 cm. ,

(1) calculate the perimeter of Δ XYZ

(2) determine the type of the figure XYCZ



4 On lattice , draw Δ ABC where A (1 , 2) , B (4 , 2) , C (4 , 4) , then map its image by the transformation $(x , y) \longrightarrow (y , -x)$

The answer of worksheet 8

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ ⓑ ⓒ ⓓ

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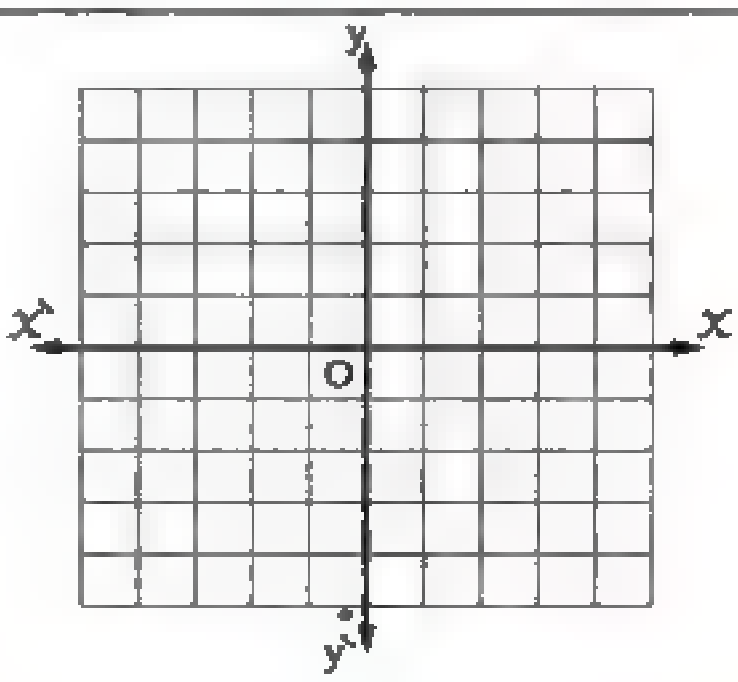

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Worksheet 9 till lesson 9 unit 3

Answer the following questions :

1 Choose the correct answer from those given :

(1) The number of axes of symmetry of the square is ....
(a) 1 (b) 2 (c) 3 (d) 4

(2) The image of the point (3 , – 5) by the reflection in X-axis is ....
(a) (3 , 5) (b) (– 3 , 5) (c) (– 3 , – 5) (d) (3 , – 5)

(3) ABC is a triangle in which m (∠ A) = 60° , m (∠ B) = 2 m (∠ C) , then m (∠ B) = ...
(a) 40° (b) 60° (c) 70° (d) 80°

(4) The image of the point (2 , 3) by the reflection in y-axis is ....
(a) (– 2 , 3) (b) (2 , – 3) (c) (– 2 , – 3) (d) (3 , 2)

(5) The sum of measures of the exterior angles of the octagon = ...
(a) 540° (b) 360° (c) 720° (d) 1080°

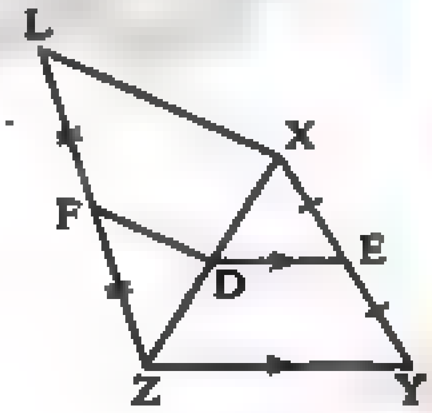
(6) ABC is a right - angled triangle at B in which AB = 3 cm. and BC = 4 cm. , then AC = ... cm.
(a) 2.5 (b) 5 (c) 16 (d) 9

2 Complete the following :

(1) The measure of the exterior angle of the triangle equals ....
(2) The image of the point (– 5 , 4) by the transformation $(X , y) \longrightarrow (-y , X)$ is ....
(3) If ABCD is a rhombus , then .... ⊥ ....
(4) The reflection in a straight line keeps .... , ... , ... , ...

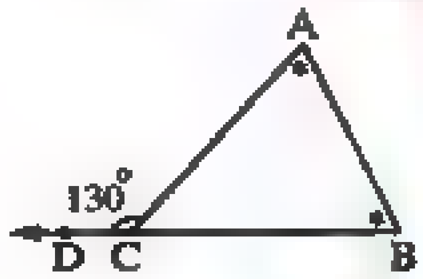
3 [a] In the opposite figure :

XE = EY , $\overleftrightarrow{ED}$ // $\overline{YZ}$
F is the midpoint of $\overline{ZL}$
Prove that : $\overline{DF}$ // $\overline{XL}$



[b] In the opposite figure :

ABC is a triangle , D ∈ $\overrightarrow{BC}$
m (∠ ACD) = 130°
Find : m (∠ B)



4 On lattice , find the image of Δ ABO where
A (2 , 2) , B (4 , 2) , O (0 , 0) by reflection in y-axis.

The answer of worksheet 9

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1) ⓐ	ⓑ	ⓒ	ⓓ
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(3) ⓐ	ⓑ	ⓒ	ⓓ
(4) ⓐ	ⓑ	ⓒ	ⓓ
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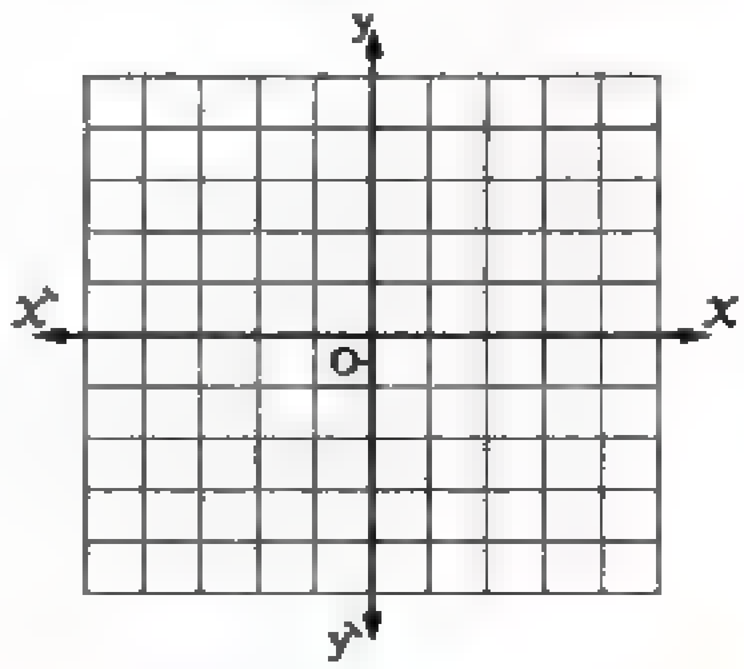

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Worksheet 10 till lesson 10 unit 3

Answer the following questions :

1 Choose the correct answer from those given :

(1) The point $(2, -4)$ is the image of the point .... by reflection in the origin point.
(a) $(2, 4)$ (b) $(-2, 4)$ (c) $(-2, -4)$ (d) $(2, -4)$

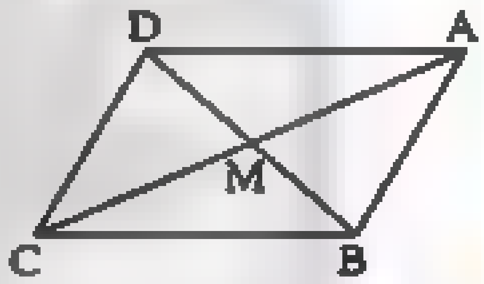
(2) The measure of the exterior angle of a triangle is .... the measure of any interior angle of the triangle except its adjacent angle.
(a) $>$ (b) $<$ (c) $=$ (d) $\leq$

(3) If ABCD is a rectangle , then AC = ...
(a) AB (b) BC (c) BD (d) CD

(4) The parallelogram whose two diagonals are perpendicular and the measure of one of its angles is 90° is called ....
(a) rectangle. (b) rhombus. (c) square. (d) trapezium.

(5) The number of axes of symmetry of the rectangle is ....
(a) 1 (b) 2 (c) 3 (d) 4

(6) **In The opposite figure :**



If ABCD is a parallelogram , then the image of Δ AMD by reflection in the point M is Δ ....
(a) AMB (b) DMC
(c) CMB (d) DMA

2 Complete the following :

(1) If the reflection in a straight line transforms the figure to itself , then this straight line is called ....

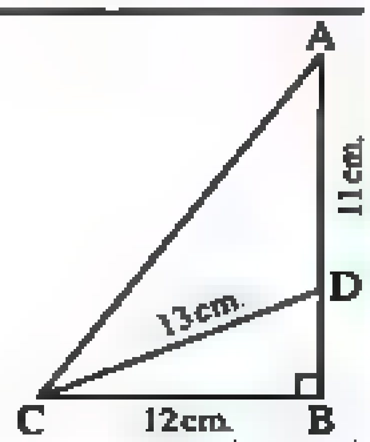
(2) The sum of the measures of the accumulative angles at a point is ....

(3) The ray drawn from the midpoint of a side of a triangle parallel to another side .... the third side.

(4) **In the opposite figure :**

The value of a = ...

3 In the opposite figure :



ABC is a triangle in which $m(\angle B) = 90°$
, $D \in \overline{AB}$, where AD = 11 cm. , BC = 12 cm.
and DC = 13 cm.
Find the length of : $\overline{BD}$ and $\overline{AC}$

4 Draw on a lattice Δ ABC , where A $(5, 1)$, B $(2, 1)$ and C $(5, 3)$, then draw its image by reflection in the origin point.

The answer of worksheet 10

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

(1)	(a)	(b)	(c)	(d)
(2)	(a)	(b)	(c)	(d)
(3)	(a)	(b)	(c)	(d)
(4)	(a)	(b)	(c)	(d)
(5)	(a)	(b)	(c)	(d)
(6)	(a)	(b)	(c)	(d)

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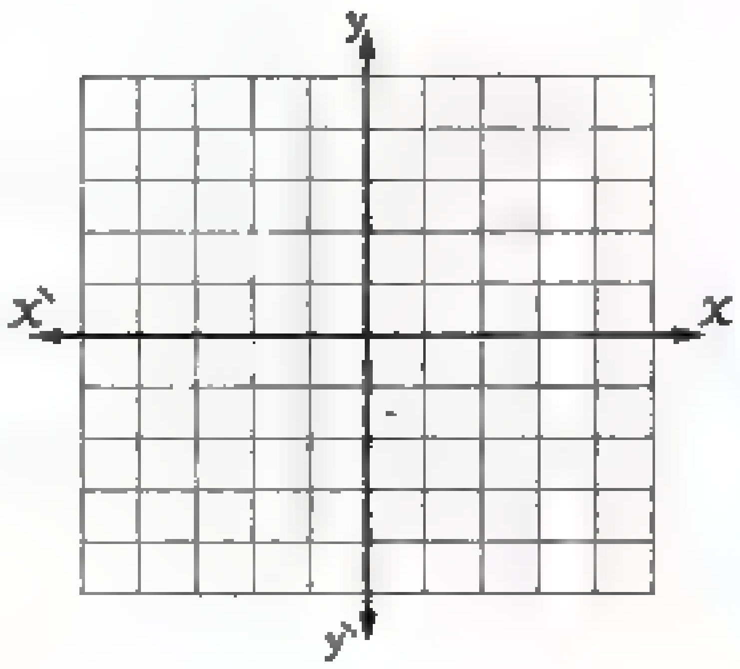

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Worksheet 11 till lesson 11 unit 3

Answer the following questions :

1 Choose the correct answer from those given :

(1) ABC is a triangle in which m (∠ A) = 4 X° , m (∠ B) = 2 X° and m (∠ C) = 3 X° , then ∠ A is ....
(a) acute. (b) right. (c) obtuse. (d) reflex.

(2) The quadrilateral in which each two opposite sides are equal in length is ....
(a) a parallelogram. (b) a trapezium. (c) a square. (d) a rectangle.

(3) The image of the point (– 2 , 3) by a translation of magnitude 5 units in the positive direction of the X-axis is ....
(a) (– 2 , – 2) (b) (– 7 , 3) (c) (– 2 , 8) (d) (3 , 3)

(4) The image of the point (0 , 5) by reflection in the X-axis is ....
(a) (0 , 5) (b) (0 , – 5) (c) (5 , 0) (d) (– 5 , 0)

(5) The image of the point (X , – y) by reflection in the origin point is ....
(a) (X , y) (b) (– X , y) (c) (X , – y) (d) (– X , – y)

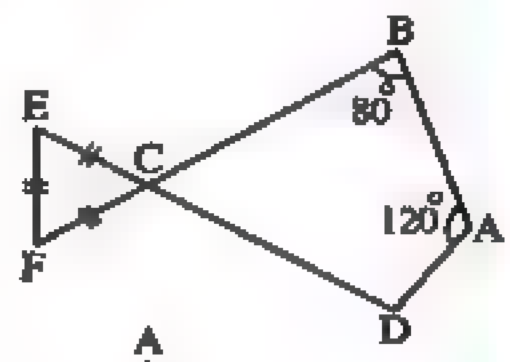
(6) If A̍ (3 , 5) is the image of the point A by the translation (X , y) ⟶ (X + 4 , y – 2) , then the point A is ....
(a) (7 , 3) (b) (7 , 7) (c) (– 1 , 7) (d) (– 1 , 3)

2 Complete the following :

(1) If the measure of an exterior angle of a regular polygon = 30° , then the number of sides of the polygon = ...
(2) The line segment which joins the two midpoints of the two sides of a triangle .... and ....
(3) The translation keeps .... , ...
(4) The translation is determined by two things , they are .... and ....

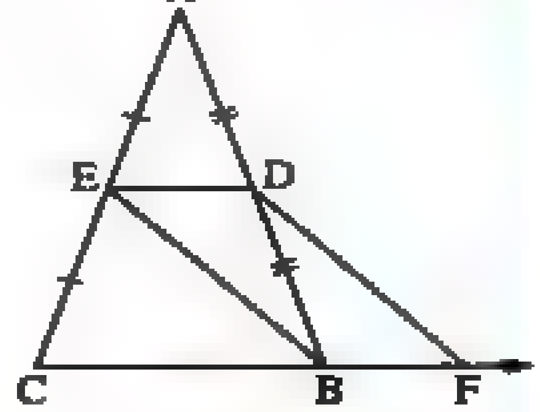
3 On lattice , draw Δ ABC where A (– 3 , 2) , B (– 1 , 1) and C (– 2 , 4) then draw its image.
(1) by the translation (X , y) ⟶ (X + 4 , y – 3)
(2) by reflection in X-axis.

4 [a] In the opposite figure :



ABCD is a quadrilateral in which
m (∠ A) = 120° , m (∠ B) = 80°
Δ CEF is an equilateral
Find by proof : m (∠ D)

[b] In the opposite figure :



D and E are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively ,
F ∈ $\overrightarrow{CB}$ where BF = $\frac{1}{2}$ BC
Prove that : the figure BEDF is a parallelogram.

The answer of worksheet 11

Total mark 15

1 Shade the circle that represents your choice for the correct answer :

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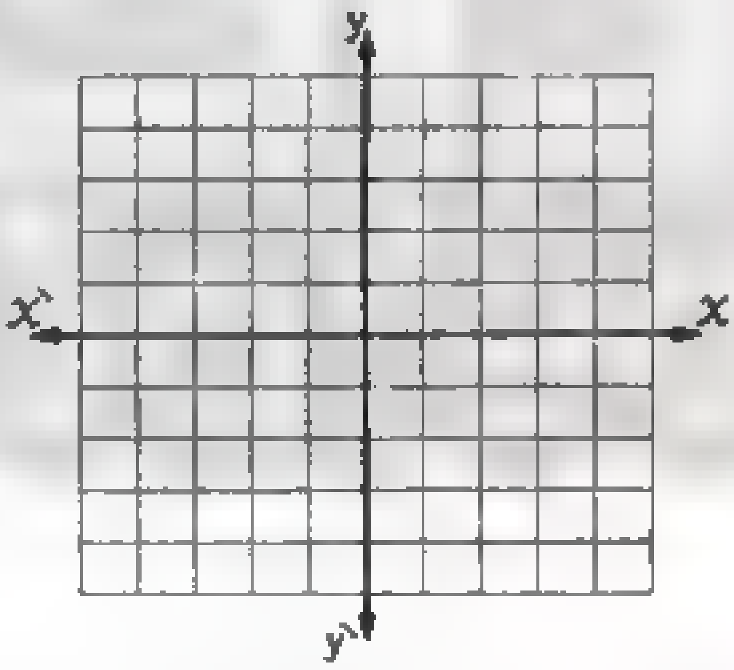

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Model Examinations of the school book

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

(1) The sum of the measure of interior angle of a triangle equals ……

(a) 90° (b) 180° (c) 270° (d) 360°

(2) The image of the point (– 1 , 3) by translation (4 , – 2) is ………

(a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)

(3) The measure of the exterior angle of the equilateral triangle is ……

(a) 30° (b) 45° (c) 60° (d) 120°

(4) In a parallelogram if the adjacent sides are equal in the length , then the shape is ……

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

(5) The number of the diagonals of a pentagon is ………

(a) 3 (b) 5 (c) 7 (d) 9

2 Complete the following :

(1) The image of the point (2 , 1) by reflection in X-axis is ………

(2) The image of the point (2 , – 1) by rotation about the origin point with an angle of measure 180° is ………

(3) The square is a rectangle in which ………

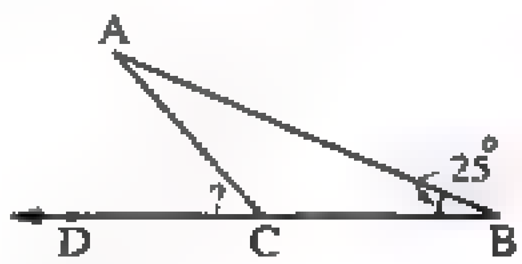
(4) ABCD is a parallelogram in which m (∠ A) = 60° , then m (∠ B) = ………

(5) The image of the point (5 , 3) by translation : $(X , y) \longrightarrow (X + 3 , y – 1)$ is ……

3 [a] In the opposite figure :

m (∠ A) = m (∠ B) = 25°

Find : m (∠ ACD)

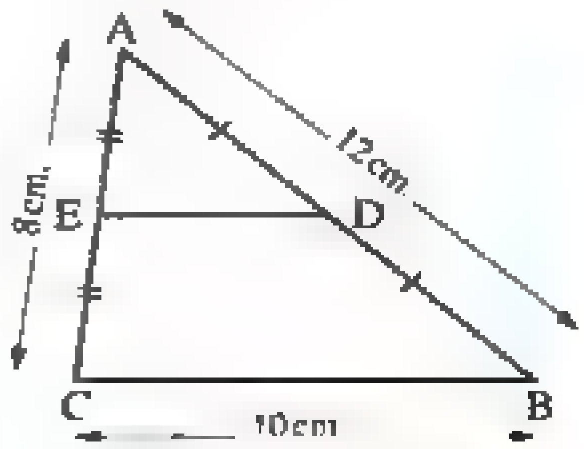


[b] In the opposite figure :

Δ ABC in which : AB = 12 cm. ,

BC = 10 cm. , AC = 8 cm.

Find the perimeter of : Δ ADE

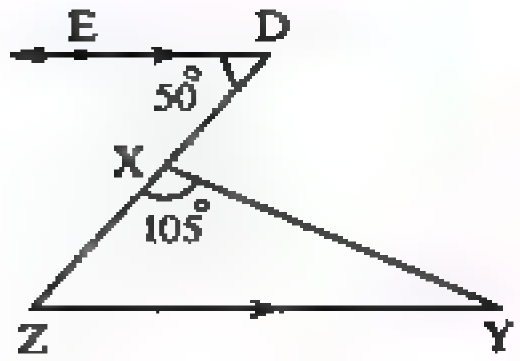


4 **[a] In the opposite figure :**

$\overrightarrow{DE} \parallel \overrightarrow{YZ}$, m ($\angle$ ZDE) = 50°

, m ($\angle$ YXZ) = 105°

Find : m ($\angle$ Z) , m ($\angle$ Y) , m ($\angle$ YXD)

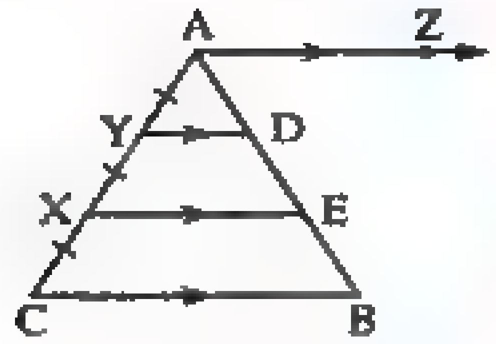


[b] In the opposite figure :

$\overrightarrow{AZ} \parallel \overline{YD} \parallel \overline{XE} \parallel \overline{CB}$,

AY = YX = XC , AB = 18 cm. ,

Find the length of : $\overline{EB}$



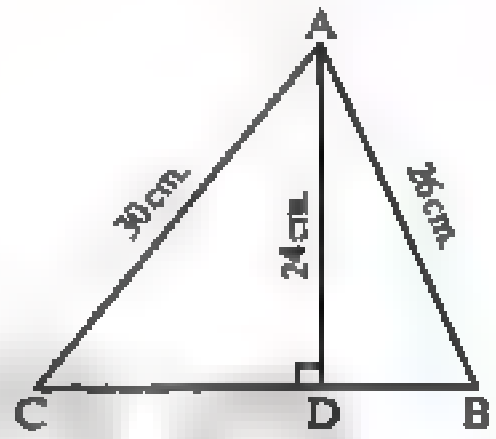
5 **In the opposite figure :**

$\overline{AD} \perp \overline{BC}$, if AD = 24 cm. ,

AB = 26 cm. , AC = 30 cm.

(1) **Find the length of :** $\overline{BC}$

(2) **Find the area of :** Δ ABC



Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) The image of the point (2 , – 5) by reflection in X-axis is ....

(a) (2 , – 5) (b) (2 , 5) (c) (– 2 , – 5) (d) (5 , 2)

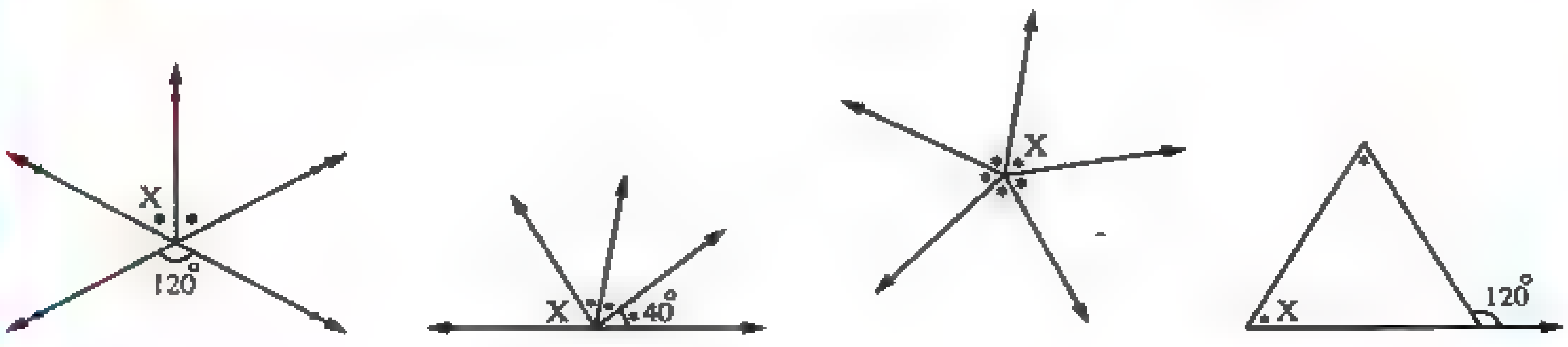
(2) The measure of each angle of regular hexagon equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

(3) The two diagonals are equal in the length and not perpendicular in ....

(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

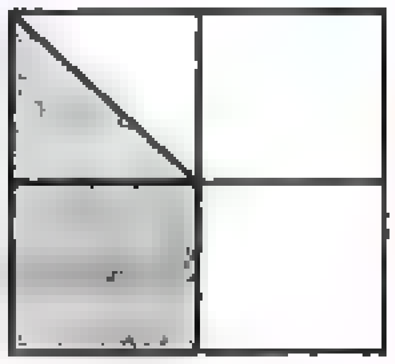
(4) All the following shapes m ($\angle$ X) = 60° except the shape ....



(a) (b) (c) (d)

(5) **In the opposite figure :**

The area of shaded part from the area of all shape equal ....



(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

2 **Complete the following :**

(1) The sum of the measures of the angles of the quadrilateral equals ......

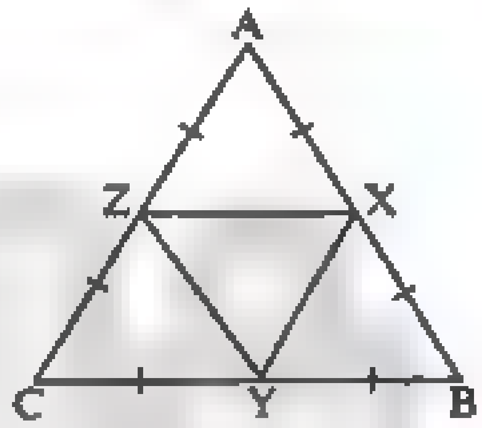
(2) The image of the point (2 , 3) by translation MN , in direction $\overrightarrow{MN}$, where M (2 , − 1) , N (5 , 1) is ....

(3) ABCD is parallelogram in which m (∠ A) = 60° , then m (∠ B) = ...

(4) The ray drawn parallel to one side of a triangle and passing through the midpoint of another side ....

(5) **In the opposite figure :**

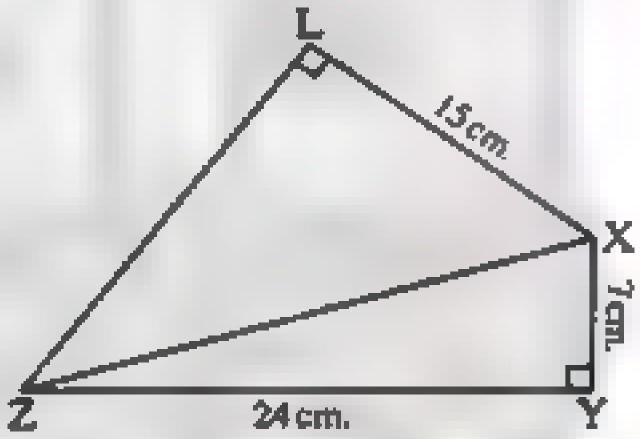
The image of the triangle XBY by translation XZ in direction $\overrightarrow{XZ}$ is ....



3 [a] **In the opposite figure :**

XYZL is quadrilateral in which m (∠ Y) = m (∠ L) = 90° , XY = 7 cm. , YZ = 24 cm. , XL = 15 cm.

Find the length of each of : $\overline{XZ}$, $\overline{LZ}$



[b] Using the square lattice , draw $\overline{AB}$ where A (4 , 3) , B (− 1 , 1) then find the image of $\overline{AB}$ by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

4 [a] Draw the image of triangle ABC where A (1 , 1) , B (3 , 4) , C (5 , 2) by reflection in x-axis.

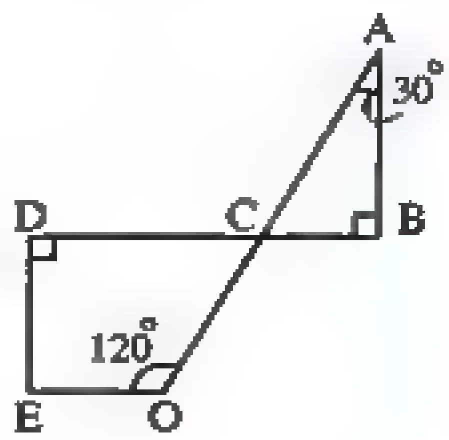
[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{ED}$, are perpendicular to $\overline{BD}$, $\overline{BD} \cap \overline{AO} = \{C\}$

m (∠ A) = 30°

, m (∠ EOC) = 120° ,

Find : m (∠ E)



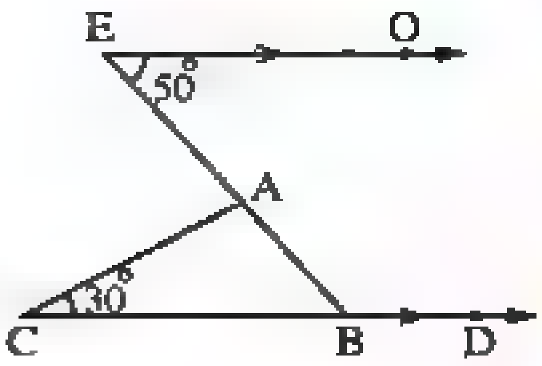
5 **[a] In the opposite figure :**

$\overline{EO} \parallel \overline{CD}$, m ($\angle$ E) = 50°

, m ($\angle$ C) = 30° ,

Find the measures of

angles of Δ ABC , m ($\angle$ ABD)



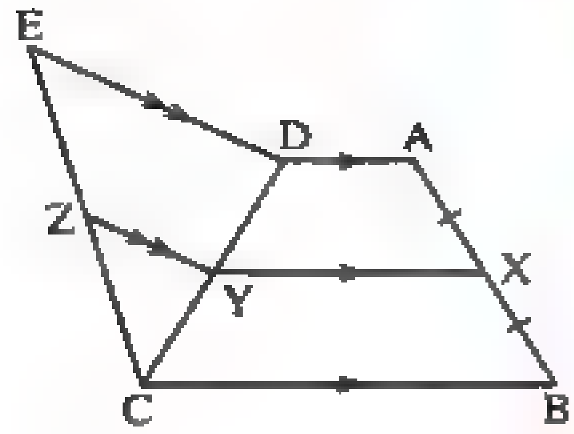
[b] In the opposite figure :

X is the midpoint of $\overline{AB}$

, Y $\in \overline{CD}$, Z $\in \overline{CE}$

, $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\overline{YZ} \parallel \overline{DE}$

is CZ = ZE ? giving reason



Model 3

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) The image of the point (3 , – 5) by reflection in y-axis is ....

(a) (3 , 5) (b) (– 3 , – 5) (c) (– 3 , 5) (d) (– 5 , 3)

(2) The sum of the measures of interior angles of a pentagon is ....

(a) 360° (b) 450° (c) 540° (d) 720°

(3) The number of diagonals of quadrilateral is ....

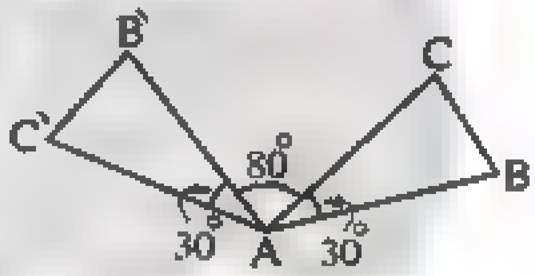
(a) 2 (b) 3 (c) 4 (d) 5

(4) **In the opposite figure :**

Δ AB̀ C̀ is the image

of Δ ABC by rotation about A

and with angle of measure ....



(a) 30° (b) 80° (c) 110° (d) 140°

(5) The diagonal of a square divided its vertex angle in two angles of the measure of each of them is ....

(a) 30° (b) 45° (c) 60° (d) 90°

2 **Complete the following :**

(1) The rhombus is a parallelogram in which ....

(2) Each opposite angles in a parallelogram are ....

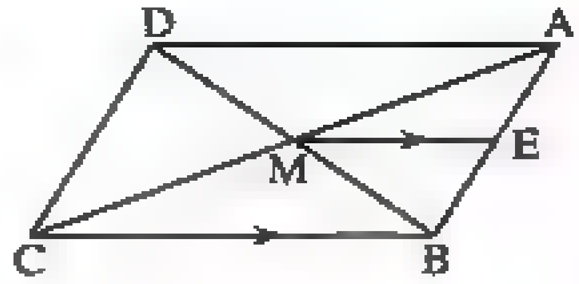
(3) (– 3 , 2) is the image of the point (3 , 2) by reflection in .... axis.

(4) The line segment joining the midpoint of two sides of a triangle is ....

(5) The image of the point (4 , 6) by geometric transformation (X , y) $\longrightarrow$ (– X , y – 7) is ....

3 [a] Using the lattice , draw Δ ABC where A (1 , 0) , B (0 , 2) and C (– 3 , 1) , then draw its image by reflection in X-axis.

[b] In the opposite figure :

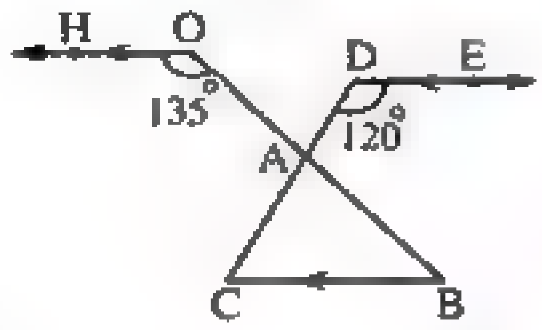


ABCD is a parallelogram , M is the intersection of its diagonals

Draw $\overrightarrow{ME} \parallel \overline{CB}$

is AE = EB ? giving reason.

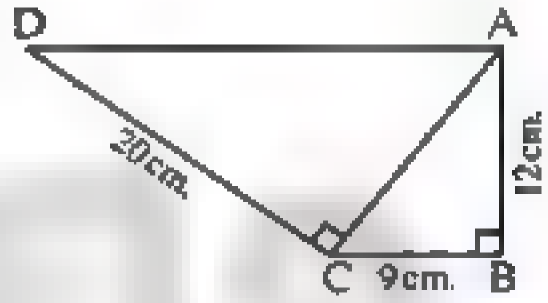
4 **[a] In the opposite figure :**



$\overleftrightarrow{DE} \parallel \overleftrightarrow{OH} \parallel \overleftrightarrow{BC}$, m ($\angle$ ADE) = 120°

, m ($\angle$ AOH) = 135° , Calculate the measures of the angles of the triangle ABC

[b] In the opposite figure :

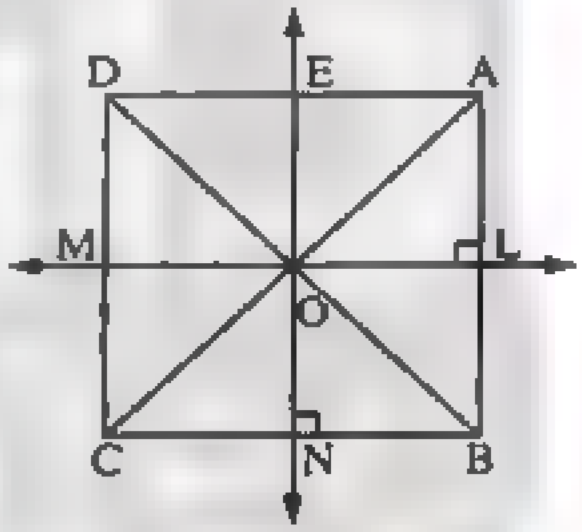


m ($\angle$ B) = m ($\angle$ ACD) = 90° ,

AB = 12 cm. , BC = 9 cm. , CD = 20 cm.

Find the length of each of : $\overline{AC}$, $\overline{AD}$

5 **In the opposite figure :**



ABCD is a square of side length 6 cm.

and the origin point its center. **Find :**

(1) The image of Δ AOL by translation 3 cm. in direction $\overrightarrow{AB}$

(2) The image of Δ AOL by reflection in $\overleftrightarrow{EN}$

(3) The image of Δ AOL by rotation about O and with an angle of measure (– 90°)

Model 4

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) The parallelogram whose diagonals are perpendicular and not equal in length is called ....

(a) rhombus. (b) square. (c) rectangle. (d) trapezium.

(2) The measure of each angle of a fegular pentagon is ....

(a) 90° (b) 108° (c) 120° (d) 136°

(3) The triangle contains at least two .... angles.

(a) acute (b) obtuse (c) right (d) reflex

(4) If ABCD is a parallelogram in which BC = 8 cm. , CD = 6 cm. , then its perimeter = ...

(a) 14 cm. (b) 28 cm. (c) 48 cm. (d) 56 cm.

(5) The image of the point (2 , – 1) by reflection in X-axis is ....

(a) (2 , 1) (b) (1 , 2) (c) (– 2 , – 1) (d) (– 1 , 2)

2 **Complete the following :**

(1) The line segment joining the midpoint of two sides of a triangle is ....

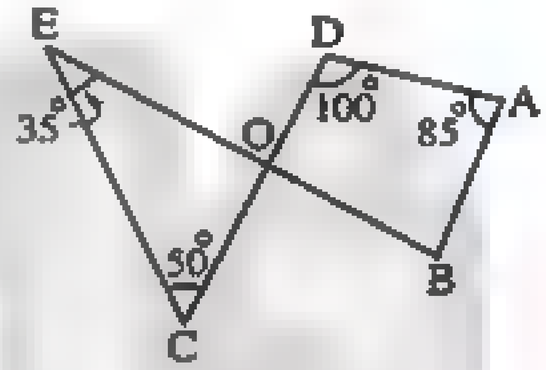
(2) In a parallelogram each opposite angles are ....

(3) The quadrilateral is a parallelogram if ....

(4) The image of the point (5 , – 3) by translation 3 units in negative direction of X-axis is ....

(5) The image of the point (3 , 2) by rotation with an angle of measure 180° about the origin is ....

3 **In the opposite figure :**



$\overline{DC} \cap \overline{BE} = \{O\}$, m (∠ A) = 85°

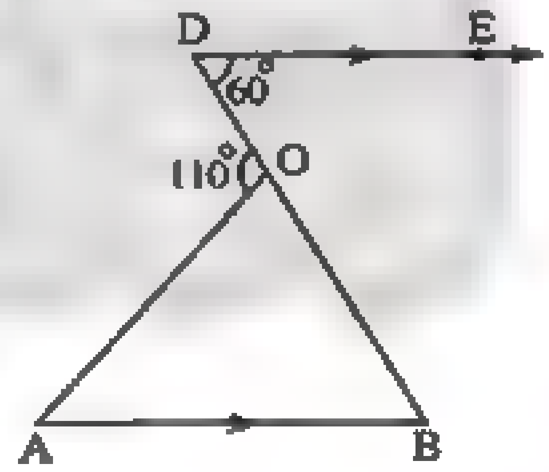
, m (∠ D) = 100° , m (∠ E) = 35° , m (∠ C) = 50°

Find with proof each of :

(1) m (∠ DOB) (2) m (∠ B)

4 [a] Find the length of the diagonal of a rectangle whose area 48 cm^2 and of width 6 cm. ?

[b] **In the opposite figure :**



$\overline{AB} \parallel \overline{DE}$,

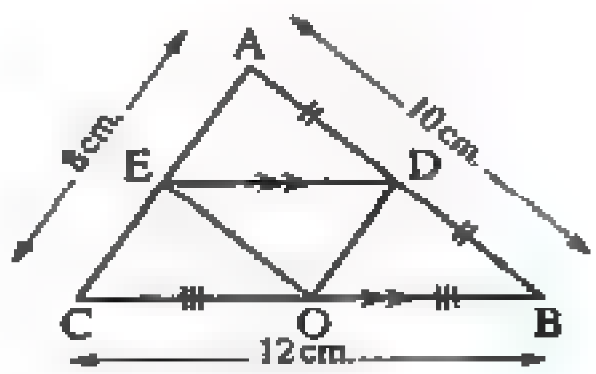
m (∠ D) = 60°

, m (∠ AOD) = 110°

Find with proof : m (∠ A)

5 [a] If the point A is the image of point B (– 1 , 2) by reflection in y-axis , find the image of A by translation (– 1 , 2)

[b] **In the opposite figure :**



ABC is a triangle in which D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{BC}$, E ∈ $\overline{AC}$ Such that $\overline{DE} \parallel \overline{BC}$,

AB = 10 cm. , BC = 12 cm. , AC = 8 cm.

(1) **Prove that :** DBOE is a parallelogram.

(2) **Find the perimeter of :** Δ EDO

Model 5

Answer the following questions :

1 Choose the correct answer from those given :

(1) The image of the point (– 3 , 4) by reflection in y-axis is ....

(a) (3 , – 4) (b) (3 , 4) (c) (– 3 , – 4) (d) (4 , – 3)

(2) The sum of the measure of the exterior angles of a triangle equals ....

(a) 90° (b) 108° (c) 180° (d) 360°

(3) The diagonals which are equal in the length and perpendicular in ....

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

(4) The image of the point (– 3 , 5) by rotation about the origin and with an angle of measure 90° is ....

(a) (5 , 3) (b) (– 5 , 3) (c) (3 , 5) (d) (– 5 , – 3)

(5) The measure of each angle of regular octagon equals ....

(a) 108° (b) 120° (c) 135° (d) 144°

2 Complete the following :

(1) The parallelogram whose diagonals are perpendicular is ....

(2) If the measure of an interior angle of a triangle is equal to the sum of the measures of the other two interior angles , then the triangle is ....

(3) Any triagnle has at least two .... interior angles.

(4) The triagnle of the point (2 , – 4) by reflection in x-axis is ....

(5) The image of the point (3 , – 2) by translation $(x , y) \longrightarrow (x - 1 , y + 6)$ is ....

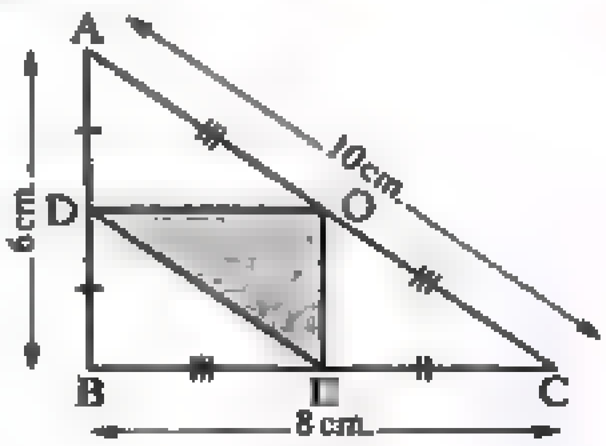
3 [a] Prove that the ray drown parallel to one side of a triangle and passing through the midpoint of another side bisects the third side of the triangle.

[b] In the opposite figure :

D , E , O are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ resectively , AB = 6 cm. ,

BC = 8 cm. , AC = 10 cm.

Find the perimeter of : Δ DEO

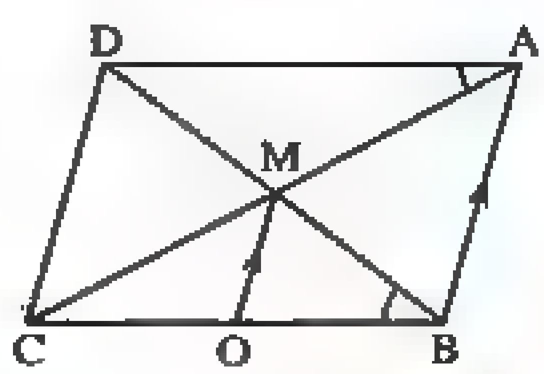


4 [a] In the opposite figure :

ABCD is a parallelogram its diagonals are intersect at M , $\overrightarrow{MO}$ // $\overline{AB}$

, $\overrightarrow{MO} \cap \overline{BC} = \{O\}$

Prove that : BO = OC

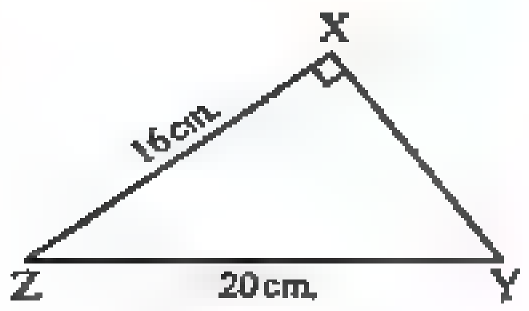


[b] In the opposite figure :

XYZ is a triangle in which m (∠ X) = 90° ,

YZ = 20 cm. , XZ = 16 cm.

Find the length of : $\overline{XY}$

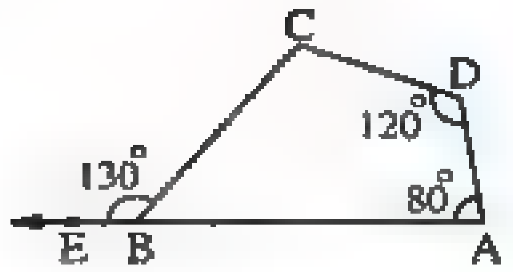


5 **[a] In the opposite figure :**

m (∠ A) = 80° , m (∠ D) = 120°

, m (∠ CBE) = 130°

Find : m (∠ C)



[b] On a square lattice , draw the triangle whose vertices are A (4 , 4) B (4 , 2) , C (1 , 2) then determine each of the following :

(1) The coordinates of the image of Δ ABC by translation 2AB in direction $\overrightarrow{AB}$

(2) The image of Δ ABC by rotation about B and with an angle of measure 180°

Model 6

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) The measure of each angle of the regular pentagon is ………

(a) 90° (b) 108° (c) 120° (d) 144°

(2) The smallest number of the acute angle in any triangle is ………

(a) zero (b) 1 (c) 2 (d) 3

(3) The rhombus of diagonals are equal in length is ………

(a) square. (b) rectangle. (c) parallelogram. (d) trapezium.

(4) The image of the point (2 , – 1) by reflection in X-axis is ………

(a) (2 , 1) (b) (1 , 2) (c) (– 2 , – 1) (d) (– 1 , 2)

(5) The image of the square by rotation about the origin point with an angle of measure 90° is ………

(a) rectangle. (b) square. (c) rhombus. (d) trapezium.

2 **Complete the following :**

(1) The measure of the exterior angle of a triangle is ………

(2) The parallelogram whose diagonals are equal in length and perpendicular is ………

(3) ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = ………

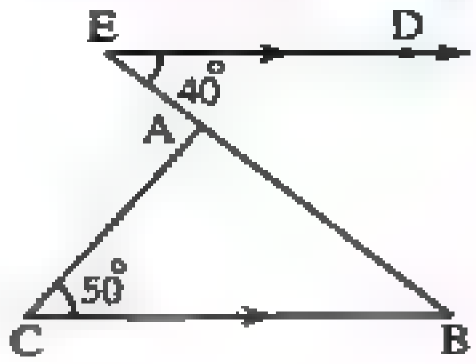
(4) The image of the point (– 4 , 5) by translation (2 , – 3) is ………

3 **[a] In the opposite figure :**

$\overleftrightarrow{ED} \parallel \overleftrightarrow{CB}$, m ($\angle$ C) = 50°

, m ($\angle$ E) = 40°

Prove that : $\overline{AC} \perp \overline{BE}$



[b] Draw Δ OBC on a square lattice where O (0 , 0) , B (3 , 0) , C (0 , 4) then find its image by rotation about the origin point with an angle of measure 180°

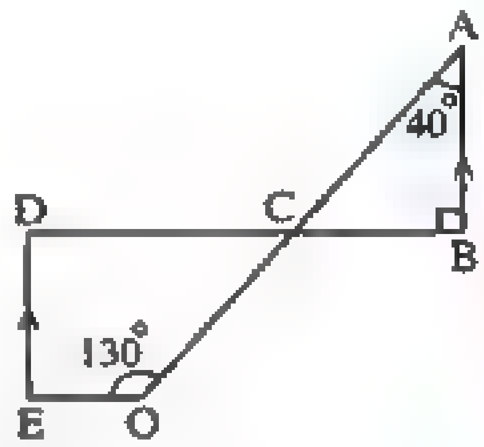
4 **In the opposite figure :**

$\overline{BD} \cap \overline{AO} = \{C\}$, $\overline{AB} \parallel \overline{DE}$,

m ($\angle$ A) = 40° , m ($\angle$ B) = 90° ,

m ($\angle$ COE) = 130°

Find : m ($\angle$ E)

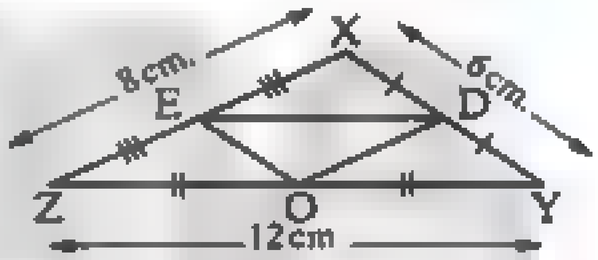


5 **[a] In the opposite figure :**

XYZ is a triangle in which D , E , O are midpoints of $\overline{XY}$, $\overline{XZ}$, $\overline{YZ}$ respectively

, XY = 6 cm. , XZ = 8 cm. , YZ = 12 cm. ,

Find the perimeter of : Δ DOE

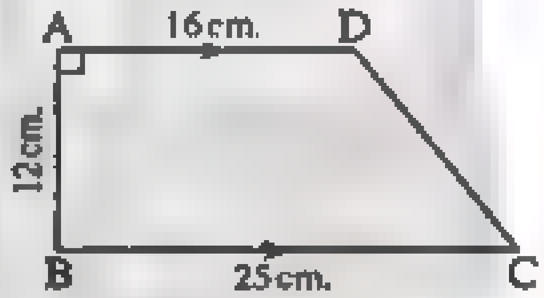


[b] In the opposite figure :

ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$

, m ($\angle$ A) = 90° , AB = 12 cm. , BC = 25 cm. , AD = 16 cm.

Find the length of : $\overline{DC}$



Model 7

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) The measure of each angle of the regular hexagon is ....

(a) 108° (b) 120° (c) 136° (d) 144°

(2) Any triangle has at least two .... interior angles.

(a) acute (b) right (c) obtuse (d) straight

(3) The diagonals of the rectangle are ....

(a) parallel. (b) perpendicular.

(c) equal in the length. (d) equal in length & perpendicular.

(4) The image of a triangle by rotation about the origin point with an angle of measure 180° is ……

(a) triangle. (b) line segment. (c) point. (d) straight line.

(5) The image of the point (3 , – 2) by reflection in y-axis ………

(a) (3 , 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (– 2 , 3)

2 **Complete the following :**

(1) The sum of the measures of the interior angles of a pentagon equals ………

(2) The measure of the exterior angle of an equilateral triangle = ……… at one of its vertices.

(3) The rectangle is a parallelogram in which one of its angles is ………

(4) The image of the point (– 4 , 5) by translation (2 , – 3) is ………

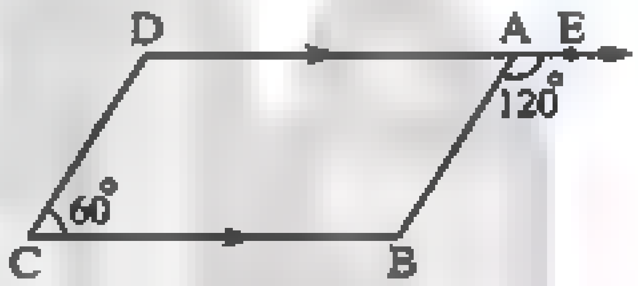
(5) If the image of the point (– 4 , 0) by rotation about the origin point is (0 , – 4) , then the measure of rotation angle is ………

3 **[a] In the opposite figure :**

$E \in \overrightarrow{DA}$, m (∠ EAB) = 120°

m (∠ C) = 60° , $\overrightarrow{DA} \parallel \overline{CB}$

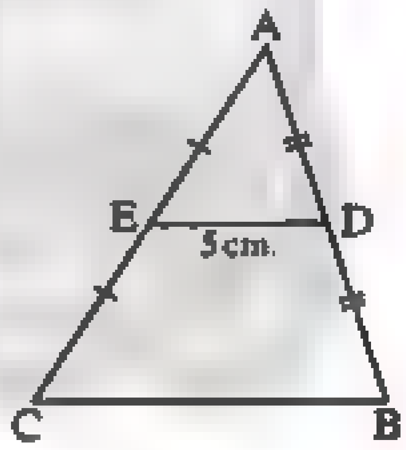
Prove that : ABCD is a parallelogram.



[b] In the opposite figure :

ABC is a triangle in which D , E are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively , DE = 5 cm.

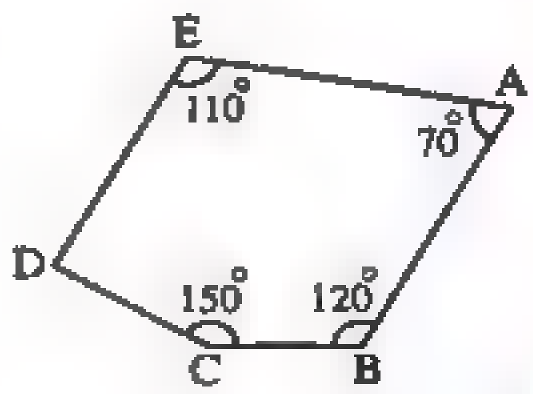
Find the length of : $\overline{BC}$



4 **[a] In the opposite figure :**

ABCDE is a pentagon in which m (∠ A) = 70° , m (∠ B) = 120° , m (∠ C) = 150° , m (∠ E) = 110°

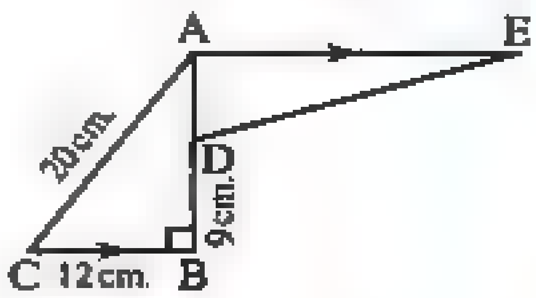
Find : m (∠ D) with proof.



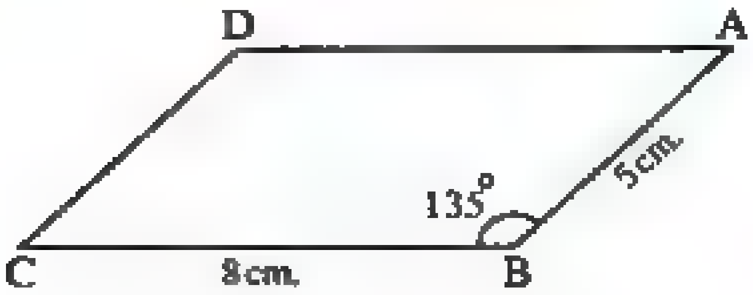
[b] In the opposite figure :

Δ ABC in which m (∠ B) = 90° , $\overline{AE} \parallel \overline{BC}$, if BC = 12 cm. , AC = 20 cm. , $D \in \overline{AB}$ such that : BD = 9 cm. , AE = 2 BC ,

Find the length of each : $\overline{AD}$, $\overline{ED}$



5 **[a] In the opposite figure :**



ABCD is a parallelogram in which

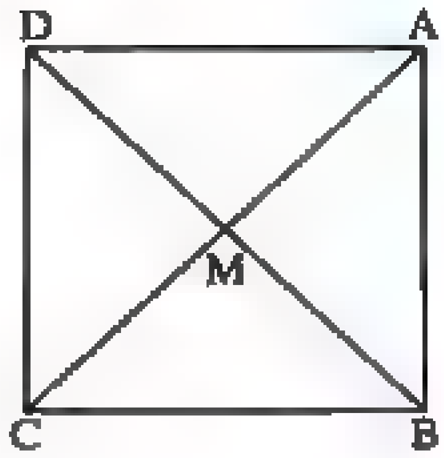
AB = 5 cm. , BC = 8 cm. , m (∠ B) = 135°

Find :

(1) m (∠ C)

(2) The perimeter of parallelogram ABCD

[b] In the opposite figure :



ABCD is a square , whose diagonals intersect at M.

Find :

(1) The image of Δ ABC by reflection in $\overleftrightarrow{AC}$

(2) The image of Δ MAB by rotation about M with measure (– 90°)

Model 8

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) The diagonal of square makes angle of measure .... with any of its sides.

(a) 45° (b) 60° (c) 90° (d) 120°

(2) The measure of any of the exterior angle of an equilateral triangle equals ....

(a) 60° (b) 90° (c) 120° (d) 180°

(3) In Δ ABC if m (∠ A) > m (∠ B) + m (∠ C) then the angle A is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

(4) The sum of the measures of any two consecutive angles in a parallelogram equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

(5) The image of the point (5 , – 3) by rotation about the origin point is itself , then the measure of rotation angle is ....

(a) 90° (b) 180° (c) 270° (d) 360°

2 **Complete the following :**

(1) The length of a line segment joining the midpoints of two sides of a triangle equals ....

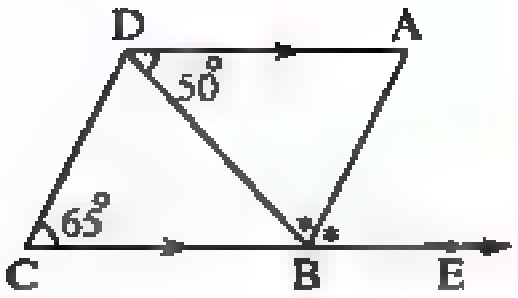
(2) The parallelogram whose diagonals are perpendicular and of equal in length is ....

(3) The parallelogram whose perimeter 24 cm. and the length of one of its sides is 7 cm. , then the length of its adjacent side equals ....

(4) The image of the point (3 , 4) by reflection in the X-axis is .... and its image by reflection in the y-axis is ....

(5) If the image of the point (– 1 , 3) by a translation is (1 , 4) then the image of the point (3 , – 2) by the same translation is ....

3 **[a] In the opposite figure :**



$\overrightarrow{DA} \parallel \overleftrightarrow{BE}$, $\overrightarrow{BA}$ bisects $\angle$ DBE

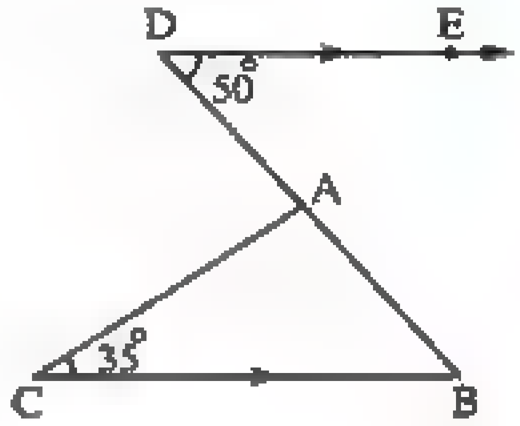
, m ($\angle$ ADB) = 50° , m ($\angle$ C) = 65°

Prove that : ABCD is a parallelogram

[b] Draw the triangle ABC in which AB = AC = 5 cm. , BC = 6 cm. , then draw its image by rotation about A with an angle of measure 270°

---

4 **[a] In the opposite figure :**



$\overrightarrow{DE} \parallel \overrightarrow{CB}$, m ($\angle$ D) = 50°

, m ($\angle$ C) = 35°

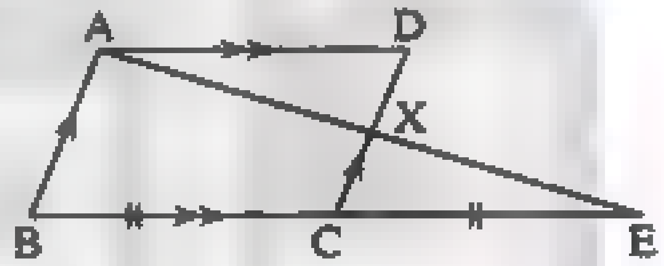
Find : (1) m ($\angle$ B)

(2) m ($\angle$ BAC)

[b] Draw the triangle ABC in which AB = 3 cm. , BC = 4 cm. , m ($\angle$ ABC) = 90° , then draw its image by reflection in straight line $\overleftrightarrow{BC}$

---

5 **[a] In the opposite figure :**

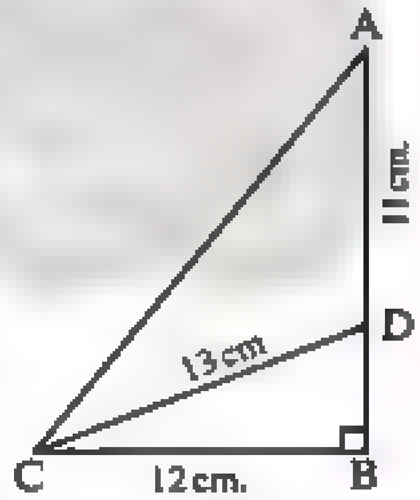


ABCD is a parallelogram , E $\in \overrightarrow{BC}$

Such that : CE = BC , AE $\cap$ $\overline{DC}$ = {X}

Prove that : AX = XE

[b] In the opposite figure :



ABC is a triangle in which m ($\angle$ B) = 90° , D $\in \overline{AB}$

Such that : AD = 11 cm.

, if BC = 12 cm. , DC = 13 cm. ,

Find the length of each of : $\overline{BD}$, $\overline{AC}$

School Examinations

1 Cairo Governorate — El-Sahel Educational Zone

Gawad Ali Hosni Exp. Language School

Answer the following questions :

1 Choose the correct answer :

(1) The sum of measures of the interior angles of the hexagon = ...

(a) 540° (b) 720° (c) 900° (d) 1080°

(2) The identity rotation about any point is with an angle of measure ......

(a) 90° (b) 180° (c) 270° (d) 360°

(3) In the parallelogram ABCD if m (∠ A) + m (∠ C) = 100° , then m (∠ B) = ...

(a) 50° (b) 80° (c) 130° (d) 40°

(4) The image of the point (2 , – 3) by translation (1 , – 1) is ......

(a) (3 , – 4) (b) (1 , – 2) (c) (2 , 3) (d) (3 , 2)

(5) In the triangle ABC , if m (∠ A) = m (∠ C) = 65° , then m (∠ B) = ...

(a) 50° (b) 65° (c) 120° (d) 130°

2 Complete each of the following :

(1) The length of the line segment joining the midpoints of two sides of a triangle is equal to ......

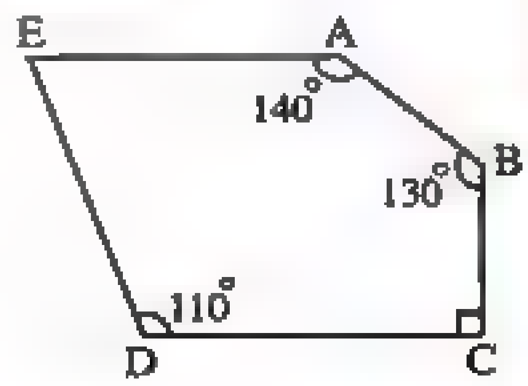
(2) Each two opposite angles of a parallelogram ......

(3) A parallelogram whose two diagonals are equal in length is called ......

(4) The sum of measures of the interior angles of a triangle equals ....°

(5) The image of a point (0 , 5) by reflection in y-axis is ....

3 [a] In the opposite figure :



m (∠ A) = 140° , m (∠ B) = 130°

m (∠ C) = 90° , m (∠ D) = 110°

Find : m (∠ E)

[b] Draw on square lattice Δ ABC

Where : A (1 , 3) , B (3 , 5) , C (5 , 0)

Find the image of Δ ABC by reflection in y-axis

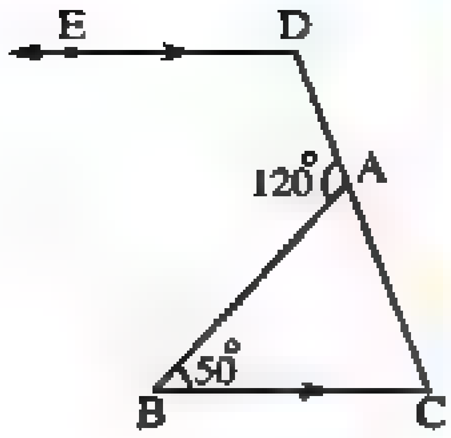
4 [a] **In the opposite figure :**

$\overrightarrow{DE} \parallel \overline{CB}$

m ($\angle$ DAB) = 120°

, ($\angle$ B) = 50°

Find by proof : m ($\angle$ D)

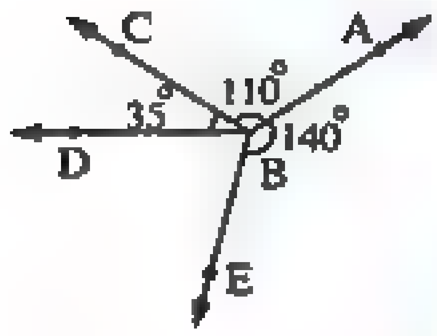


[b] **In the opposite figure :**

m ($\angle$ ABC) = 110° , m ($\angle$ CBD) = 35° ,

m ($\angle$ ABE) = 140° ,

Find : m ($\angle$ EBD)

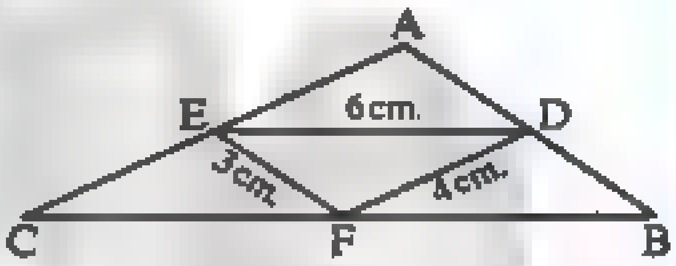


5 [a] Draw Δ ABC in which AB = 5 cm. , BC = 3 cm. and AC = 3 cm. , then find its image by reflection in $\overleftrightarrow{AB}$

[b] **In the opposite figure :**

Δ ABC in which : D , F and E are midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively , such that : DF = 4 cm. , FE = 3 cm. , DE = 6 cm.

Calculate the perimeter of : Δ ABC



Additional question

1 **Complete the following :**

(1) The image of the point (– 3 , 5) by reflection in the origin point is ....

(2) The translation in a plane keeps ....

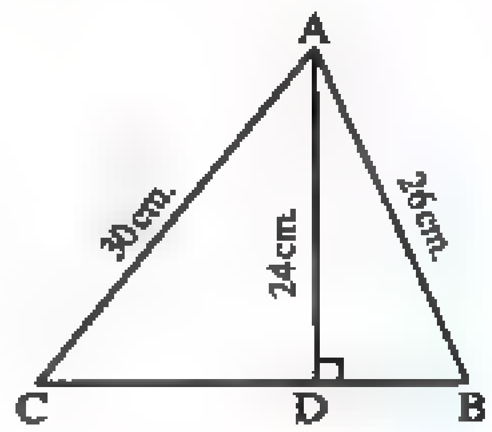
(3) The number of axes of symmetry of the rhombus is ....

2 **In the opposite figure :**

ABC is a triangle in which $\overline{AD} \perp \overline{BC}$, if AD = 24 cm. , AB = 26 cm. , AC = 30 cm.

(1) **Find :** BC

(2) **Calculate the area of :** Δ ABC



2 Cairo Governorate — Amal Language Schools

Maadi Educational Directorate

Answer the following questions :

1 Choose the correct answer from those given :

(1) XYZL is a parallelogram if m (∠ X) = 70° , then m (∠ Y) = ……

(a) 70° (b) 90° (c) 20° (d) 110°

(2) The image of the point (– 2 , 1) by reflection in y-axis is ……

(a) (2 , 1) (b) (2 , – 1) (c) (– 2 , – 1) (d) (1 , 2)

(3) The measure of an interior angle of a regular octagon is ……

(a) 120° (b) 135° (c) 108° (d) 72°

(4) The sum of the measures of the interior angles of a triangle is ……

(a) 90° (b) 150° (c) 100° (d) 180°

(5) The measure of the exterior angle of the equilateral triangle = ……

(a) 60° (b) 90° (c) 120° (d) 150°

2 Complete :

(1) The image of the point (3 , – 5) by rotation about the origin point with an angle of measure 90° is ……

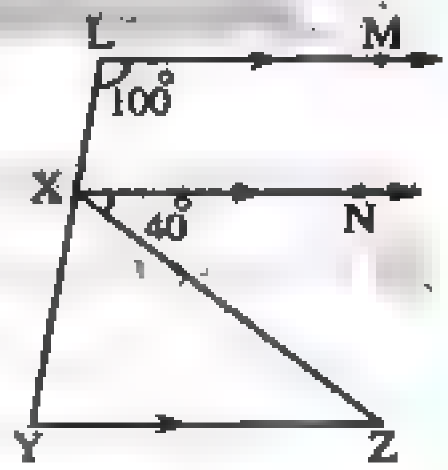
(2) In the rhombus , the two diagonals are …… and ……

(3) The ray drawn from the midpoint of a side of a triangle parallel to another side ……

(4) A quadrilateral in which only two opposite sides are parallel is called a ……

(5) The image of the point (1 , 4) by translation $(x , y) \longrightarrow (x - 1 , y - 2)$ is ……

3 [a] In the opposite figure :



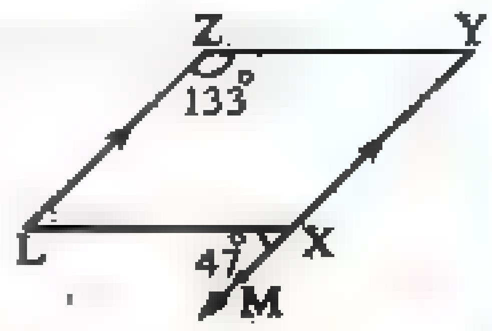
$\overrightarrow{LM}$ // $\overrightarrow{XN}$ // $\overrightarrow{YZ}$

, m (∠ L) = 100°

m (∠ NXZ) = 40°

Find the measures of angles of : Δ XYZ

[b] In the opposite figure :



$\overline{LZ}$ // $\overrightarrow{YX}$, m (∠ Z) = 133°

m (∠ LXM) = 47°

Prove that : XYZL is a parallelogram.

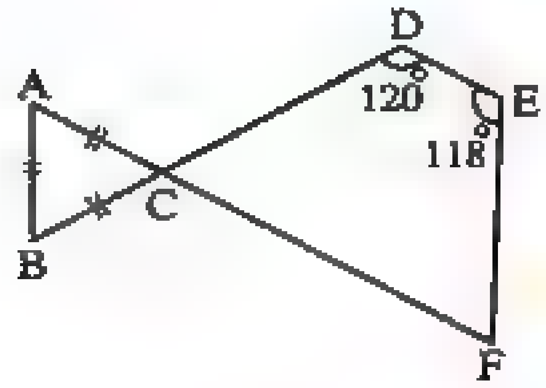
4 [a] **In the opposite figure :**

DEFC is a quadrilateral

, Δ ABC is an

equilateral triangle where $\overline{DB} \cap \overline{AF} = \{C\}$

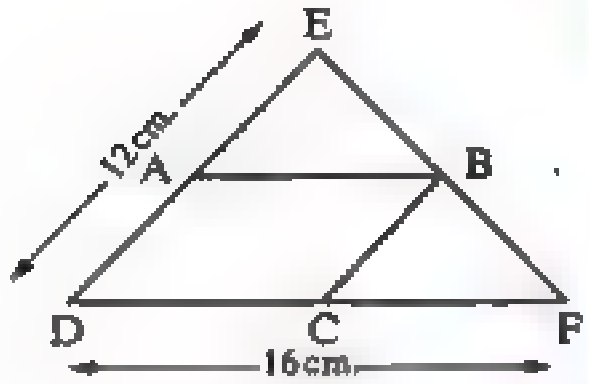
Find with proof : m (∠ F)



[b] **In the opposite figure :**

EDF is a triangle in which A , B , C are the midpoints of $\overline{ED}$, $\overline{EF}$ and $\overline{DF}$ respectively

Find the primeter of the quadrilateral : ABCD



5 Using the square lattice draw Δ LMN in which L = (4 , 4) , M = (4 , 1) , N = (1 , 1) then find its image by rotation about the origin point with an angle of measure 180°

Additional question

1 **Complete the following :**

(1) In the right-angled triangle, the square of the length of the hypotenuse equals ……….

(2) The number of axes of symmetry of the square is ……….

(3) The point (5 , – 3) is the image of the point ………. by reflection in the origin point.

2 Draw a triangle ABC where AB = BC = 5 cm. and AC = 6 cm. , then find its image by reflection in the point B

3 Giza Governorate — Dokki Educational Directorate

La Rose De Lisieux Language

Answer the following questions :

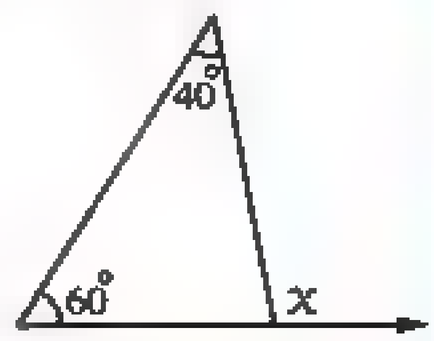
1 **Choose the correct answer from the given ones :**

(1) The image of the point (– 3 , 5) by reflection in X-axis is ……

(a) (5 , – 3) (b) (3 , – 5) (c) (3 , 5) (d) (– 3 , – 5)

(2) **In the opposite figure :**

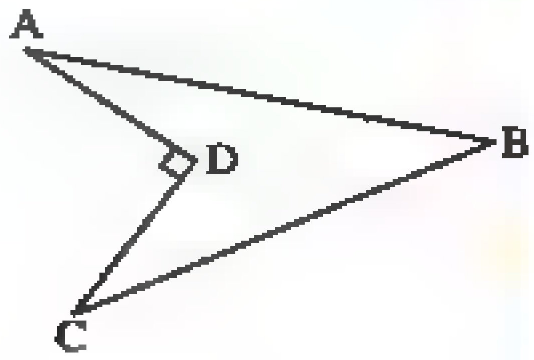
X = ………



(a) 60 – 40 (b) 60 × 40

(c) 60 + 40 (d) (60 ÷ 40)

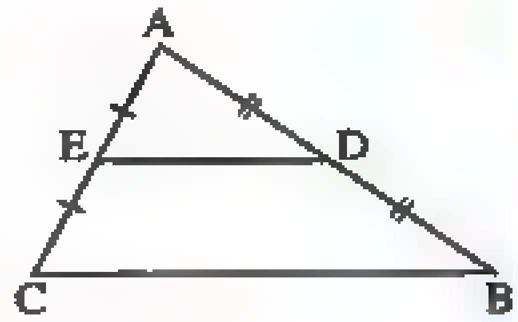
(3) ABCD is called a concave polygon because one of its interior angles is ....



(a) acute. (b) Right.
(c) obtuse. (d) reflex.

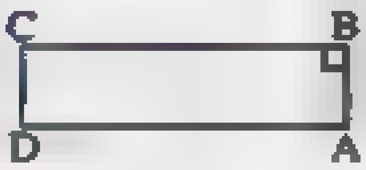
(4) **In the opposite figure :**

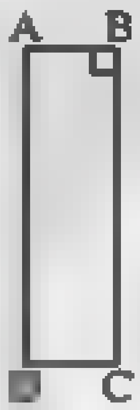
If D , E are the midpoints if $\overline{AB}$, $\overline{AC}$ respectively and the perimeter if Δ AED = 24 cm. then the perimeter if Δ ABC = ...

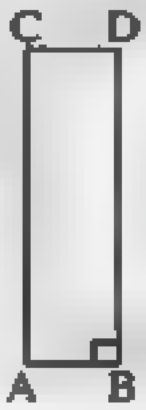


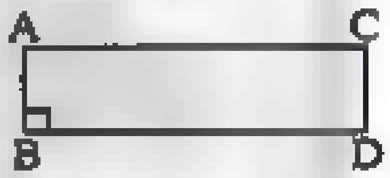
(a) 12 (b) 18 (c) 48 (d) 24

(5) The image of the opposite figure by rotation about C with angle of measure 90° is ....

(a)


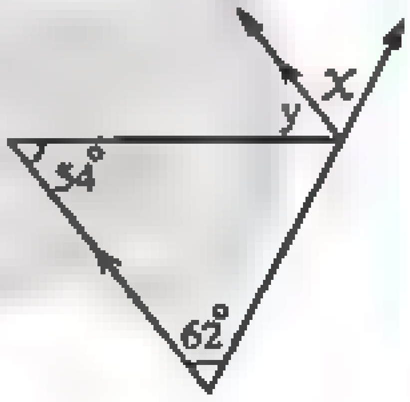
(b)


(c)


(d)


2 Complete the following :

(1) In the opposite figure : $x + y =$ ....



(2) The image of the point (5 , – 3) by rotation about the origin point with an angle of measure 90° is ....

(3) The measure of the interior angle of the regular hexagon = ...

(4) Each two opposite sides of a parallelogram are .... , and ....

(5) A square is rectangle in which the two diagonals are ....

3 [a] Using the square lattice draw Δ ABC in which :

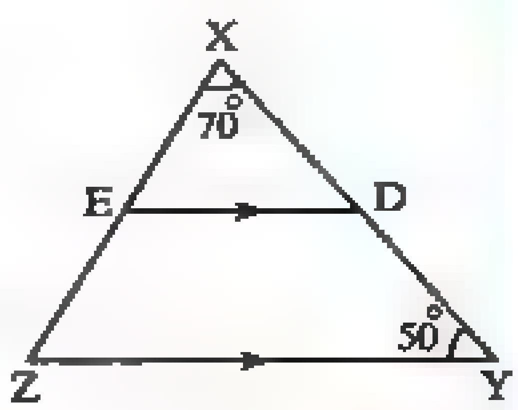
A (5 , – 2) , B (2 , 2) and C (1 , – 3) then find its image by translation (– 3 , 2)

[b] In the opposite figure :

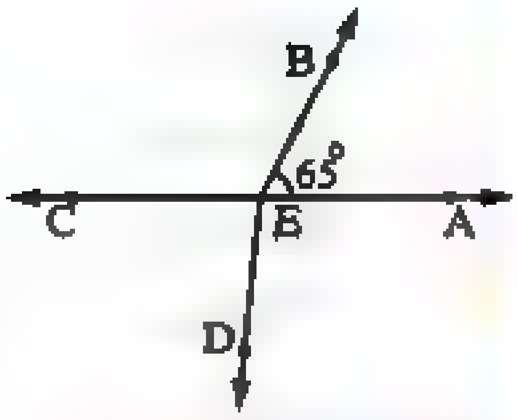
Δ XYZ in which m (X) = 70° ,

m (Y) = 50° , $\overline{ED}$ // $\overline{ZY}$

Find : m (∠ XED)



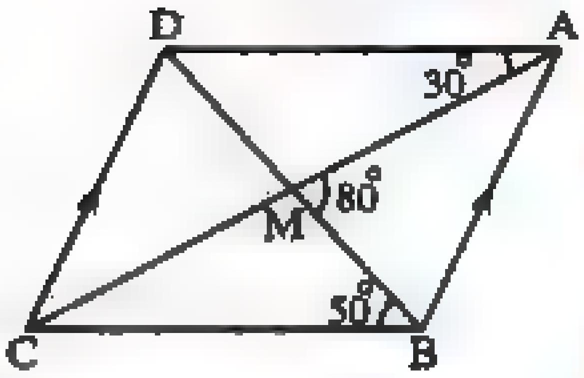
4 [a] $\overleftrightarrow{AC} \cap \overrightarrow{ED} \cap \overrightarrow{ED} = \{E\}$



(1) **Find :** m (∠ BEC)

(2) If m (CED) = 85° are the points B , E and D on the same straight line ?

[b] In the opposite figure :



$\overline{AB} // \overline{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$,

m (∠ DAC) = 30° , m (∠ DBC) = 50° ,

and m (∠ AMB) = 80°

Find : m (∠ MCB) then prove that ABCD is a parallelogram.

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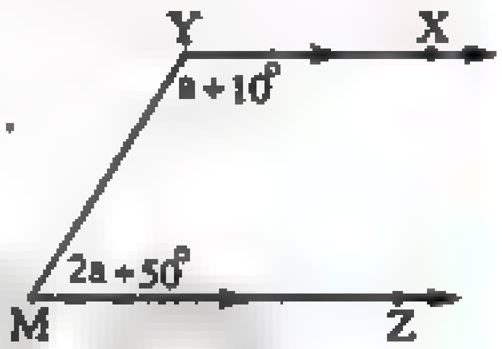
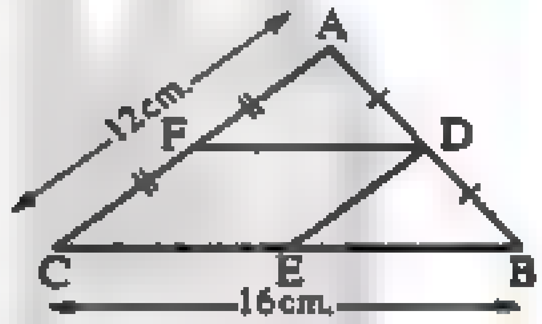
5 [a] **In the opposite figure :**



If $\overline{XY} // \overline{MZ}$

Find the value of a

[b] In the opposite figure :



ABC is a triangle in which D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ respectively , BC = 16 cm. , AC = 12 cm.

Find the perimeter of the quadrilateral DECF with proof.

Additional question

1 **Complete the following :**

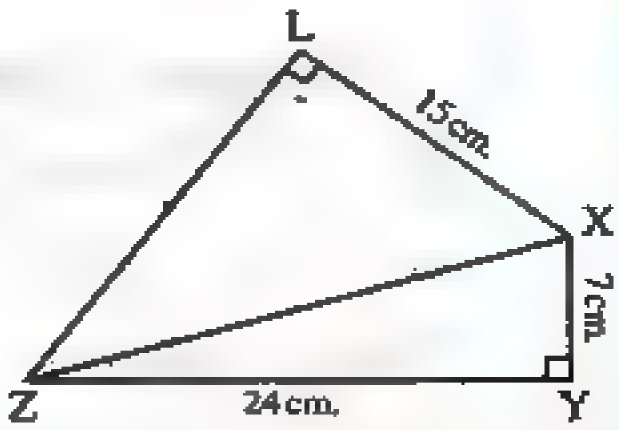
(1) The length of the diagonal of the rectangle whose dimensions are 8 cm. and 6 cm. equals ....

(2) **In the opposite figure :**

The image of $\overline{AB}$ by reflection in M is ....

---

2 **In the opposite figure :**



m (∠ XYZ) = m (∠ XLZ) = 90°

, XY = 7 cm. , YZ = 24 cm.

, and XL = 15 cm.

Find the length of : $\overline{XZ}$, $\overline{LZ}$

4 Giza Governorate — Boulak Educational Zone

Queen International School

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) The image of the point (2 , – 3) by reflection in X-axis is ………

(a) (1 , 2) (b) (2 , 3) (c) (– 3 , 2) (d) (– 2 , – 3)

(2) The image of the point (– 2 , 5) by rotation about the origin with an angle of measure 90° is ………

(a) (– 2 , 5) (b) (– 2 , – 5) (c) (– 5 , – 2) (d) (2 , – 5)

(3) The image of the point (– 1 , – 2) by translation of magnitude 3 units in the positive direction of X-axis is ………

(a) (2 , – 2) (b) (– 1 , 1) (c) (3 , 5) (d) (2 , – 4)

(4) The measure of the interior angle of the regular hexagon = ………

(a) 102° (b) 180° (c) 120° (d) 360°

(5) The identity rotation about any point is with an angle of measure ………

(a) 90° (b) 180° (c) 270° (d) 360°

2 Complete each of the following :

(1) The point (5 , – 3) is the image of the point (3 , 2) by translation ………

(2) If the measure of an angle in a triangle equals the sum of measures of the two other angles , then the triangle is ………

(3) The line segment which joins two midpoints of two sides in a triangle ………

(4) The square is a ……… one of its angles is right.

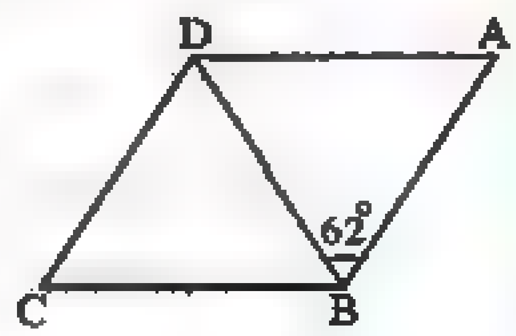
(5) The polygon which has a reflex angle is called ………

3 [a] In the opposite figure :

ABCD is a rhombus , $\overline{BD}$ is a diagonal ,

m (∠ ABD) = 62°

Find by proof : m (∠ BAD)



[b] Find the number of sides of the regular polygon if the measure of one of its interior angles is 135°

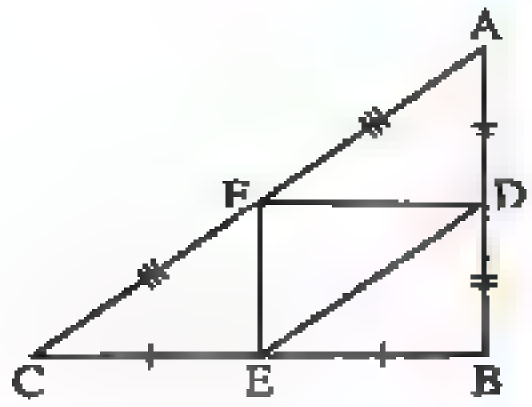
4 **[a] In the opposite figure :**

ABC is a triangle in which : AB = 6 cm.

BC = 8 cm. , AC = 10 cm. and D , E , E

are the midpoints of the sides

Calculate the perimeter of the Δ DEF

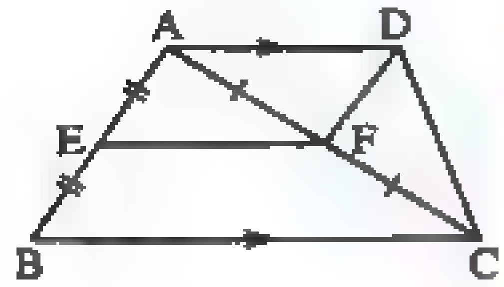


[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, BC = 2 AD , E is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{AC}$

Prove that : AEFD is a parallelogram.



5 On a square lattice draw Δ ABC where A (4 , 1) , B (1 , 1) and C (4 , 4) , then **find :**

(1) The image of Δ ABC by rotation about O with an angle of measure 90°

(2) The image of Δ ABC by reflection in the *X*-axis.

Additional question

1 **Choose the correct answer from those ones :**

(1) The point whose image by reflection in the origin point is itself is ……

(a) (0 , 1) (b) (1 , 0) (c) (0 , 0) (d) (– 1 , 0)

(2) ABC is a right-angled triangle at B , if AB = 3 cm. and AC = 5 cm.

, then BC = ……… cm.

(a) 16 (b) 5 (c) 3 (d) 4

2 **In the opposite figure :**

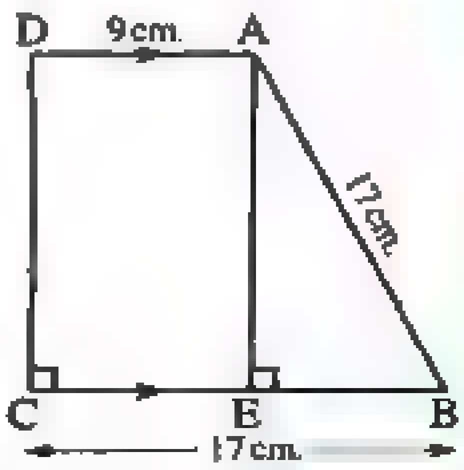
$\overline{AD}$ // $\overline{BC}$, m (∠ DCB) = 90°

, $\overline{AE}$ ⊥ $\overline{BC}$, AB = BC = 17 cm.

and AD = 9 cm.

(1) **Find the length of :** $\overline{DC}$

(2) **Calculate the area of the trapezium :** ABCD



5 Alexandria Governorate — East Educational Zone

Al-Thghr E.L.S. for boys

Answer the following questions :

1 Choose the correct answer :

(1) The point (3 , 4) is the image of the point (1 , – 2) by translation ………

(a) (– 2 , – 6) (b) (2 , 6) (c) (– 2 , 6) (d) (2 , – 6)

(2) The image of the (2 , – 1) by reflection in X-axis is ………

(a) (2 , – 1) (b) (– 2 , 1) (c) (– 2 , – 1) (d) (2 , 1)

(3) The sum of measures of interior angles of a pentagon = ………

(a) 108° (b) 540° (c) 360° (d) 180°

(4) In triangle ABC if m (∠ X) + m (∠ Y) < m (∠ Z) then (∠ Z) is ……… angle.

(a) acute (b) obtuse (c) right (d) straight

(5) The measure of one exterior angle of a regular hexagon = ………

(a) 30° (b) 60° (c) 90° (d) 120°

2 Complete each of the following to get correct statement :

(1) The rectangle is a ……… with a right angle.

(2) The image of point (– 1 , 4) under rotation with an angle of measure 180° about the origin is ………

(3) The line segment joining two midpoints of two sides of a triangle is ……… to the third side.

(4) The sum of measures of the accumulative angles at a point is ………°

(5) If ABCD is a parallelogram , m (∠ A) + m (∠ C) = 130° , then m (∠ B) = ………°

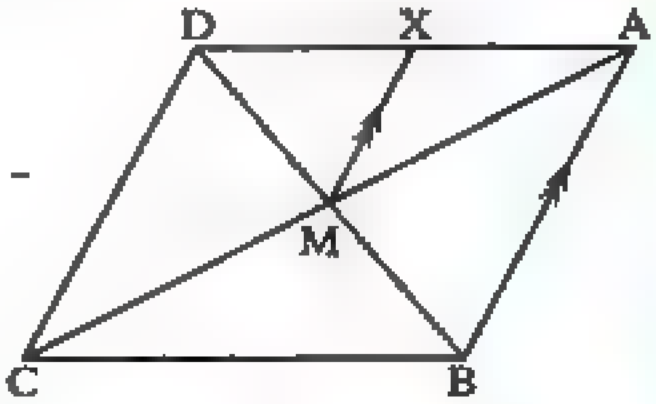
3 [a] Draw Δ ABC where A (– 1 , 2) , B (1 , 3) and C (1 , – 2) then draw its image by reflection in y-axis.

[b] In the opposite figure :

ABCD is a parallelogram ,

$\overline{MX} \parallel \overline{AB}$

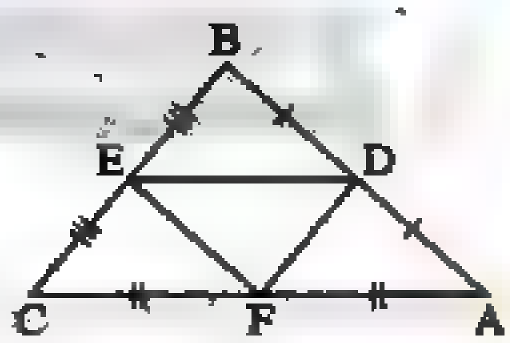
Prove that : X is the midpoint of $\overline{AD}$



4 **[a] In the opposite figure :**

AB = 8 cm. , BC = 7 cm. and CA = 10 cm.

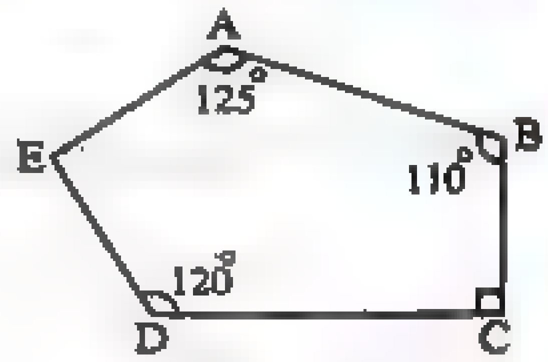
If D , E and F are midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively. Find with proof perimeter of Δ DEF



[b] Find the number of sides of a regular polygon if the measure of one of its interior angles is 135°

5 **[a] In the opposite figure :**

m (∠ E) = ...

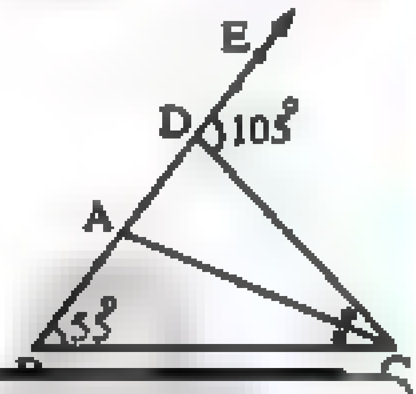


[b] In the opposite figure :

m (∠ EDC) = 105° , m (∠ B) = 55°

, $\overrightarrow{AC}$ bisects ∠ DCB

Find with proof : m (∠ CAD)



Additional question

1 **Complete the following :**

(1) If Δ XYZ is a right-angled triangle at Z , then $(XZ)^2$ = ...

(2) The image of the point (– 4 , 2) by reflection in the origin point is ....

(3) The number of axes of symmetry of the isosceles triangle is ....

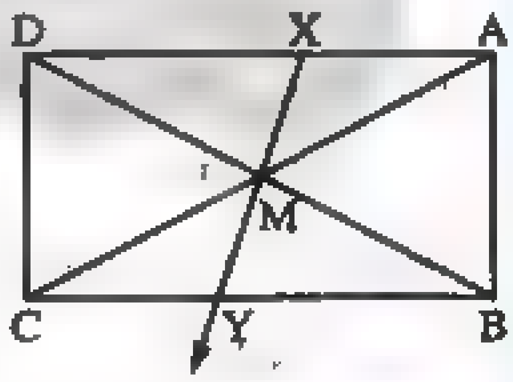
2 **In the opposite figure :**

ABCD is a rectangle , M is the point of intersection of its diagonals , X ∈ $\overline{AD}$ and $\overleftrightarrow{XM}$ ∩ $\overline{BC}$ = {Y}

Prove that :

(1) Y is the reflected image of X in M

(2) The figure AXCY is a parallelogram.



6 Alexandria Governorate — El Nasr Boys School

Preparatory Department

Answer the following questions :

1 **Choose the correct answer :**

(1) In Δ ABC if m (∠ A) = m (∠ B) = 50° , then m (∠ C) = ...

(a) 40° (b) 60° (c) 80° (d) 100°

(2) The image of the point $(-3 , 5)$ by reflection in y-axis is ....

(a) $(3 , -5)$ (b) $(3 , 5)$ (c) $(-3 , -5)$ (d) $(-3 , 5)$

(3) In Δ ABC , if m ($\angle$ B) = m ($\angle$ A) + m ($\angle$ C) , then $\angle$ B is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

(4) The measure of each interior angle of a regular pentagon = ...

(a) 72° (b) 80° (c) 120° (d) 108°

(5) The image of point $(-2 , 3)$ by reflection on the X-axis is ....

(a) $(2 , 3)$ (b) $(-2 , -3)$ (c) $(-3 , -2)$ (d) $(3 , 2)$

2 Complete :

(1) The quadrilateral in which only two sides are parallel is called ....

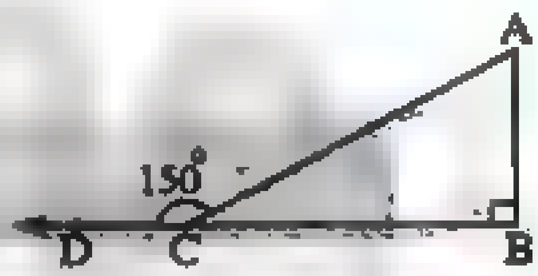
(2) The image of point $(4 , 0)$ by translation $(1 , -3)$ is ....

(3) In a parallelogram , every two opposite sides are .... and ....

(4) **In the opposite figure :**

$\overline{AB} \perp \overrightarrow{BC}$, D $\in \overrightarrow{BC}$

, then m ($\angle$ A) = ...



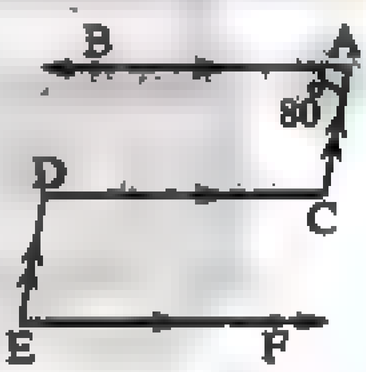
(5) The rotation about the origin point with an angle of measure .... called the identify rotation.

3 [a] In the opposite figure :

$\overline{AB} \parallel \overline{DC} \parallel \overline{EF}$, m ($\angle$ A) = 80°

$\overline{AC} \parallel \overline{DE}$, then

m ($\angle$ E) = ...°

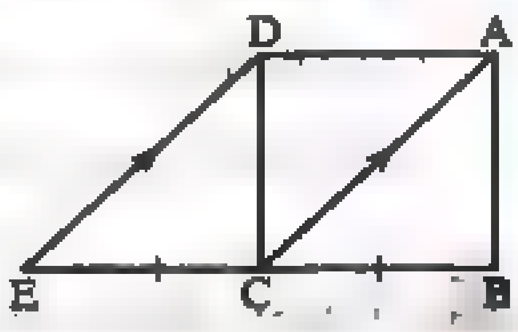


[b] ABCD is a square

E $\in \overrightarrow{BC}$, and $\overline{AC} \parallel \overline{DE}$

(1) **Prove that :** ACED is a parallelogram.

(2) **Find :** m ($\angle$ ACE)



4 [a] Calculate the sum of measures of the interior angles of hexagon.

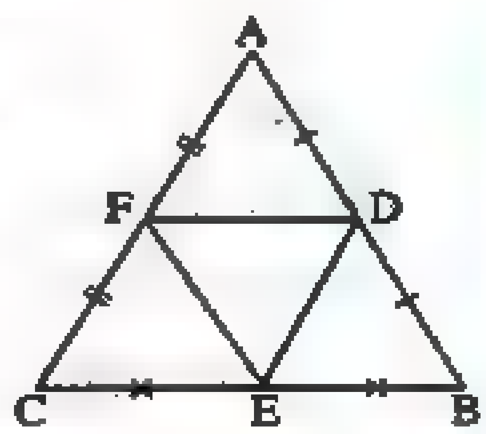
[b] In the opposite figure :

AB = 5 cm. , BC = 8 cm. , CA = 7 cm.

D , E and F are the midpoints of

$\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively

Calculate the perimeter of Δ DEF



5 On the lattice draw ΔABC , where A (4 , 3) , B (2 , 0) , C (0 , 2) , then draw its image $\Delta \acute{A}\acute{B}\acute{C}$ by rotation about the origin with an angle of measure 180°

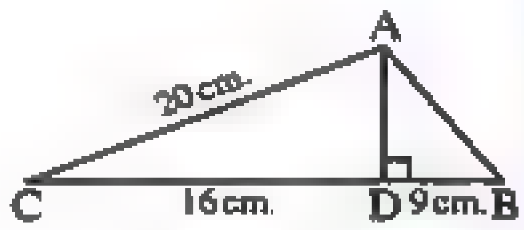
Additional question

1 **Complete the following :**

(1) The image of the point (2 , – 7) by reflection in the origin point is ·········

(2) If the axis of symmetry of ΔABC , where A (0 , 3) and B (2 , – 1) is the y-axis , then C (········· , ·········)

2 **In the opposite figure :**



$\overline{AD} \perp \overline{BC}$, BD = 9 cm. ,

DC = 16 cm. , and AC = 20 cm.

Find the length of : $\overline{AD}$, $\overline{AB}$

7 El-Kalyoubia Governorate — Benha Educational Zone

Math inspection

Answer the following questions :

1 **Complete :**

(1) The sum of measures of the interior angles of a convex polygon of n sides equals ·········

(2) A quadrilateral in which there are only two sides are parallel is called ·········

(3) The ray drawn from the midpoint of a side of a triangle parallel to another side ·········

(4) The image of the point (4 , 3) by the translation $(x , y) \longrightarrow (x + 1 , y - 2)$ is ·········

(5) The image of the point (0 , – 3) by reflection in x-axis is ·········

2 **Choose the correct answer :**

(1) The two diagonals of the rhombus are ·········

(a) perpendicular.
(b) parallel.
(c) equal in length.
(d) perpendicular and equals in length.

(2) The length of line segment which joins the two midpoints of two sides in a triangle equal to ········· of the third side.

(a) the length
(b) $\frac{1}{4}$ the length
(c) twice the length
(d) $\frac{1}{2}$ the length

(3) In the triangle ABC if m (∠ A) = m (∠ B) = 50° , then m (∠ C) = ...

(a) 130° (b) 100° (c) 80° (d) 50°

(4) A rhombus with perimeter 40 cm. , then its side length ....

(a) 10 cm. (b) 5 cm. (c) 4 cm. (d) 8 cm.

(5) The image of the point (2 , -3) by rotation about the origin point with an angle of measure 90° is ....

(a) (−2 , 3) (b) (−2 , −3) (c) (2 , 3) (d) (3 , 2)

3 **On an orthogonal Cartesian coordinates plane :**

Draw Δ LMN where : L (2 , 4) , M (4 , 2) and N (3 , 1) then map its image :

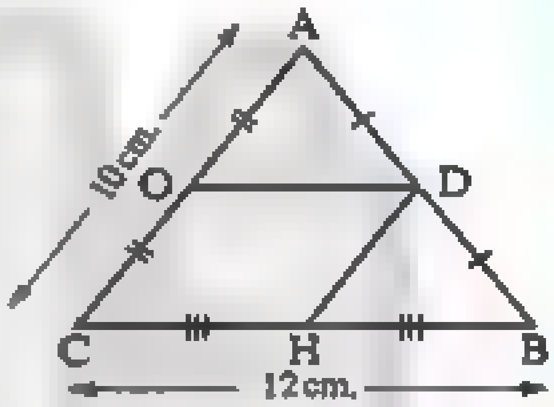
(1) By reflection in y-axis

(2) By rotation about the origin point with an angle of measure 180°

4 **[a] In the opposite figure :**

ABC is a triangle in which D , H and O are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively , BC = 12 cm. and AC = 10 cm.

Find the perimeter of the figure DHCO



[b] Find the number of sides of the convex regular polygon if the measure of one of its interior angles is 108°

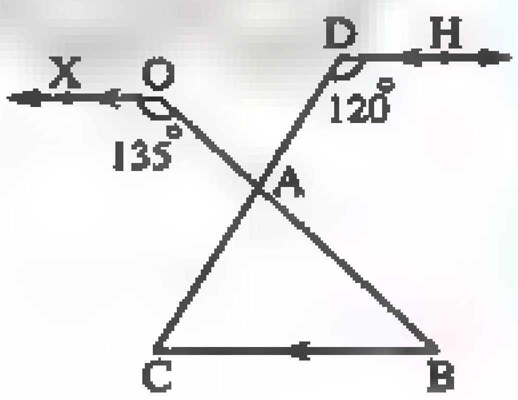
5 **[a] In the opposite figure :**

$\overrightarrow{DH}$ // $\overrightarrow{OX}$ // $\overrightarrow{BC}$,

m (∠ HDC) = 120° and

m (∠ XOB) = 135°

Find the measures of the angles of the triangle ABC



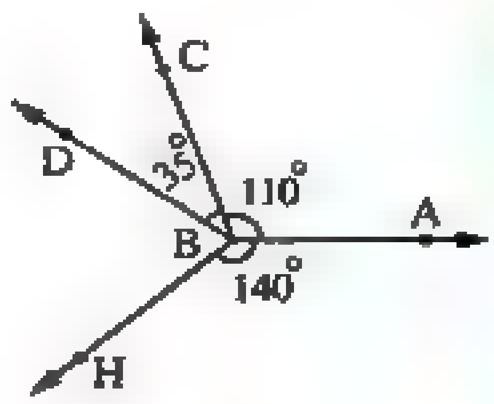
[b] In the opposite figure :

m (∠ ABC) = 110°

, m (∠ CBD) = 35°

and m (∠ ABH) = 140°

Find : m (∠ HBD)



Additional question

1 Complete the following :

(1) The point (– 1 , 9) is the image of the point (1 , – 9) by reflection in ……

(2) If the lengths of the two diagonals of a rhombus are 6 cm. and 8 cm. , then its side length = ………

2 Draw the square ABCD whose side length is 5 cm. , then draw its image by reflection in the point M where M is the point of intersection of its diagonals. What do you observe ?

8 El-Kalyoubia Governorate — Shebin El-Anater

Math Inspection

Answer the following questions :

1 Complete :

(1) The measure of the interior angle of the regular hexagon = ………°

(2) The rhombus which has a right angle is called ………

(3) The image of the point (– 2 , 4) by reflection in X-axis is ………

(4) The image of the point (– 1 , 5) by translation (1 , – 3) is ………

(5) The sum of measures of the interior angles of a triangle = ………°

2 Choose the correct answer :

(1) If two adjacent sides are equal in a parallelogrm , then the figure is ………

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

(2) The point (5 , 3) is the image of the point (– 3 , 5) by rotation about the origin point with an angle of measure ………

(a) 90° (b) – 90° (c) 180° (d) 360°

(3) ABCD is a square m (∠ ACB) = ………

(a) 90° (b) 60° (c) 45° (d) 30°

(4) The sum of measures of the interior angles of the pentagon = ………

(a) 360° (b) 450° (c) 540° (d) 720°

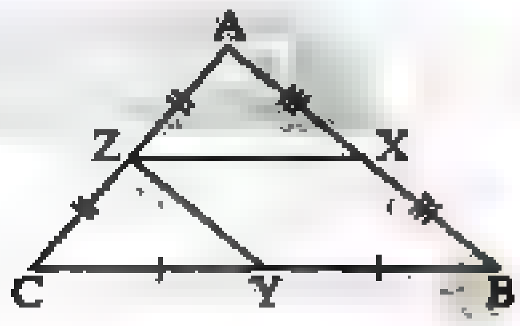
(5) The ray drawn from the midpoint of a side of a triangle parallel to another side ……… the third side.

(a) equals (b) parallel (c) perpendicular to (d) bisects

3 **[a] In the opposite figure :**

Δ ABC in which X , Y and Z are the midpoint of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively , AB = 8 cm. , BC = 10 cm.

Find the perimeter of the figure : XBYZ



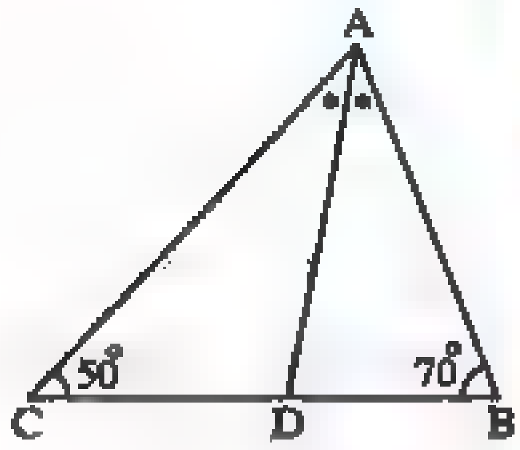
[b] In the opposite figure :

$\overrightarrow{AD}$ bisects $\angle$ BAC ,

m ($\angle$ B) = 70°

and m ($\angle$ C) = 50°

Find : m ($\angle$ ADB)



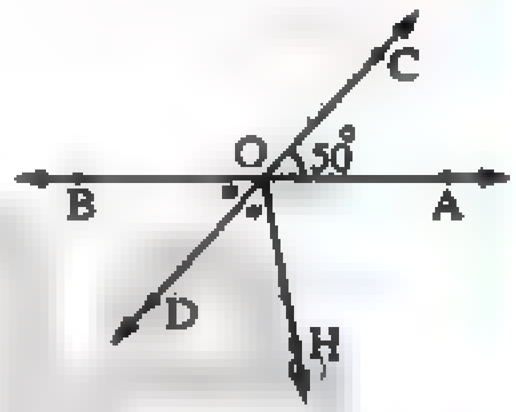
4 **[a] In the opposite figure :**

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{O\}$,

m ($\angle$ AOC) = 50° ,

$\overrightarrow{OD}$ bisects $\angle$ HOB

Find : m ($\angle$ AOH)



[b] In the opposite figure :

m ($\angle$ BAH) = 45° , m ($\angle$ AHB) = 70° ,

m ($\angle$ C) = 115° and m ($\angle$ D) = 65°

Prove that : ABCD is a parallelogram.

5 Draw on an orthogonal lattice Δ ABC where A (4 , 2) , B (2 , 4) and C (1 , – 2) then find the image of Δ ABC by reflection in y-axis.

Additional question

1 **Complete the following :**

(1) In the right-angled triangle , the square of the length of the hypotenuse equals ....

(2) If (2 X , 4) is the image of (X – 9 , – 4) by reflection in the origin point , then X = ...

2 By using geometric instruments , draw the rectangle ABCD , where AB = 3 cm. and BC = 4 cm. locate À as the reflected image of A by reflection in $\overleftrightarrow{CD}$ and locate C̀ as the reflected image of C by reflection in $\overleftrightarrow{AB}$

Prove that : (1) m ($\angle$ C̀AC) = 2 m ($\angle$ CAB)

(2) $\overleftrightarrow{AC̀}$ // $\overleftrightarrow{ÀC}$

9 Dakahlia Governorate — Fakhr Language School

Answer the following questions :

1 Choose the correct answer :

(1) The sum of measures of 5 angles of a polygon of 6 sides is 610°, then the measure of the remaining angle is ....

(a) 60° (b) 180° (c) 110° (d) 120°

(2) The image of (3 , 0) by reflection in X-axis is ....

(a) (– 3 , 0) (b) (3 , 0) (c) (0 , – 3) (d) (0 , 3)

(3) In the parallelogram ABCD, if m (∠ A) + m (∠ C) = 160° , then m (∠ B) = ...

(a) 110° (b) 100° (c) 80° (d) 20°

(4) In the parallelogram, if the diagonals are orthogonal and equal in length , then it is a ....

(a) rhombus. (b) rectangle. (c) trapezium. (d) square.

(5) ABC is a triangle, m (∠ A) = $2X$°, m (∠ C) = X° and m (∠ B) = 3 X° , then ∠ B is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

2 Complete the following :

(1) The length of the line segment joining the midpoints of two sides of a triangle is equal to ....

(2) A rhombus with a right angle is called a ....

(3) In the parallelogram, if two adjacent sides are equal in length, then it becomes a ....

(4) The sum of measures of exterior angles of a triangle equals ....°

(5) The image of the point (– 2 , 3) by reflection into the origin is ....

3 [a] Draw the image of Δ ABC , where A (1 , 3) , B (0 , 5) and C (– 2 , 4) by reflection in the X-axis.

[b] In the opposite figure :

ABCD is a parallelogram ,

m (∠ CAD) = 55° , m (∠ ACD) = 25°

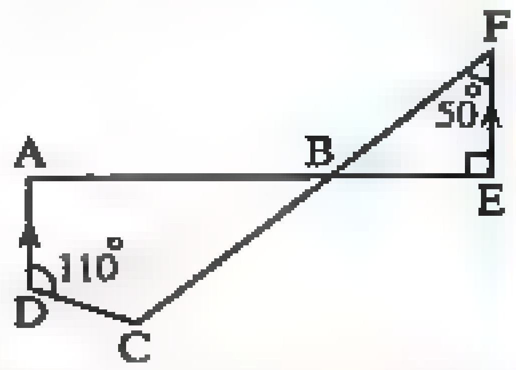
Find with proof : m (∠ B)

4 **[a] In the opposite figure :**

$\overline{AD}$ // $\overline{EF}$, m (∠ E) = 90°,

m (∠ F) = 50° , m (∠ D) = 110°

Find : m (∠ C)



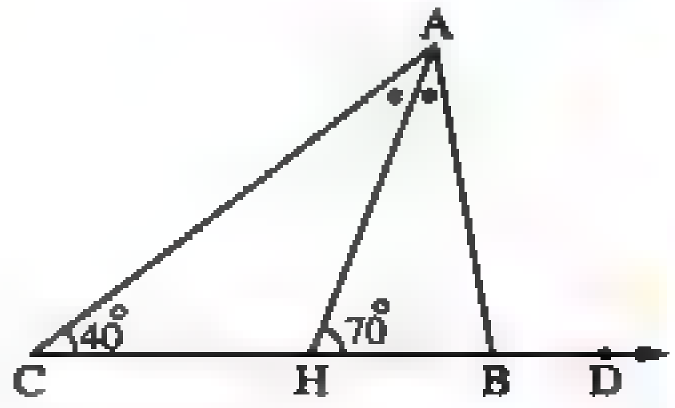
[b] In the opposite figure :

$\overrightarrow{AH}$ bisects $\angle$ BAC , D $\in \overrightarrow{CB}$,

m ($\angle$ AHB) = 70° , and m ($\angle$ ACH) = 40°

Find : (1) m ($\angle$ BAC)

(2) m ($\angle$ ABD)

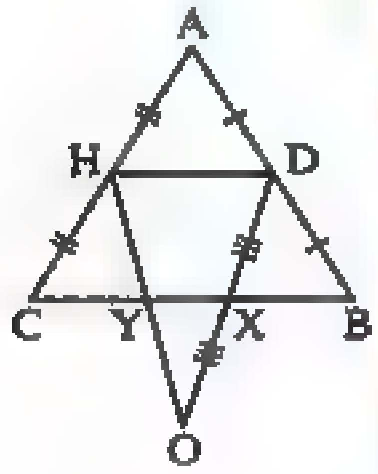


5 In the opposite figure :

D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$ and X is the midpoint of $\overline{DO}$

Prove that : (1) Y is the midpoint of $\overline{HO}$

(2) If : BC = 12 cm. , then find XY



Additional question

1 Complete the following :

(1) The image of the point (4 , – 1) by reflection in the origin point is ………

(2) The number of axes of symmetry of the equilateral triangle is ………

2 In the opposite figure :

ABC is a triangle in which

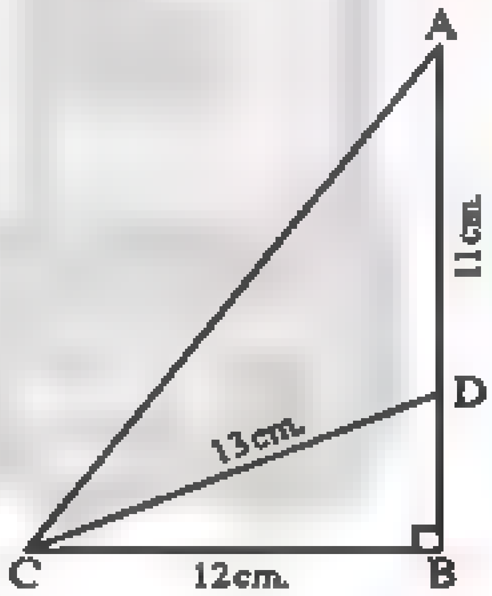
m ($\angle$ B) = 90° ,

D $\in \overline{AB}$, where AD = 11 cm.

, if BC = 12 cm.

and DC = 13 cm.

Find the length of : $\overline{BD}$ and $\overline{AC}$



10 Kafr El-Sheik Governorate — Kallin Administration Zone

Kallin Exp. Lang. School

Answer the following questions :

1 Choose the correct answer :

(1) The sum of the measures of the accumulative angles at a point is ………

(a) 90° (b) 180° (c) 270° (d) 360°

(2) (–1 , – 3) is the image of (3 , – 1) by rotation about the origin with an angle of measure ………

(a) 90° (b) – 90° (c) 180° (d) – 180°

(3) ABCD is a parallelogram in which m (∠ B) + m (∠ D) = 130° , then m (∠ A) = ………

(a) 50° (b) 65° (c) 115° (d) 150°

(4) The measure of the interior angle of the regular hexagon = ………

(a) 90° (b) 135° (c) 120° (d) 60°

(5) A parallelogram which its two diagonals are equal is = ………

(a) rectangle. (b) square. (c) a , b together. (d) rhombus.

2 **Complete the following :**

(1) Each two opposite angles of a parallelogram ………

(2) The two diagonals are perpendicular in ……… and ………

(3) If the polygon does not have reflex angle then it called ……… polygon.

(4) The point (2 , – 1) is the image of the point (– 1 , 2) by translation (……… , ………).

(5) The ray drawn from the midpoint of a side of a triangle parallel to another side ……….

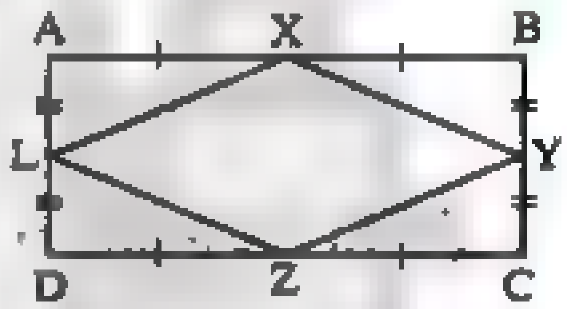
3 [a] Find the measure of each interior angle of regular octagon (with 8 sides)

[b] In the opposite figure :

ABCD is a rectangle and X , Y , Z and L are the midpoint of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively prove that :

(1) XYZL is a rhombus.

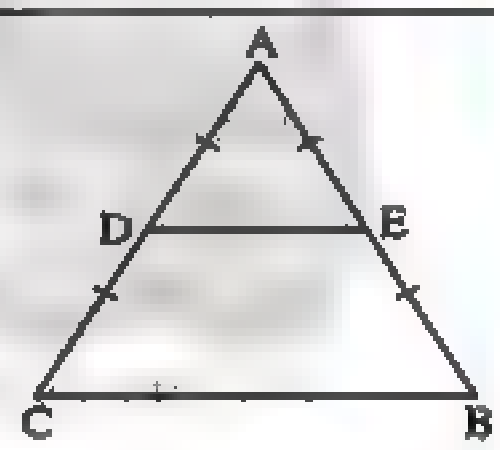
(2) The perimeter of the rhombus = 2 BD



4 **[a] In the opposite figure :**

ABC is an equilateral triangle of side length 12 cm.

, then find the perimeter of ADE



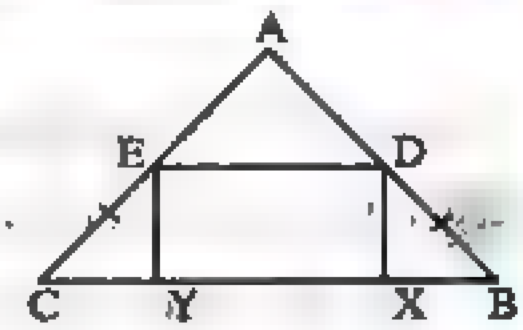
[b] On lattice draw ABC where A (4 , 0) , B (0 , 3) and C (2 , 5)

Find the image of Δ ABC by rotation around the origin point with an angle of measure 180°

5 **[a] In the opposite figure :**

EC = BD and DXYE is a rectangle

Prove that : m (∠ ADE) = m (∠ AED)

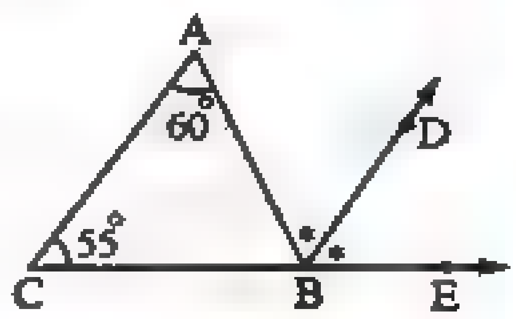


[b] In the opposite figure :

ABC is a triangle in which m (∠ A) = 60°

m (∠ C) = 55° , and $\overrightarrow{BD}$ bisects ∠ ABE , E ∈ $\overrightarrow{CB}$

Find : m (∠ DBE)



Additional question

1 Complete the following :

(1) The image of the point (a , – b) by reflection in the origin point is ·······

(2) In the right-angled triangle , the square of the length of the hypotenuse is equal to ·······

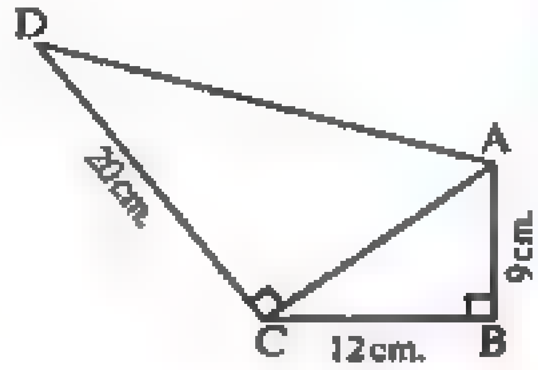
2 In the opposite figure :

m (∠ B) = m (∠ ACD) = 90°

, AB = 9 cm. , BC = 12 cm.

, CD = 20 cm.

Find the perimeter of the figure : ABCD



11 El-Gharbia Governorate West Mahalla El Kobra Admin.

Salah-Deen Exp. Language School

Answer the following questions :

1 Choose the correct answer :

(1) Á (2 , – 3) is the image of A by translation (X + 1 , y + 1) , then A is ·······

(a) (1 , 0) (b) (1 , – 4) (c) (2 , – 2) (d) (3 , – 2)

(2) The point (3 , 5) is the image of the point (5 , – 3) by rotation about origin point with an angle of measure ·······

(a) 90° (b) 180° (c) –90° (d) 360°

(3) ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 110° , then m (∠ B) = ·······

(a) 50° (b) 125° (c) 130° (d) 45°

(4) The image of the point (5 , 0) by reflection in X-axis is ·······

(a) (5 , 0) (b) (0 , 5) (c) (0 , – 5) (d) (– 5 , 0)

(5) The sum of measures of interior angles of pentagon is ·······

(a) 540° (b) 720° (c) 900° (d) 360°

2 Complete :

(1) The line segment joining two midpoints of two sides of triangle is ······· and ·······.

(2) ABCD is a parallelogram in which m (∠ A) = 130° , then m (∠ B) = ······· °

(3) The image of point (3 , 1) by reflection in y-axis is ·······

(4) The rhombus whose diagonals are equal in length is ·······

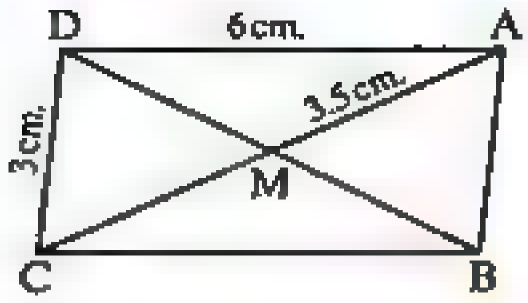
(5) The image of the point (3 , – 5) by rotation of angle 360° about origin point is ·······

3 **[a] In the opposite figure :**

ABCD is a parallelogram,

AD = 6 cm. , DC = 3 cm. , AM = 3.5 cm.

Find perimeter of triangle : ABC

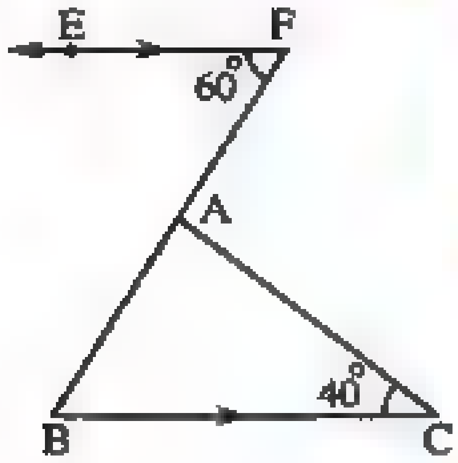


[b] In the opposite figure :

$\overline{FE} \parallel \overline{BC}$, m (∠ F) = 60° ,

m (∠ C) = 40°

Find : m (∠ BAC)



4 [a] ABCDE is a pentagon in which m (∠ A) = 140° ,

m (∠ B) = 130° , m (∠ C) = 90° , m (∠ D) = 110°

Find : m (∠ E)

[b] Draw the equilateral triangle ABC of side length 5 cm., find the image of triangle ABC by rotation 60° about C

5 [a] Draw on square lattice the triangle ABC where : A (1 , 3) , B (3 , 5) and C (5 , 0)

Find the image of the triangle ABC by reflection in y-axis.

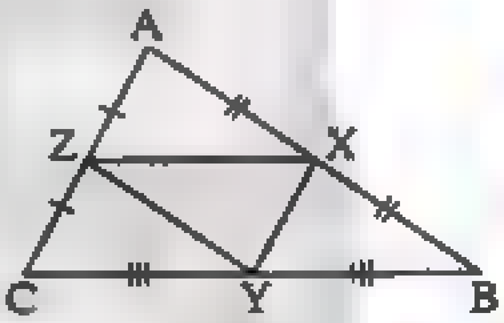
[b] In the opposite figure :

ABC is a triangle in which , X , Y , Z are midpoints

of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$,

AB = 7 cm. , BC = 9 cm. , AC = 6 cm.

Find the perimeter of triangle : XYZ



Additional question

1 **Complete the following :**

(1) Δ XYZ is a right-angled triangle at Y , if XY = 4 cm. and YZ = 3 cm. , then XZ = ...

(2) If the reflection in a straight line maps the figure to itself , then this straight line is called ....

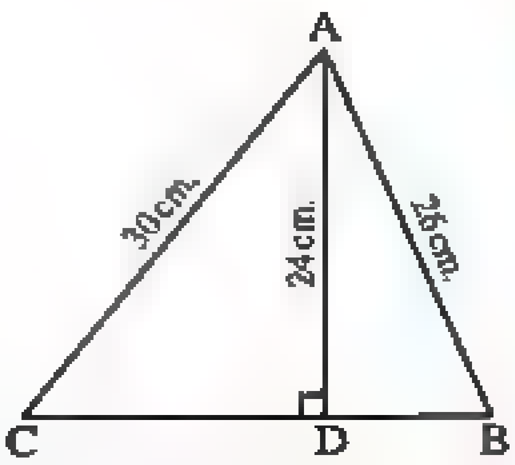
2 **In the opposite figure :**

ABC is a triangle in which $\overline{AD} \perp \overline{BC}$,

if AD = 24 cm. , AB = 26 cm. ,

AC = 30 cm.

Calculate the length of : $\overline{BC}$



12 Damietta Governorate — Damietta Lang. School

Answer the following questions :

1 Choose the correct answer :

(1) The sum of measures of the interior angles of a triangle = ……

(a) 180° (b) 90° (c) 270° (d) 360°

(2) ABCD is parallelogram, in which m (∠ B) = 150°, then m (∠ A) = ……

(a) 30° (b) 75° (c) 105° (d) 210°

(3) The image of the point (5 , – 4) by reflection in the y-axis is the point ……

(a) (5 , 4) (b) (– 5 , – 4) (c) (4 , 5) (d) (– 4 , 5)

(4) The parallelogram whose diagonals are perpendicular and not equal is a ……

(a) square. (b) rectangle. (c) trapezium. (d) rhombus.

(5) **In the opposite figure :**

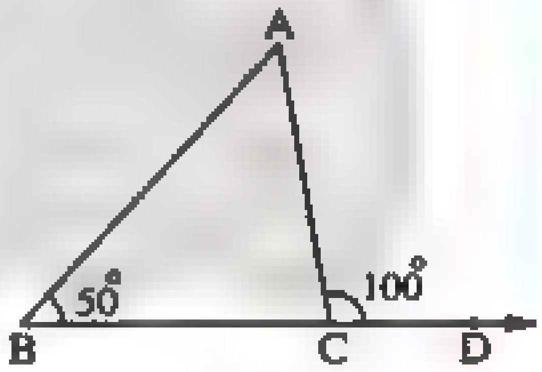
If B is the midpoint of $\overline{AC}$, then the image of $\overline{AC}$ by rotation about B with an angle of measure 180° is ………

(a) {B} (b) $\overline{AB}$ (c) $\overline{CA}$ (d) $\overline{CB}$

2 Complete the following :

(1) If two straight lines intersect , then each two vertically opposite angles are ………

(2) **In the opposite figure :**



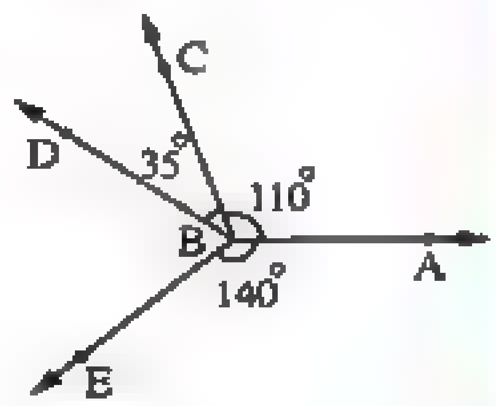
m (∠ CAB) = ………°

(3) The ray drawn from the midpoint of a side of triangle parallel to another side ……… the third side.

(4) The measure of the interior angle of the regular hexagon = ………

(5) The image of the point A (2 , – 1) is the point A` (5 , 1) by translation ………

3 [a] In the opposite figure :

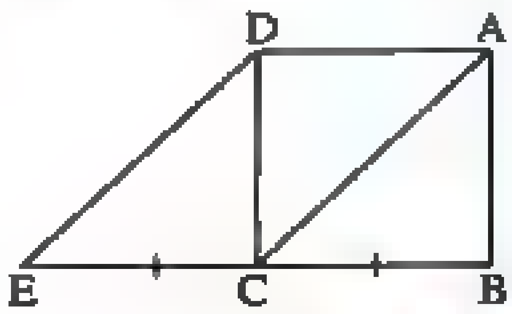


m (∠ ABC) = 110°, m (∠ CBD) = 35° ,

and m (∠ ABE) = 140°

Find by proof : m (∠ EBD)

[b] In the opposite figure :



ABCD is a square , E ∈ $\overrightarrow{BC}$, BC = CE

(1) **Prove that :** The figure CED is a parallelogram.

(2) **Find :** m (∠ E)

13 Fayoum Governorate — Fayoum West Education Adm.

Azza Zidan E.L.S

Answer the following questions :

1 Choose the correct answer :

(1) The sum of measures of accumulative angles at a point is ………

(a) 360° (b) 180° (c) 90° (d) 270°

(2) The image of (– 2 , 3) by rotation about the origin point with an angle of measure 180° is ………

(a) (– 3 , 2) (b) (3 , – 2) (c) (– 3 , – 2) (d) (2 , – 3)

(3) The sum of measures of the interior angles of an octagon = ………

(a) 540° (b) 720° (c) 1080° (d) 900°

(4) If ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = ………

(a) 70° (b) 180° (c) 90° (d) 110°

(5) The image of (1 , – 2) by reflection in X-axis is ………

(a) (1 , 2) (b) (– 1 , – 2) (c) (2 , – 1) (d) (– 1 , 2)

2 Complete each of the following :

(1) The line segment which joins two midpoints of two sides of a triangle is ………

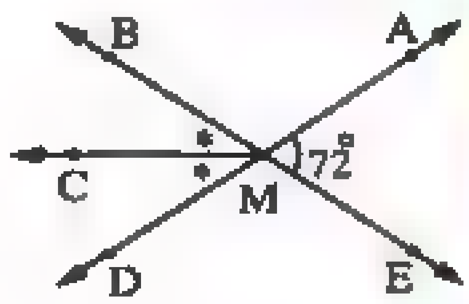
(2) The image of the point (3 , 1) by translation (X – 2 , y – 1) is ………

(3) If the two diagonals of a parallelogram are equal in length and not perpendicular then it is called ………

(4) The measure of each angle of a regular pentagon = ………°

(5) The sum of measures of the interior angles of the triangle = ………°

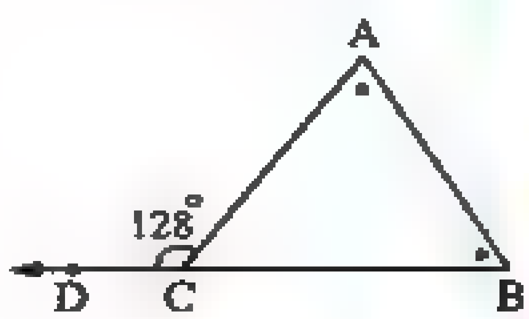
3 [a] In the opposite figure :



$\overleftrightarrow{DA} \cap \overleftrightarrow{EB} = \{M\}$, $\overrightarrow{MC}$ is a bisector of (∠ BMD) and m (∠ AME) = 72°

Find : m (∠ CMD)

[b] In the opposite figure :

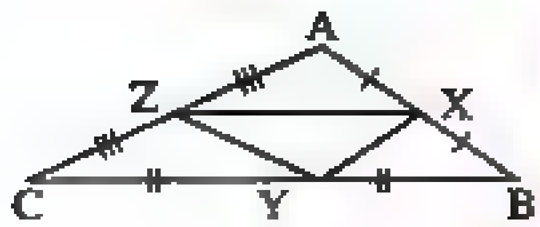


D ∈ $\overrightarrow{BC}$, m (∠ ABC) = m (∠ BAC) ,

and m (∠ ACD) = 128°

Find : m (∠ ABC)

4 **[a] In the opposite figure :**



XYZ is a triangle in which XZ = 4 cm. ,

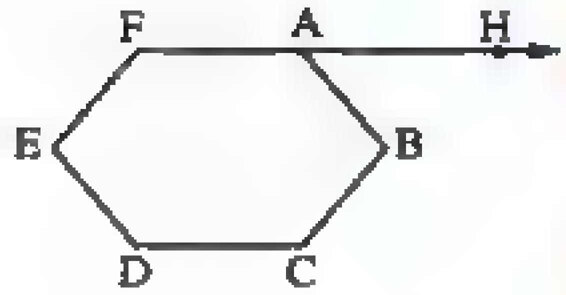
XY = 2.5 cm. and ZY = 3 cm. , where

X , Y and Z are the midpoints of $\overline{AB}$,

$\overline{BC}$ and $\overline{CA}$

Find the perimeter of : Δ ABC

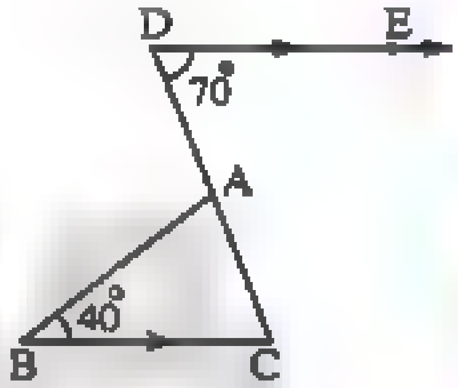
[b] In the opposite figure :



ABCDEF is a regular hexagon , H $\in \overrightarrow{FA}$

Find : m ($\angle$ HAB)

5 **[a] In the opposite figure :**



$\overrightarrow{DE}$ // $\overrightarrow{BC}$

m ($\angle$ D) = 70° , and m ($\angle$ B) = 40°

Find : m ($\angle$ DAB)

[b] In an orthogonal coordinates system with two dimensions :

Draw Δ ABC in which A (– 2 , 4) , B (5 , 0) and C (3 , – 3) , then find the image of Δ ABC by reflection in y-axis.

Additional question

1 **Choose the correct answer :**

(1) The point (2 , – 4) is the image of the point .... by reflection in the origin point.

(a) (2 , 4)　(b) (– 2 , 4)　(c) (– 2 , – 4)　(d) (2 , – 4)

(2) **In the opposite figure :**

If the area of Δ ABC is 24 cm^2.

and AB = 6 cm. ,

then AC = ... cm.

(a) 10　(b) 100　(c) 64　(d) 8

2 Using the square lattice , draw the image of Δ ABC by reflection in the origin point where A (5 , 1) , B (2 , 1) and C (5 , 3)

14 Beni Suef Governorate — Ministry of Education Zone

Saad Bn ABe Wakas School

Answer the following questions :

1 Choose the correct answer :

(1) The sum of measures of the interior angles of a heptagon = ...

(a) 540° (b) 720° (c) 900° (d) 1080°

(2) ABCD is a parallelogram in which : m (∠ B) + m (∠ D) = 150°

, then m (∠ A) = ...

(a) 105° (b) 210° (c) 75° (d) 360°

(3) The point (5 , – 3) is the image of the point (3 , 5) by rotation about the origin point with an angle of measure ....

(a) 90° (b) – 90° (c) 180° (d) 360°

(4) Δ XYZ in which E and F and two midpoints of $\overline{XY}$ and $\overline{XZ}$ respectively , if YZ = 10 cm. , then the length of $\overline{EF}$ = ... cm.

(a) 5 (b) 10 (c) 6 (d) 8

(5) Á (2 , – 3) is the image of A by translation (X + 1 , Y + 1) , then A is ....

(a) (1 , 0) (b) (1 , – 4) (c) (2 , – 2) (d) (3 , – 2)

2 Complete :

(1) If two straight lines intersect, then the measure of each two vertically opposite angles ....

(2) The image of point (0 , 4) by reflection in the y-axis is ....

(3) If two opposite sides in the quadrilateral are parallel , then it is called ....

(4) The two diagonals are perpendicular in .... and ....

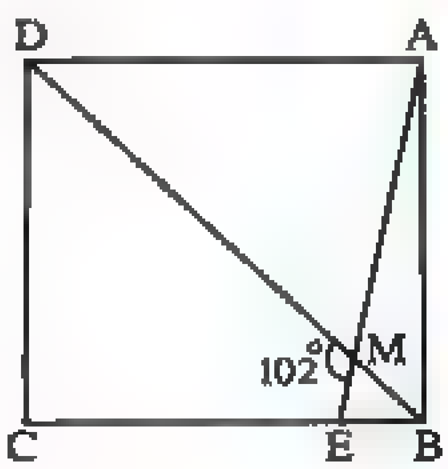
(5) The sum of measures of accumulative angles at a point = ...

3 [a] In the opposite figure :

$\overline{BD}$ is a diagonal in the square ABCD ,

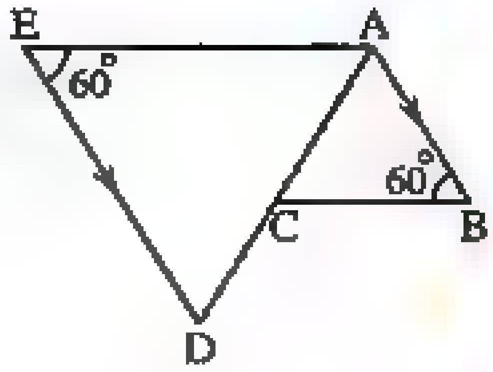
m (∠ DME) = 102°

Find with proof : m (∠ MEC)



[b] Find the number of sides of the regular polygon if the measure of one of its interior angles is 135°

4 **[a] In the opposite figure :**

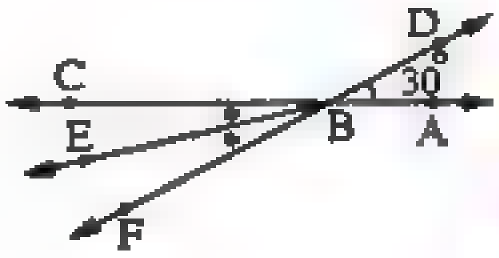


$\overline{AB} \parallel \overline{ED}$ and

, m ($\angle$ ABC) = m ($\angle$ AED) = 60°

Prove that : $\overline{BC} \parallel \overline{AE}$

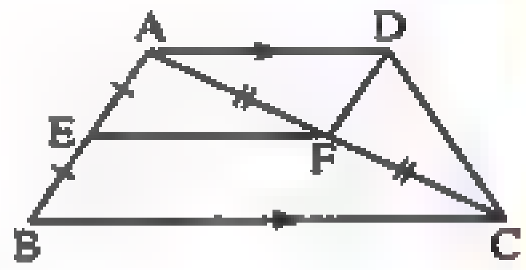
[b] In the opposite figure :



$\overleftrightarrow{AC} \cap \overleftrightarrow{DF} = \{B\}$, m ($\angle$ ABD) = 30° , $\overrightarrow{BE}$ bisects $\angle$ CBF

Find : m ($\angle$ ABE).

5 **[a] In the opposite figure :**



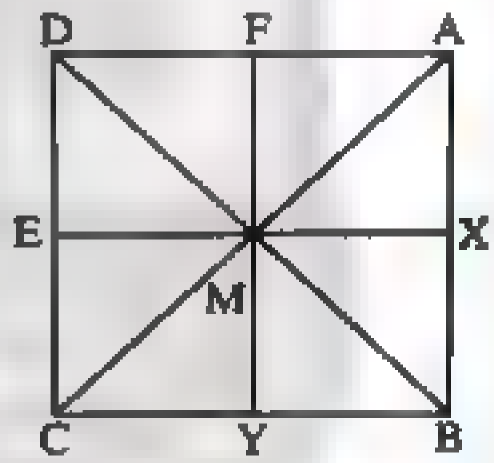
$\overline{AD} \parallel \overline{BC}$, BC = 2 AD

Prove that : AEFD is parallelogram.

[b] Use the lattice to find the image of the triangle LMN by the reflection in X-axis , where L (– 4 , – 1) , M (– 1 , – 3) , N (0 , – 1)

Additional question

1 **In the opposite figure :**



ABCD is a square of side length 6 cm.

M is the point of intersection of its diagonals.

X , Y , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

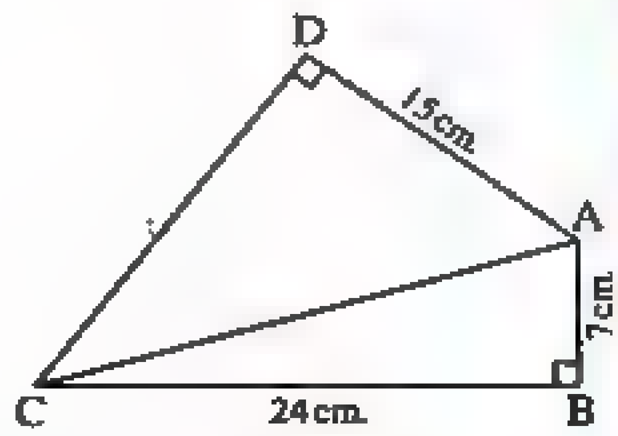
Complete the following :

(1) The image of the point A by reflection in M is ....

(2) The image of $\overline{AB}$ by reflection in M is ....

(3) The image of Δ AFM by reflection in M is ....

2 **In the opposite figure :**



m ($\angle$ B) = m ($\angle$ D) = 90° ,

AB = 7 cm. , BC = 24 cm.

and AD = 15 cm.

Find the length of : $\overline{DC}$

15 Souhag Governorate — Juhina Educational Administration

Juhina Experimental Complex for Languages

Answer the following questions :

1 Complete the following :

(1) The measure of each interior angle of the regular pentagon is ....°

(2) If ABCD is a parallelogram and m (∠ A) = 70° , then m (∠ B) = ... °

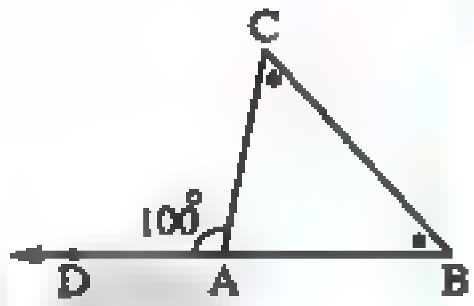
(3) The length of the line segment joining the two midpoints of two sides of a triangle equals ....

(4) The image of the point (3 , – 1) by translation (3 , 2) is ....

(5) **In the opposite figure :**

, D ∈ $\overrightarrow{BA}$, m (∠ B) = m (∠ C) ,

m (∠ CAD) = 100° , then m (∠ C) = ...



2 Choose the correct answer :

(1) In Δ ABC , m (∠ B) = m (∠ A) + m (∠ C) , then m (∠ B) = ...

(a) 30° (b) 60° (c) 90° (d) 150°

(2) The sum of measure of the interior angles of hexagon is ....

(a) 180° (b) 540° (c) 120° (d) 720°

(3) The image of the point (– 3 , 2) by reflection in y-axis is ....

(a) (3 , – 2) (b) (– 3 , – 2) (c) (3 , 2) (d) (– 3 , 2)

(4) (– 4 , 5) is the image of (5 , 4) by rotation in the origin point with an angle of measure ....

(a) 90° (b) – 90° (c) 180° (d) 360°

(5) The sum of measures of the exterior angles of the triangle is ....

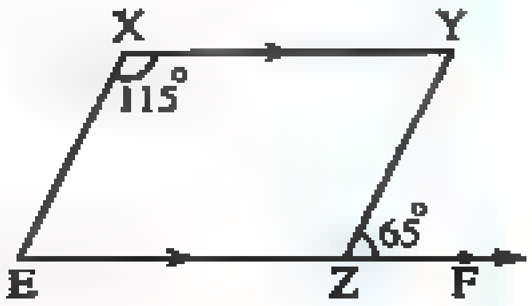
(a) 360° (b) 180° (c) 120° (d) 100°

3 [a] In the opposite figure :

$\overrightarrow{XY}$ // $\overrightarrow{EZ}$, m (∠ X) = 115° ,

and m (∠ YZF) = 65°

Prove that : XYZE is a parallelogram.



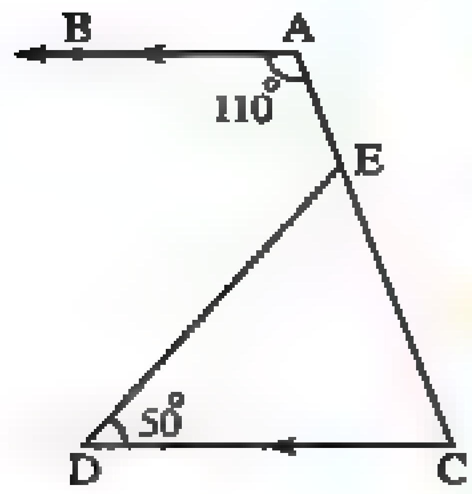
[b] In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{CD}$

, m ($\angle$ A) = 110° ,

m ($\angle$ D) = 50°

Find with proof : m ($\angle$ AED)

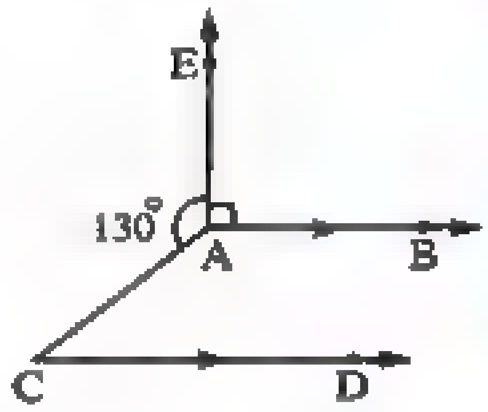


4 **[a] In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$ and m ($\angle$ EAC) = 130° ,

m ($\angle$ EAB) = 90°

Find : m ($\angle$ ACD)

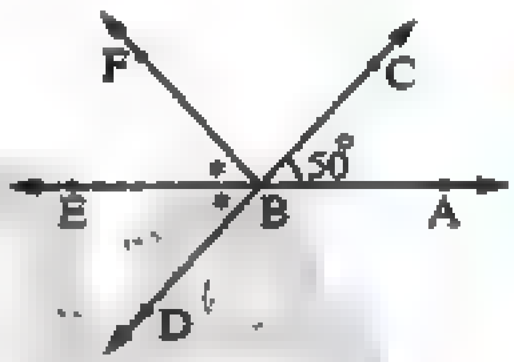


[b] In the opposite figure : $\overleftrightarrow{AE} \cap \overleftrightarrow{CD} = \{B\}$,

m ($\angle$ ABC) = 50° ,

$\overrightarrow{BE}$ bisects ($\angle$ FBD)

Find : m ($\angle$ FBC)



5 Draw Δ ABC where A (1 , 5) , B (3 , 1) and C (5 , 3), then draw its image

(1) By reflection in X-axis.

(2) By rotation about origin point with an angle of measure 180°

Additional question

1 **Complete the following :**

(1) The number of axes of symmetry of the rectangle = ...

(2) If (2 a , 3) is the image of (8 , – 3) by reflection in the origin point, then a = ...

(3) If the reflection in a straight line maps the figure to itself, then this straight line is called ....

2 **In the opposite figure :**

ABC is a triangle in which m ($\angle$ B) = 90° ,

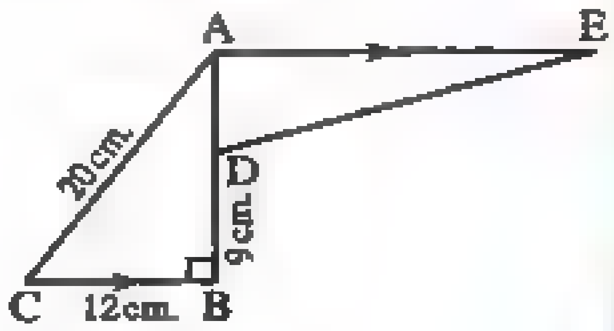
$\overline{AE} \parallel \overline{BC}$

If : BC = 12 cm. , AC = 20 cm.

, D ∈ $\overline{AB}$ where BD = 9 cm.

and AE = 2 BC

Find the length of : $\overline{AD}$, $\overline{DE}$



Model Examinations of the School Book

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

(1) The sum of the measure of interior angle of a triangle equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

(2) The image of the point (– 1 , 3) by translation (4 , – 2) is ....

(a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)

(3) The measure of the exterior angle of the equilateral triangle is ....

(a) 30° (b) 45° (c) 60° (d) 120°

(4) In a parallelogram if the adjacent sides are equal in the length , then the shape is ....

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

(5) The number of the diagonals of a pentagon is ....

(a) 3 (b) 5 (c) 7 (d) 9

2 Complete the following :

(1) The image of the point (2 , 1) by reflection in X-axis is ....

(2) The image of the point (2 , – 1) by rotation about the origin point with an angle of measure 180° is ....

(3) The square is a rectangle in which ....

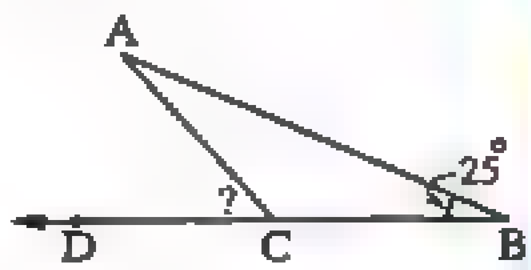
(4) ABCD is a parallelogram in which m (∠ A) = 60° , then m (∠ B) = ...

(5) The image of the point (5 , 3) by translation : $(X , y) \longrightarrow (X + 3 , y - 1)$ is ....

3 [a] In the opposite figure :

m (∠ A) = m (∠ B) = 25°

Find : m (∠ ACD)

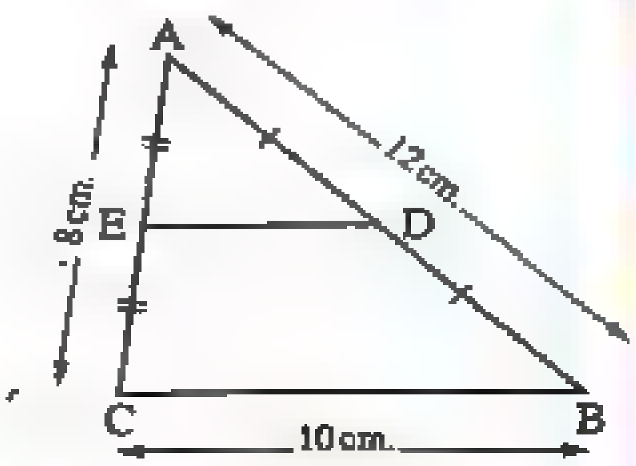


[b] In the opposite figure :

Δ ABC in which : AB = 12 cm. ,

BC = 10 cm. , AC = 8 cm.

Find the perimeter of : Δ ADE

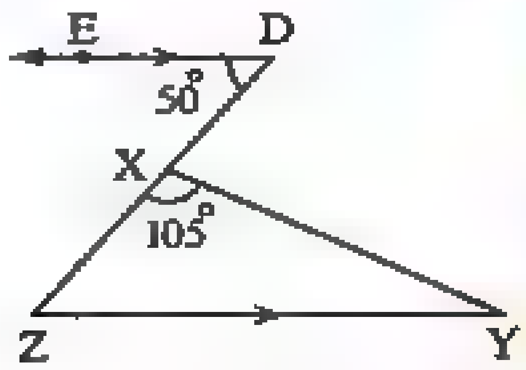


4 **[a] In the opposite figure :**

$\overleftrightarrow{DE} \parallel \overline{YZ}$, m ($\angle$ ZDE) = 50°

, m ($\angle$ YXZ) = 105°

Find : m ($\angle$ Z) , m ($\angle$ Y) , m ($\angle$ YXD)

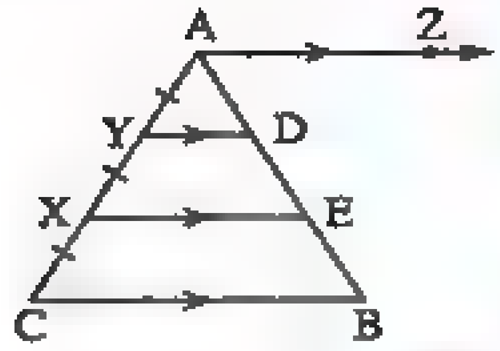


[b] In the opposite figure :

$\overrightarrow{AZ} \parallel \overline{YD} \parallel \overline{XE} \parallel \overline{CB}$,

AY = YX = XC , AB = 18 cm. ,

Find the length of : $\overline{EB}$



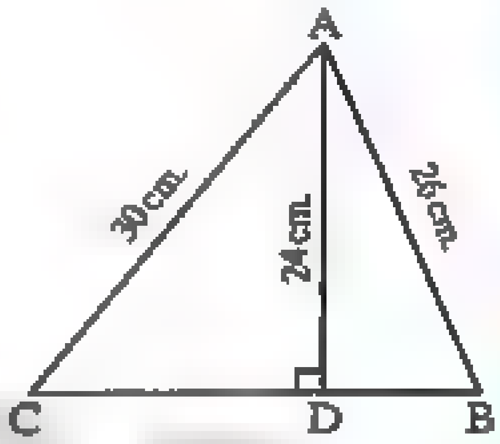
5 **[a] In the opposite figure :**

$\overline{AD} \perp \overline{BC}$, if AD = 24 cm. ,

AB = 26 cm. , AC = 30 cm.

① **Find the length of :** $\overline{BC}$

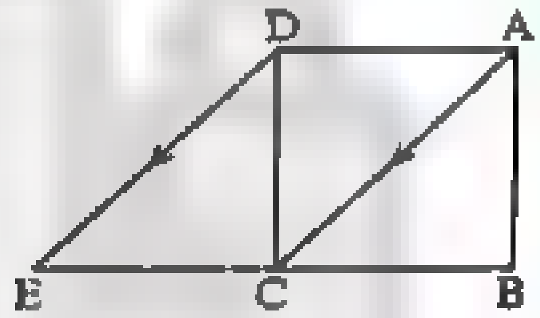
② **Find the area of :** Δ ABC



[b] ABCD is a square , E $\in \overleftrightarrow{BC}$

$\overline{AC} \parallel \overline{DE}$, Prove that

ACED is a parallelogram.



Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

① The image of the point (2 , – 5) by reflection in X-axis is ....

(a) (2 , – 5) (b) (2 , 5) (c) (– 2 , – 5) (d) (5 , 2)

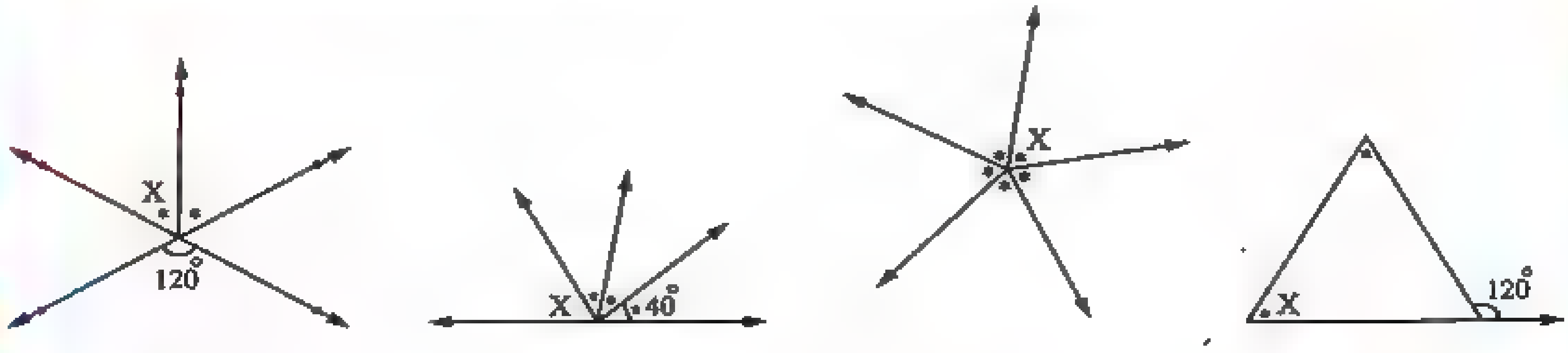
② The measure of each angle of regular hexagon equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

③ The two diagonals are equal in the length and not perpendicular in ....

(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

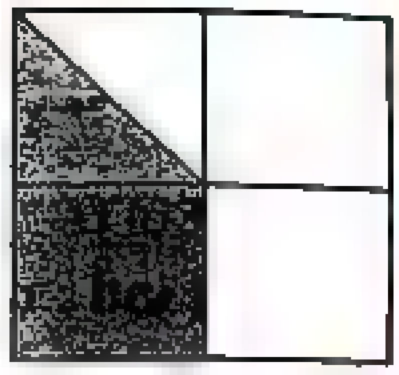
④ All the following shapes m ($\angle$ X) = 60° except the shape ....



(a) (b) (c) (d)

⑤ **In the opposite figure :**

The area of shaded part from the area of all shape equal ....



(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

2 **Complete the following :**

① The sum of the measures of the angles of the quadrilateral equals ....

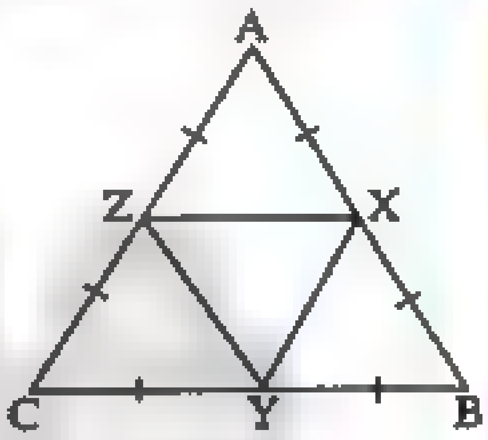
② The image of the point (2 , 3) by translation MN , in direction $\overrightarrow{MN}$, where M (2 , – 1) , N (5 , 1) is ....

③ ABCD is parallelogram in which m (∠ A) = 60° , then m (∠ B) = ...

④ The ray drawn parallel to one side of a triangle and passing through the midpoint of another side ....

⑤ **In the opposite figure :**

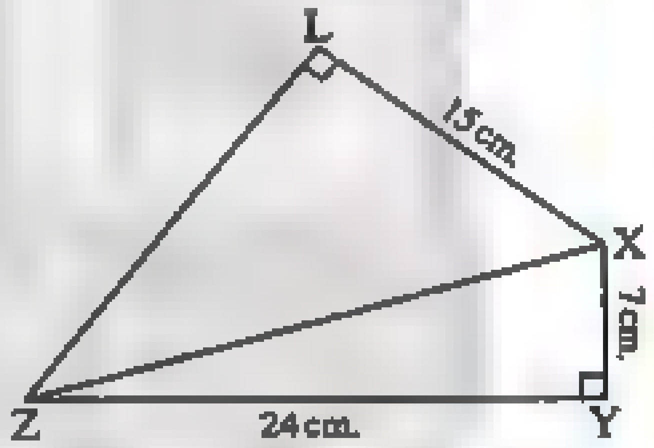
The image of the triangle XBY by translation XZ in direction $\overrightarrow{XZ}$ is ....



3 **[a] In the opposite figure :**

XYZL is quadrilateral in which m (∠ Y) = m (∠ L) = 90° , XY = 7 cm. , YZ = 24 cm. , XL = 15 cm.

Find the length of each of : $\overline{XZ}$, $\overline{LZ}$



[b] Using the square lattice , draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1) then find the image of $\overline{AB}$ by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

4 **[a]** Draw the image of triangle ABC where A (1 , 1) , B (3 , 4) , C (5 , 2) by reflection in x-axis.

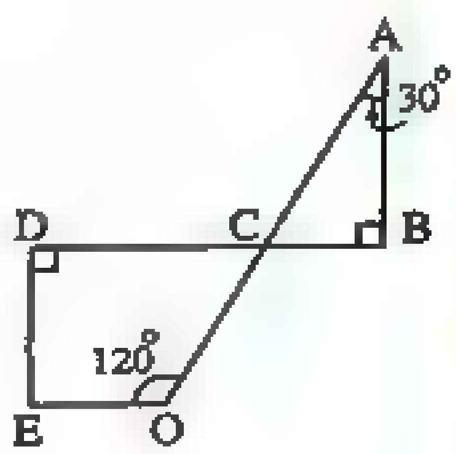
[b] In the opposite figure :

$\overline{AB}$ and $\overline{ED}$, are perpendicular to $\overline{BD}$, $\overline{BD} \cap \overline{AO} = \{C\}$,

m (∠ A) = 30°

, m (∠ EOC) = 120° ,

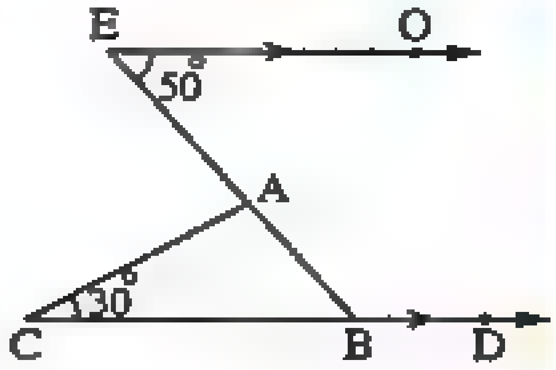
Find : m (∠ E)



5 **[a] In the opposite figure :**

$\overrightarrow{EO} // \overline{CD}$, m (∠ E) = 50° , m (∠ C) = 30° ,

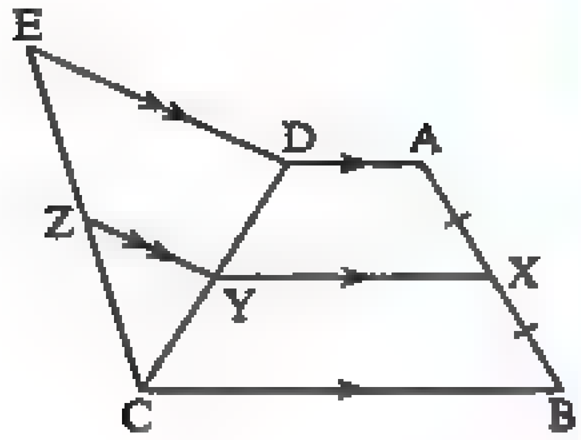
Find the measures of angles of Δ ABC , m (∠ ABD)



[b] In the opposite figure :

X is the midpoint of $\overline{AB}$, Y ∈ $\overline{CD}$, Z ∈ $\overline{CE}$, $\overline{AD} // \overline{XY} // \overline{BC}$, $\overline{YZ} // \overline{DE}$

is CZ = ZE ? giving reason



Model 3

Answer the following questions :

1 **Choose the correct answer from those given :**

① The image of the point (3 , – 5) by reflection in y-axis is ………

(a) (3 , 5) (b) (– 3 , – 5) (c) (– 3 , 5) (d) (– 5 , 3)

② The sum of the measures of interior angles of a pentagon is ………

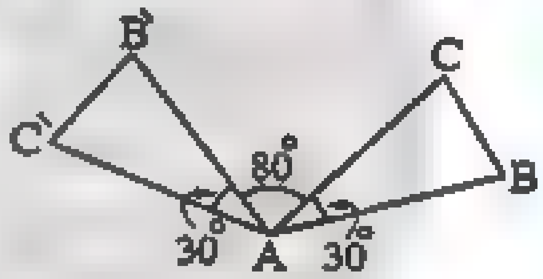
(a) 360° (b) 450° (c) 540° (d) 720°

③ The number of diagonals of quadrilateral is ………

(a) 2 (b) 3 (c) 4 (d) 5

④ **In the opposite figure :**

Δ AB̀ C̀ is the image of Δ ABC by rotation about A and with angle of measure ………



(a) 30° (b) 80° (c) 110° (d) 140°

⑤ The diagonal of a square divided its vertex angle in two angles of the measure of each of them is ………

(a) 30° (b) 45° (c) 60° (d) 90°

2 **Complete the following :**

① The rhombus is a parallelogram in which ………

② Each opposite angles in a parallelogram are ………

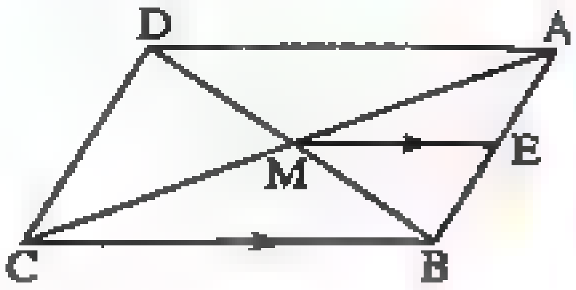
③ (– 3 , 2) is the image of the point (3 , 2) by reflection in ……… axis.

④ The line segment joining the midpoint of two sides of a triangle is ………

⑤ The image of the point (4 , 6) by geometric transformation $(x , y) \longrightarrow (-x , y-7)$ is ………

3 [a] Using the lattice , draw Δ ABC where A (1 , 0) , B (0 , 2) and C (– 3 , 1) , then draw its image by reflection in X-axis.

[b] In the opposite figure :

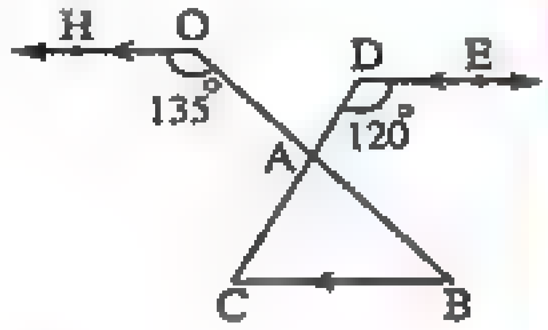


ABCD is a parallelogram , M is the intersection of its diagonals

Draw $\overrightarrow{ME}$ // $\overline{CB}$

is AE = EB ? giving reason.

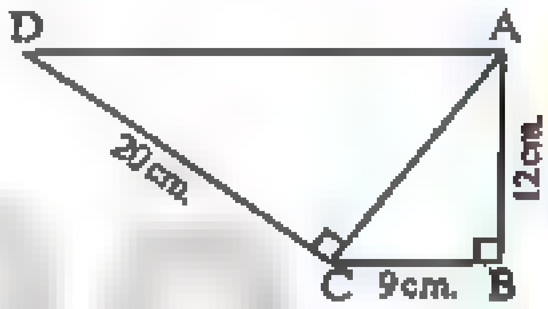
4 [a] **In the opposite figure :**



$\overleftrightarrow{DE}$ // $\overleftrightarrow{OH}$ // $\overleftrightarrow{BC}$, m ($\angle$ ADE) = 120°

, m ($\angle$ AOH) = 135° , Calculate the measures of the angles of the triangle ABC

[b] In the opposite figure :

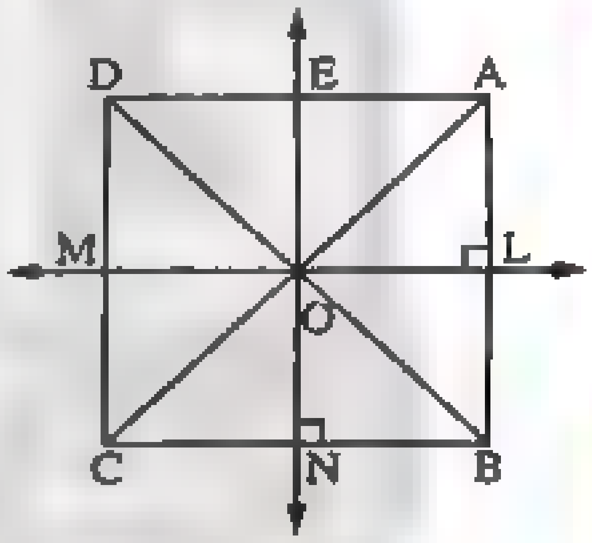


m ($\angle$ B) = m ($\angle$ ACD) = 90° ,

AB = 12 cm. , BC = 9 cm. , CD = 20 cm.

Find the length of each of : $\overline{AC}$, $\overline{AD}$

5 **In the opposite figure :**



ABCD is a square of side length 6 cm. and the origin point its center. **Find :**

(1) The image of Δ AOL by translation 3 cm. in direction $\overrightarrow{AB}$

(2) The image of Δ AOL by reflection in $\overleftrightarrow{EN}$

(3) The image of Δ AOL by rotation about O and with an angle of measure (– 90°)

Model 4

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) The parallelogram whose diagonals are perpendicular and not equal in length is called ....

(a) rhombus. (b) square. (c) rectangle. (d) trapezium.

(2) The measure of each angle of a fegular pentagon is ....

(a) 90° (b) 108° (c) 120° (d) 136°

(3) The triangle contains at least two .... angles.

(a) acute (b) obtuse (c) right (d) reflex

(4) If ABCD is a parallelogram in which BC = 8 cm. , CD = 6 cm. ,
then its perimeter = ...
(a) 14 cm. (b) 28 cm. (c) 48 cm. (d) 56 cm.

(5) The image of the point (2 , – 1) by reflection in x-axis is ....
(a) (2 , 1) (b) (1 , 2) (c) (– 2 , – 1) (d) (– 1 , 2)

2 **Complete the following :**

(1) The line segment joining the midpoint of two sides of a triangle is ....

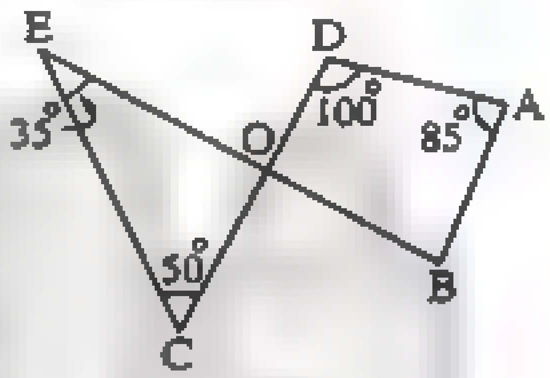
(2) In a parallelogram each opposite angles are ....

(3) The quadrilateral is a parallelogram if ....

(4) The image of the point (5 , – 3) by translation 3 units in negative direction of x-axis is ....

(5) The image of the point (3 , 2) by rotation with an angle of measure 180° about the origin is ....

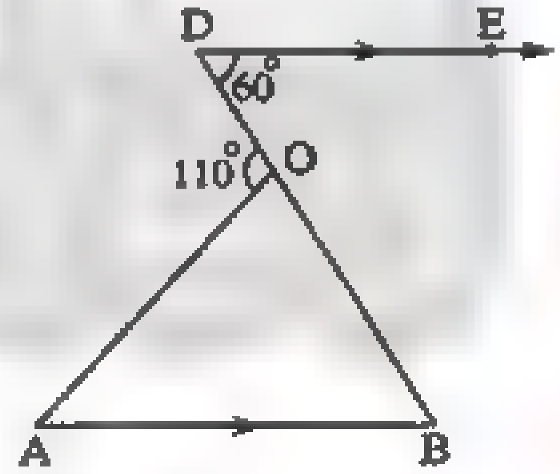
3 **In the opposite figure :**



$\overline{DC} \cap \overline{BE} = \{O\}$, m (∠ A) = 85°
, m (∠ D) = 100° , m (∠ E) = 35° , m (∠ C) = 50°
Find with proof each of :
(1) m (∠ DOB) (2) m (∠ B)

4 [a] Find the length of the diagonal of a rectangle whose area 48 cm^2 and of width 6 cm. ?

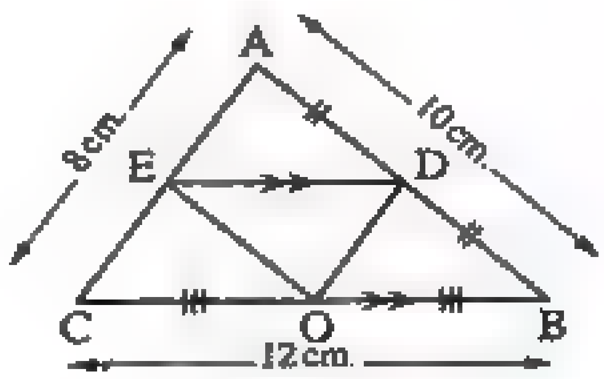
[b] In the opposite figure :



$\overrightarrow{AB} // \overrightarrow{DE}$,
m (∠ D) = 60°
, m (∠ AOD) = 110°
Find with proof : m (∠ A)

5 [a] If the point A is the image of point B (– 1 , 2) by reflection in y-axis , find the image of A by translation (– 1 , 2)

[b] In the opposite figure :



ABC is a triangle in which D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{BC}$, E ∈ $\overline{AC}$
Such that $\overline{DE} // \overline{BC}$,
AB = 10 cm. , BC = 12 cm. , AC = 8 cm.
(1) **Prove that :** DBOE is a parallelogram.
(2) **Find the perimeter of :** Δ EDO

Model 5

Answer the following questions :

1 Choose the correct answer from those given :

① The image of the point (– 3 , 4) by reflection in y-axis is ....
(a) (3 , – 4) (b) (3 , 4) (c) (– 3 , – 4) (d) (4 , – 3)

② The sum of the measure of the exterior angles of a triangle equals ....
(a) 90° (b) 108° (c) 180° (d) 360°

③ The diagonals which are equal in the length and perpendicular in ....
(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

④ The image of the point (– 3 , 5) by rotation about the origin and with an angle of measure 90° is ....
(a) (5 , 3) (b) (– 5 , 3) (c) (3 , 5) (d) (– 5 , – 3)

⑤ The measure of each angle of regular octagon equals ....
(a) 108° (b) 120° (c) 135° (d) 144°

2 Complete the following :

① The parallelogram whose diagonals are perpendicular is ....

② If the measure of an interior angle of a triangle is equal to the sum of the measures of the other two interior angles , then the triangle is ....

③ Any triagnle has at least two .... interior angles.

④ The triagnle of the point (2 , – 4) by reflection in X-axis is ... .

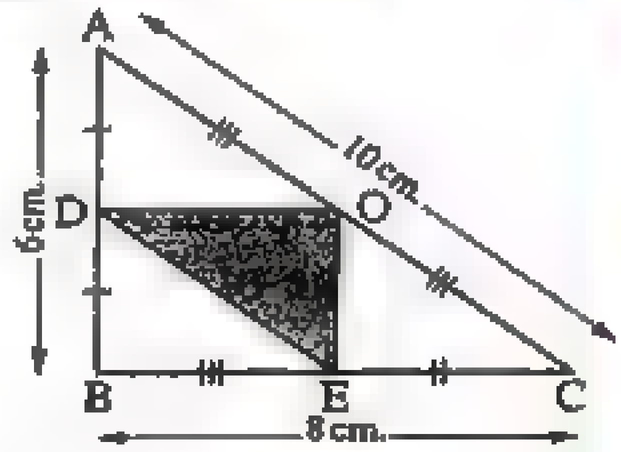
⑤ The image of the point (3 , – 2) by translation $(X , y) \longrightarrow (X - 1 , y + 6)$ is ....

3 [a] Prove that the ray drown parallel to one side of a triangle and passing through the midpoint of another side bisects the third side of the triangle.

[b] In the opposite figure :

D , E , O are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ resectively , AB = 6 cm. , BC = 8 cm. , AC = 10 cm.

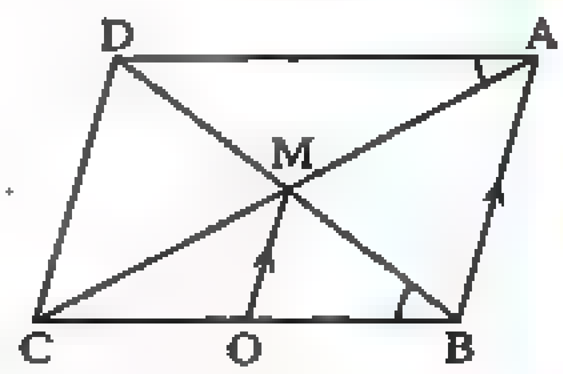
Find the perimeter of : Δ DEO



4 **[a] In the opposite figure :**

ABCD is a parallelogram its diagonals are intersect at M , $\overrightarrow{MO}$ // $\overline{AB}$, $\overrightarrow{MO} \cap \overline{BC} = \{O\}$

Prove that : BO = OC

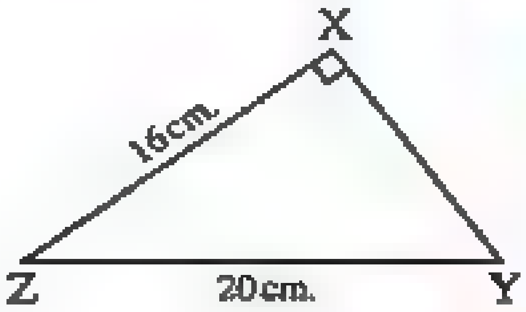


[b] In the opposite figure :

XYZ is a triangle in which m (∠ X) = 90° ,

YZ = 20 cm. , XZ = 16 cm.

Find the length of : $\overline{XY}$

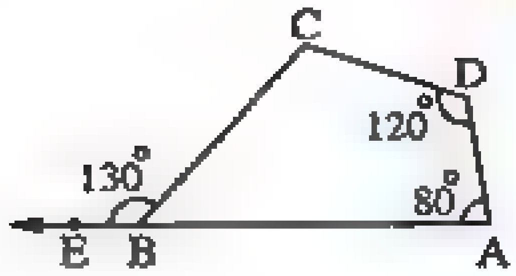


5 **[a] In the opposite figure :**

m (∠ A) = 80° , m (∠ D) = 120°

, m (∠ CBE) = 130°

Find : m (∠ C)



[b] On a square lattice , draw the triangle whose vertices are A (4 , 4) B (4 , 2) , C (1 , 2) then determine each of the following :

① The coordinates of the image of Δ ABC by translation 2AB in direction $\overrightarrow{AB}$

② The image of Δ ABC by rotation about B and with an angle of measure 180°

Model 6

Answer the following questions :

1 **Choose the correct answer from those given :**

① The measure of each angle of the regular pentagon is ....

(a) 90° (b) 108° (c) 120° (d) 144°

② The smallest number of the acute angle in any triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

③ The rhombus of diagonals are equal in length is ....

(a) square. (b) rectangle. (c) parallelogram. (d) trapezium.

④ The image of the point (2 , – 1) by reflection in X-axis is ....

(a) (2 , 1) (b) (1 , 2) (c) (– 2 , – 1) (d) (– 1 , 2)

⑤ The image of the square by rotation about the origin point with an angle of measure 90° is ....

(a) rectangle. (b) square. (c) rhombus. (d) trapezium.

2 **Complete the following :**

① The measure of the exterior angle of a triangle is ....

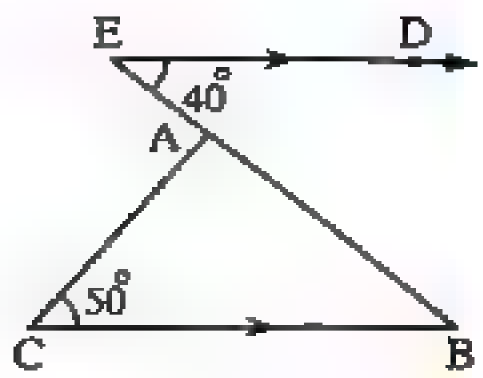
② The parallelogram whose diagonals are equal in length and perpendicular is ....

③ ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = ...

④ The image of the point (– 4 , 5) by translation (2 , – 3) is ....

⑤ Δ XYZ , m (∠ Y) = 90° , then $(XZ)^2$ = ...

3 **[a] In the opposite figure :**



$\overrightarrow{ED} \parallel \overrightarrow{CB}$, m (∠ C) = 50°

, m (∠ E) = 40°

Prove that : $\overline{AC} \perp \overline{BE}$

[b] Draw Δ OBC on a square lattice where O (0 , 0) , B (3 , 0) , C (0 , 4) then find its image by rotation about the origin point with an angle of measure 180°

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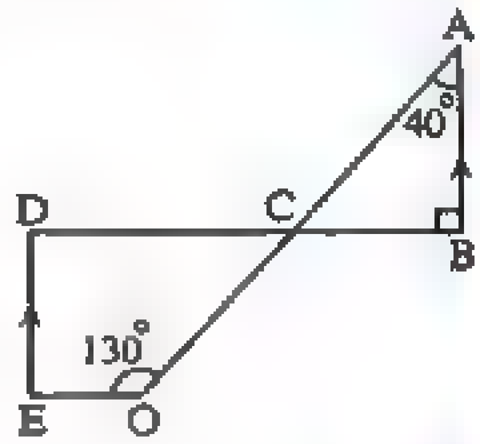
4 **[a] In the opposite figure :**



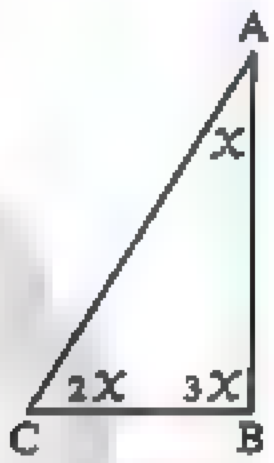
$\overline{BD} \cap \overline{AO} = \{C\}$, $\overline{AB} \parallel \overline{DE}$,

m (∠ A) = 40° , m (∠ B) = 90° ,

m (∠ COE) = 130°

Find : m (∠ E)

[b] In the opposite figure :



Find with proof the measures of the angles of Δ ABC

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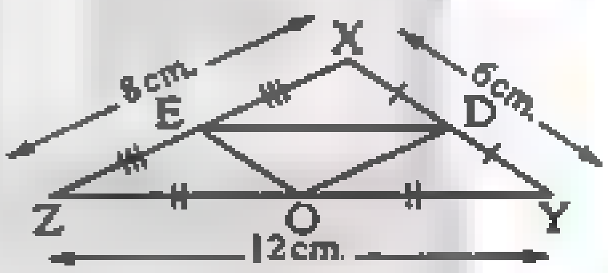
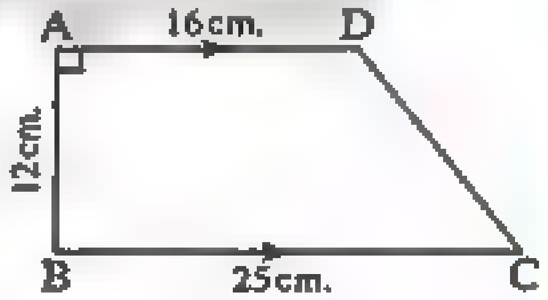
5 **[a] In the opposite figure :**



XYZ is a triangle in which D , E , O are midpoints of $\overline{XY}$, $\overline{XZ}$, $\overline{YZ}$ respectively , XY = 6 cm. , XZ = 8 cm. , YZ = 12 cm. ,

Find the perimeter of : Δ DOE

[b] In the opposite figure :



ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, m (∠ A) = 90° , AB = 12 cm. , BC = 25 cm. , AD = 16 cm.

Find the length of : $\overline{DC}$

Model 7

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) The measure of each angle of the regular hexagon is ·········

(a) 108° (b) 120° (c) 136° (d) 144°

(2) Any triangle has at least two ········· interior angles.

(a) acute (b) right (c) obtuse (d) straight

(3) The diagonals of the rectangle are ....

(a) parallel. (b) perpendicular.

(c) equal in the length. (d) equal in length & perpendicular.

(4) The image of a triangle by rotation about the origin point with an angle of measure 180° is ....

(a) triangle. (b) line segment. (c) point. (d) straight line.

(5) The image of the point (3 , – 2) by reflection in y-axis ....

(a) (3 , 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (– 2 , 3)

2 **Complete the following :**

(1) The sum of the measures of the interior angles of a pentagon equals ....

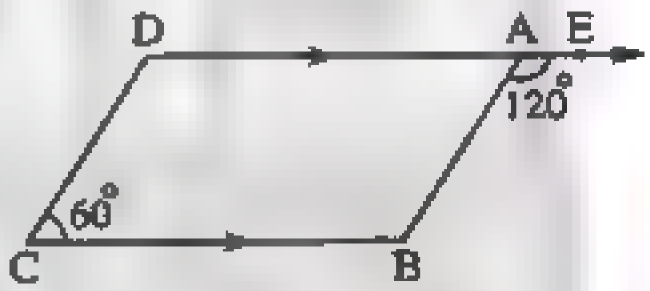
(2) The measure of the exterior angle of an equilateral triangle = ... at one of its vertices.

(3) The rectangle is a parallelogram in which one of its angles is ....

(4) The image of the point (– 4 , 5) by translation (2 , – 3) is ....

(5) If the image of the point (– 4 , 0) by rotation about the origin point is (0 , – 4) , then the measure of rotation angle is ....

3 **[a] In the opposite figure :**

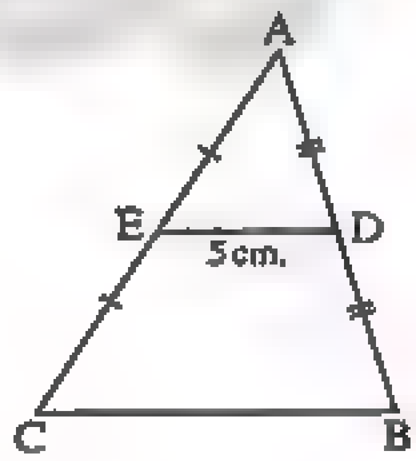


$E \in \overrightarrow{DA}$, m (∠ EAB) = 120°

m (∠ C) = 60° , $\overrightarrow{DA} \parallel \overline{CB}$

Prove that : ABCD is a parallelogram.

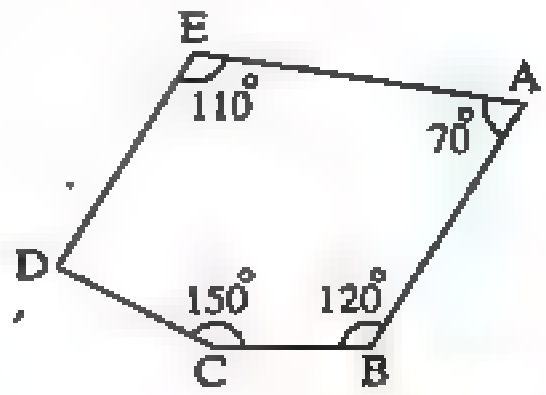
[b] In the opposite figure :



ABC is a triangle in which D , E are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively ; DE = 5 cm.

Find the length of : $\overline{BC}$

4 **[a] In the opposite figure :**

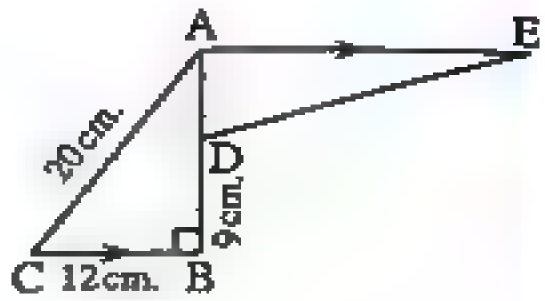


ABCDE is a pentagon in which m (∠ A) = 70° , m (∠ B) = 120° , m (∠ C) = 150° , m (∠ E) = 110°

Find : m (∠ D) with proof.

[b] In the opposite figure :

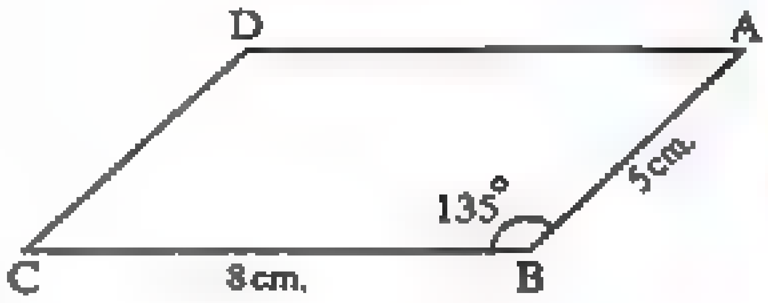
Δ ABC in which m ($\angle$ B) = 90° , $\overline{AE}$ // $\overline{BC}$,
if BC = 12 cm. , AC = 20 cm. , D $\in \overline{AB}$
such that : BD = 9 cm. , AE = 2 BC ,
Find the length of each : $\overline{AD}$, $\overline{ED}$



5 **[a] In the opposite figure :**

ABCD is a parallelogram in which
AB = 5 cm. , BC = 8 cm. , m ($\angle$ B) = 135°
Find :

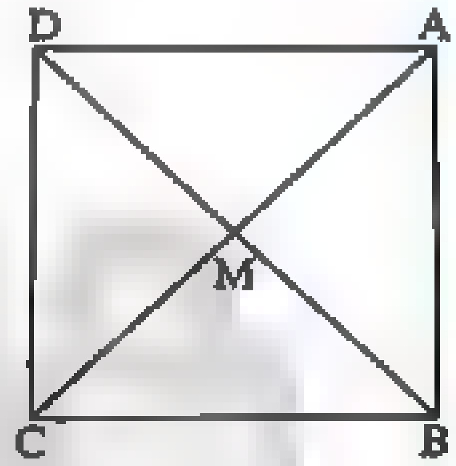
① m ($\angle$ C)
② The perimeter of parallelogram ABCD



[b] In the opposite figure :

ABCD is a square , whose diagonals intersect at M.
Find :

① The image of Δ ABC by reflection in $\overleftrightarrow{AC}$
② The image of Δ MAB by rotation about M with measure (– 90°)



Model 8

Answer the following questions :

1 **Choose the correct answer from those given :**

① The diagonal of square makes angle of measure .... with any of its sides.
(a) 45° (b) 60° (c) 90° (d) 120°

② The measure of any of the exterior angle of an equilateral triangle equals ....
(a) 60° (b) 90° (c) 120° (d) 180°

③ In Δ ABC if m ($\angle$ A) > m ($\angle$ B) + m ($\angle$ C) then the angle A is ....
(a) acute. (b) right. (c) obtuse. (d) straight.

④ The sum of the measures of any two consecutive angles in a parallelogram equals ....
(a) 90° (b) 180° (c) 270° (d) 360°

⑤ The image of the point (5 , – 3) by rotation about the origin point is itself , then the measure of rotation angle is ....
(a) 90° (b) 180° (c) 270° (d) 360°

2 **Complete the following :**

(1) The length of a line segment joining the midpoints of two sides of a triangle equals ········

(2) The parallelogram whose diagonals are perpendicular and of equal in length is ·········

(3) The parallelogram whose perimeter 24 cm. and the length of one of its sides is 7 cm. , then the length of its adjacent side equals ·········

(4) The image of the point (3 , 4) by reflection in the X-axis is ········· and its image by reflection in the y-axis is ·········

(5) If the image of the point (– 1 , 3) by a translation is (1 , 4) then the image of the point (3 , – 2) by the same translation is ·········

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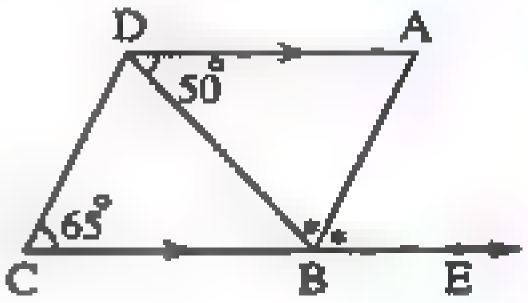
3 [a] **In the opposite figure :**



$\overrightarrow{DA}$ // $\overleftrightarrow{BE}$, $\overrightarrow{BA}$ bisects ∠ DBE

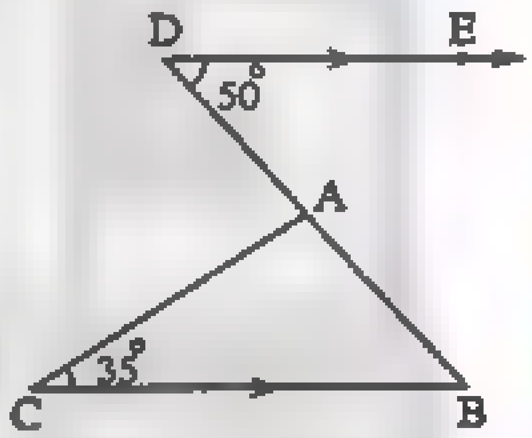
, m (∠ ADB) = 50° , m (∠ C) = 65°

Prove that : ABCD is a parallelogram

[b] Draw the triangle ABC in which AB = AC = 5 cm. , BC = 6 cm. , then draw its image by rotation about A with an angle of measure 270°

---

4 [a] **In the opposite figure :**



$\overrightarrow{DE}$ // $\overrightarrow{CB}$, m (∠ D) = 50°

, m (∠ C) = 35°

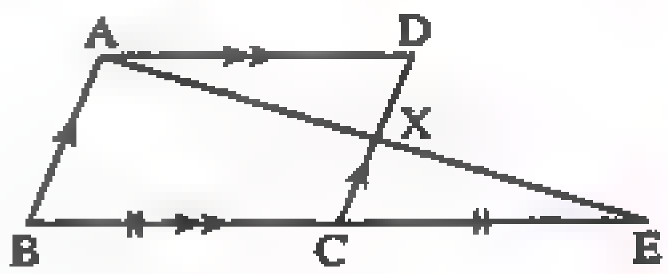
Find : (1) m (∠ B)

(2) m (∠ BAC)

[b] Draw the triangle ABC in which AB = 3 cm. , BC = 4 cm. , m (∠ ABC) = 90° , then draw its image by reflection in straight line $\overleftrightarrow{BC}$

---

5 [a] **In the opposite figure :**

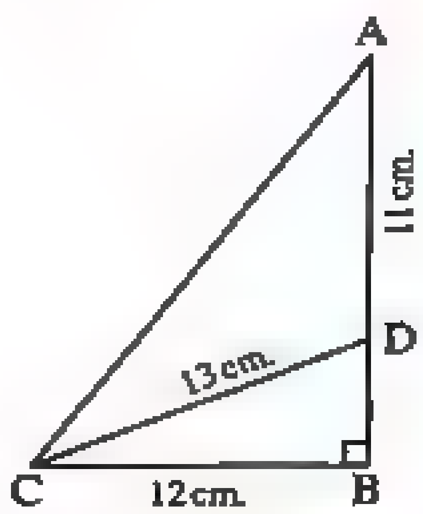


ABCD is a parallelogram , E ∈ $\overrightarrow{BC}$

Such that : CE = BC , $\overline{AE} \cap \overline{DC}$ = {X}

Prove that : AX = XE

[b] **In the opposite figure :**



ABC is a triangle in which m (∠ B) = 90° , D ∈ $\overline{AB}$

Such that : AD = 11 cm.

, if BC = 12 cm. , DC = 13 cm. ,

Find the length of each of : $\overline{BD}$, $\overline{AC}$

School Examinations

1 Cairo Governorate — El-Wayli Educational Zone

Sakr Koreish (E.L.S)

Answer the following questions :

1 Choose the correct answer :

(1) The sum of the measures of the exterior angles of a polygon = ...

(a) 180° (b) 360° (c) 540° (d) 720°

(2) The sum of the measures of the interior angles of a right-angled triangle = ...

(a) 180° (b) 90° (c) 60° (d) 30°

(3) The image of the point (8 , – 2) by reflection on X-axis is ....

(a) (8 , 2) (b) (8 , – 2) (c) (– 8 , – 2) (d) (– 8 , 2)

(4) ABCD is a rectangle in which m (∠ BAC) = 40° , then m (∠ DAC) = ...

(a) 360° (b) 90° (c) 50° (d) 40°

(5) In Δ ABC : if X , Y are the midpoints of $\overline{AC}$ and $\overline{BC}$ respectively , then $\overline{XY}$ // ....

(a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$ (d) $\overline{CY}$

2 Complete each of the following :

(1) The image of the point (– 5 , 0) by reflection on X-axis is ....

(2) If XYZL is a rhombus , then .... ⊥ ....

(3) Any triangle has at least two .... interior angles.

(4) The ray drawn from the midpoint of a side of a triangle parallel to another side .... the third side.

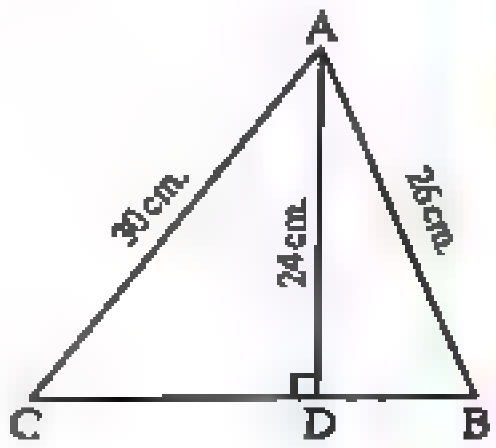
(5) The image of the point (7 , 5) is (5 , – 7) by rotation about the origin with an angle of measure ....

3 [a] In the opposite figure :

$\overline{AD} \perp \overline{BC}$ if AD = 24 cm. , AB = 26 cm. ,

AC = 30 cm. , **find :** the length of $\overline{BC}$, **then**

Find : the area of Δ ABC

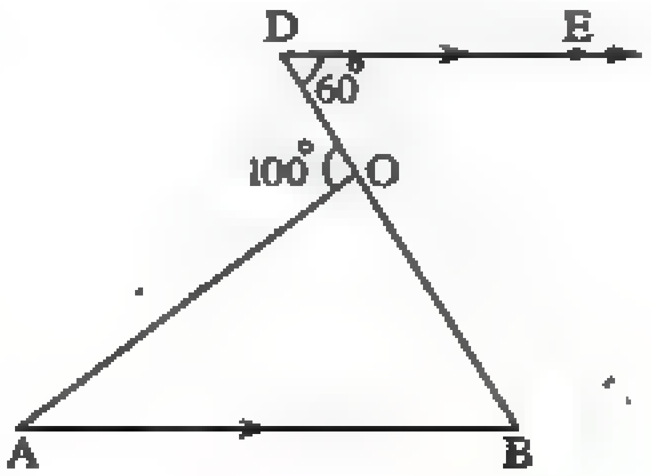


[b] In the opposite figure :

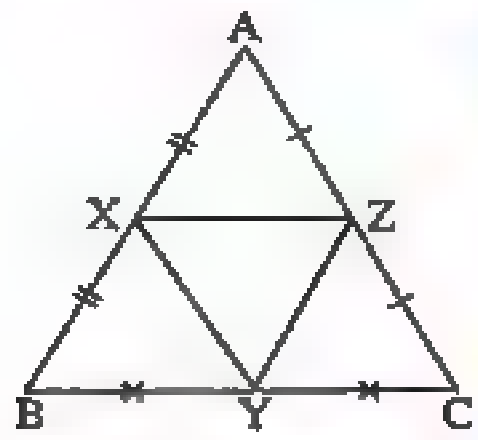
$\overline{AB}$ // $\overrightarrow{DE}$, m (∠ D) = 60°

, m (∠ AOD) = 100°

Find with proof : m (∠ A)



4 **[a] In the opposite figure :**



$XY = 14$ cm. , $YZ = 12$ cm. and $XZ = 10$ cm. ,

X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively. **Calculate the perimeter of :** Δ ABC.

[b] On the lattice. Draw Δ ABC where A (– 1 , 3) , B (– 4 , 2) and C (– 2 , 5) , then draw its image under the translation of magnitude MN in the direction of $\overrightarrow{MN}$ in which M (3 , – 2) aand N (8 , 1)

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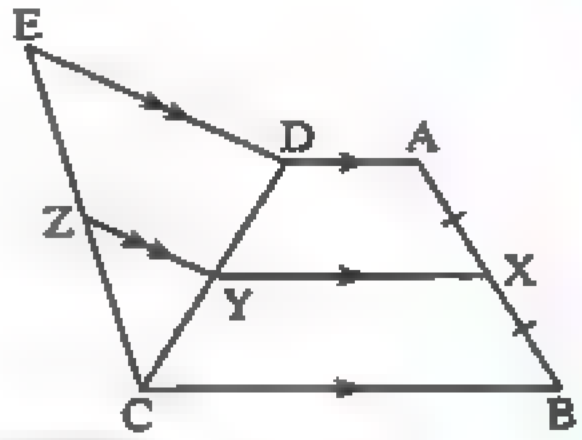
5 **[a] In the opposite figure :**



X is the midpoints of $\overline{AB}$,

$Y \in \overline{CD}$, $Z \in \overline{CE}$,

$\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$

, $\overline{YZ} \parallel \overline{DE}$

Prove that : CZ = ZE

[b] Draw the square ABCD in which A (1 , 1) , B (4 , 1) , C (4 , 4) , and D (1 , 4) Then draw its image by rotation with angle – 90° about the origin.

2 Cairo Governorate — El-Nozha Directorate

Sunrise Language Schools

Answer the following questions :

1 **Choose the correct answer :**

① The measure of each angle of regular hexagon = ...

(a) 60° (b) 108° (c) 120° (d) 135°

② The image of the point (– 3 , 5) by rotation about the origin and with an angle with measure 90° is ....

(a) (5 , 3) (b) (– 5 , 3) (c) (3 , 5) (d) (– 5 , – 3)

③ The measure of any of the exterior angle of an equilateral triangle equals ....

(a) 60° (b) 90° (c) 120° (d) 180°

④ The diagonal of square divided its vertex angle into two angles of the measure of each of them is ....

(a) 30° (b) 45° (c) 60° (d) 90°

⑤ The sum of the measure of any two consecutive angles in a parallelogram equals ....

(a) 180° (b) 90° (c) 270° (d) 360°

2 **Complete each of the following :**

① ABC is a right-angled triangle at B if AB = 5 cm. , AC = 13 cm. , then BC = ... cm.

② The image of the point (5 , 3) by translation $(x , y) \longrightarrow (x+3 , y-1)$ is ....

③ The length of a line segment joining the midpoints of two sides of a triangle equals ....

④ (– 3 , 2) is the image of the point (3 , 2) by reflection in .... axis.

⑤ The parallelogram whose diagonals are perpendicular and not equal in length is called ....

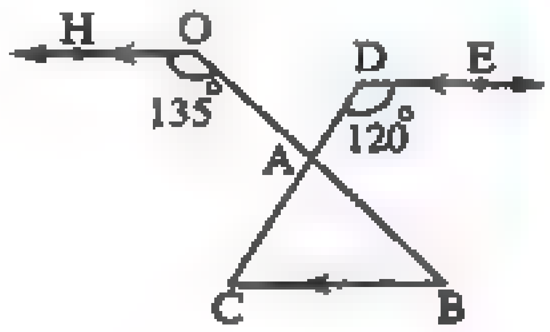
---

3 **[a] In the opposite figure :** $\overrightarrow{DE} \,//\, \overrightarrow{OH} \,//\, \overline{BC}$

, m (∠ ADE) = 120°

, m (∠ AOH) = 135°

Find the measures of the angles of : Δ ABC

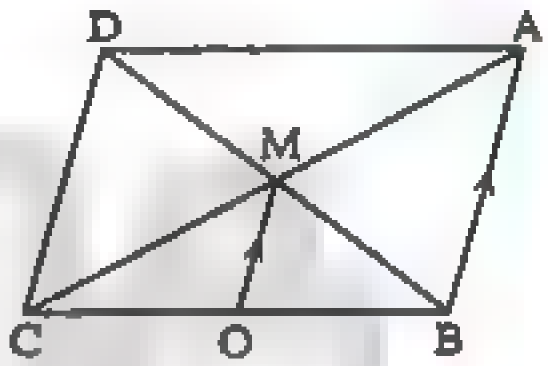


[b] In the opposite figure :

ABCD is a parallelogram its diagonals are intersect at M ,

$\overline{MO} \,//\, \overline{AB}$, $\overline{MO} \cap \overline{BC} = \{O\}$

Prove that : BO = OC



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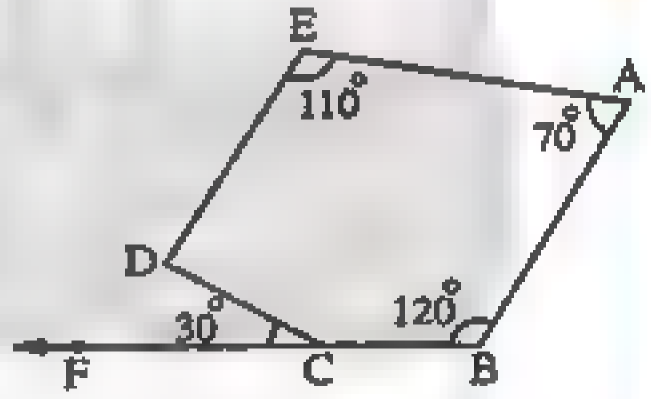
4 [a] Using the lattice. Draw Δ ABC where A (1 , 0) , B (0 , 2) and C (– 3 , 1) , then draw its image by reflection in x-axis.

[b] In the opposite figure :

ABCDE is pentagon in which m (∠ A) = 70°

, m (∠ B) = 120° , m (∠ E) = 110° , m (∠ DCF) = 30°

Find with proof : m (∠ D)



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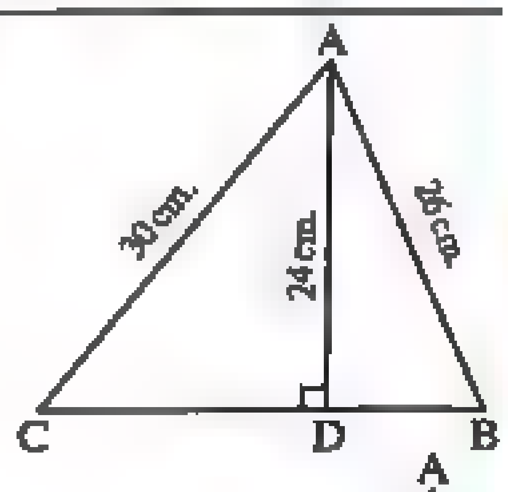
5 **[a] In the opposite figure :**

$\overline{AD} \perp \overline{BC}$, if AD = 24 cm. , AB = 26 cm. , AC = 30 cm.

Find : ① The length of $\overline{BC}$

② The area of Δ ABC



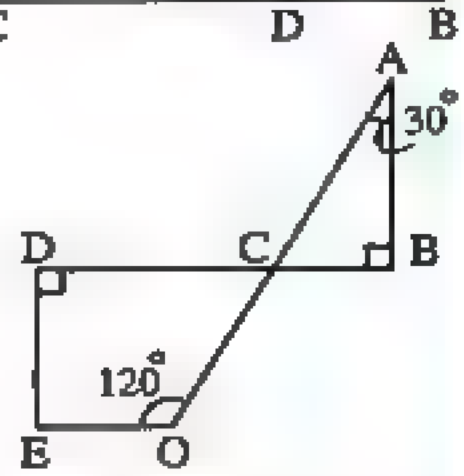
[b] In the opposite figure :

$\overline{AB}$, $\overline{ED}$ are perpendicular to $\overline{BD}$,

$\overline{BD} \cap \overline{AO} = \{C\}$, m (∠ A) = 30°

, m (∠ EOC) = 120°

Find : m (∠ E)

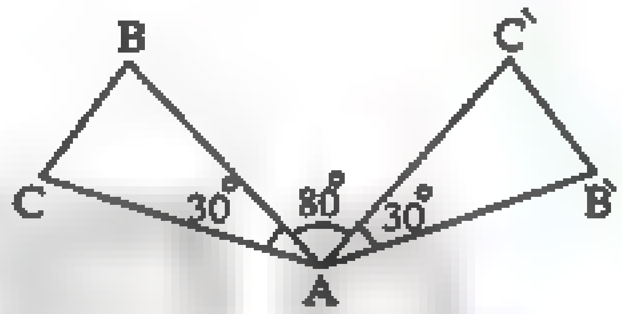


3 Cairo Governorate El-Zawia Educational Zone

Answer the following questions :

1 Choose the correct answer :

1. The image of the point (2 , –1) by reflection in y-axis is ....
   (a) (2 , 1) (b) (1 , 2) (c) (–2 , –1) (d) (–1 , 2)
2. The measure of each angle of regular hexagon equal ....
   (a) 108° (b) 120° (c) 135° (d) 144°
3. The measure of the exterior angle of the equilateral triangle is ....
   (a) 30° (b) 45° (c) 60° (d) 120°
4. **In the opposite figure :**
   Δ AB̀C̀ is the image of
   Δ ABC by rotation about A
   and with angle of measure ....



   (a) 30° (b) 80° (c) 110° (d) 140°
5. If ABCD is a square then m (∠ ACB) = ...
   (a) 90° (b) 60° (c) 45° (d) 30°

2 Complete each of the following :

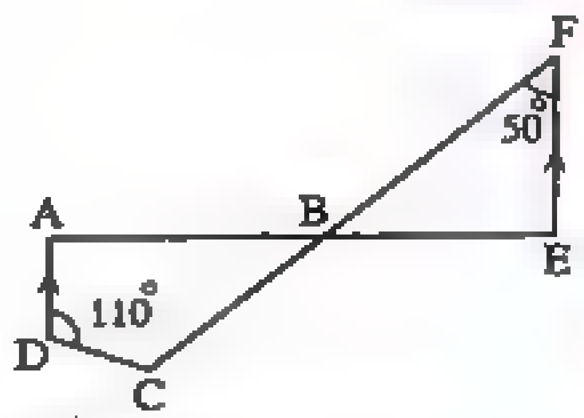
1. ABCD is a parallelogram in which m (∠ A) = 50° then m (∠ B) = ...
2. The image of the point (3 , –2) by translation $(x , y) \longrightarrow (x - 1 , y + 6)$ is ....
3. The sum of measures of the interior angles of the triangle = ...
4. The ray drawn from the midpoint of a side of triangle parallel to another side ....
5. In Δ ABC : If m (∠ A) + m (∠ C) = m (∠ B) , then m (∠ B) = ...°

3 [a] In the opposite figure :

$\overline{AD} // \overline{EF}$, m (∠ E) = 90°

m (∠ F) = 50° , m (∠ D) = 110°

Find : m (∠ C)



[b] Find the number of sides of a regular polygon if the measure of one of its interior angles is 135°

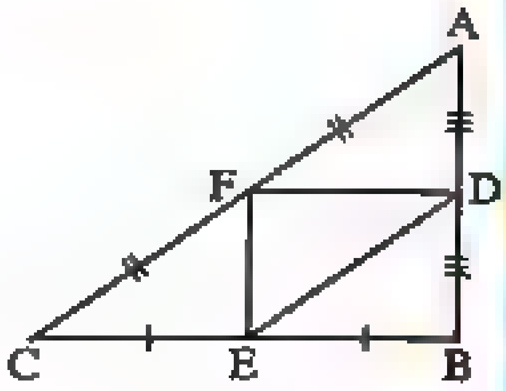
4 **[a] In the opposite figure :**

ABC is a triangle in which AB = 6 cm.

BC = 8 cm. , AC = 10 cm. and D , E , F

are the midpoints of $\overline{AB}$, $\overline{CB}$, $\overline{AC}$

Find the perimeter of Δ DEF.



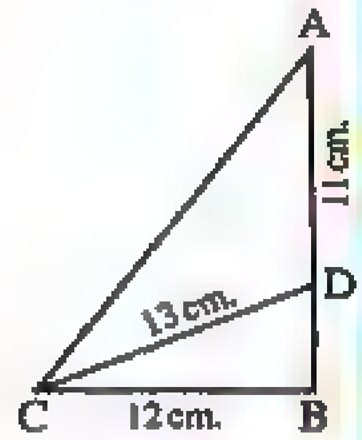
[b] In the opposite figure :

ABC is a triangle in which

m (∠ B) = 90° , D ∈ $\overline{AB}$ where AD = 11 cm. ,

BC = 12 cm. and DC = 13 cm.

Find the length of : $\overline{BD}$ and $\overline{AC}$



5 **Draw Δ ABC where A (1 , 5) , B (3 , 1) and C (5 , 3) , then draw its image :**

① By reflection in X-axis.

② By rotation about origin point with an angle of measure 180°.

4 Giza Governorate — Inspection of Mathematics

Experimental Directory

Answer the following questions :

1 **Choose the correct answer :**

① The sum of measures of accumulative angles at a point is ....

(a) 360° (b) 180° (c) 90° (d) 270°

② The image of (– 2 , 3) by rotation about the origin point with an angle of measure 180° is ....

(a) (– 3 , 2) (b) (3 , – 2) (c) (– 3 , – 2) (d) (2 , – 3)

③ The sum of measures of the interior angles of an octagon = ...

(a) 540° (b) 720° (c) 1080° (d) 900°

④ If ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = ...

(a) 70° (b) 180° (c) 90° (d) 110°

⑤ The image of (1 , – 2) by reflection in X-axis is ....

(a) (1 , 2) (b) (– 1 , – 2) (c) (2 , – 1) (d) (– 1 , 2)

2 **Complete each of the following :**

① The line segment which joins two midpoints of two sides of a triangle is ....

② The image of the point (3 , 1) by translation $(x - 2 , y - 1)$ is ....

③ If the two diagonals of a parallelogram are equal in length and not perpendicular , then it is called ....

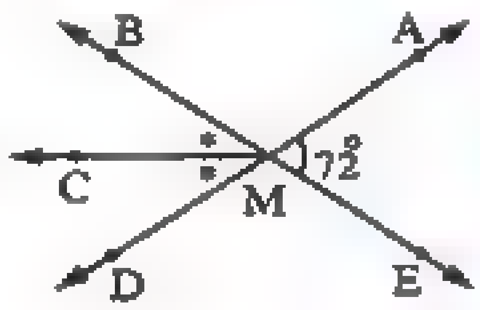
④ The measure of each angle of a regular pentagon = ...°

⑤ The sum of measures of the interior angles of the triangle = ...°

3 [a] **In the opposite figure :**

$\overleftrightarrow{DA} \cap \overleftrightarrow{EB} = \{M\}$, $\overrightarrow{MC}$ is a bisector of ($\angle$ BMD) and m ($\angle$ AME) = 72°

Find : m ($\angle$ CMD)

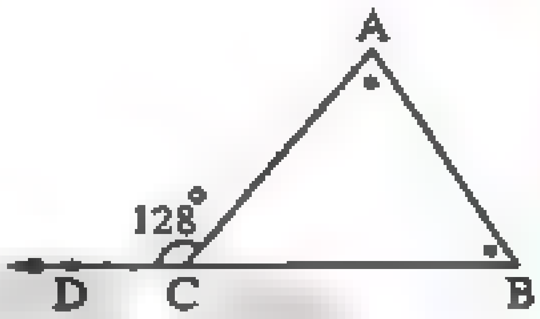


[b] **In the opposite figure :**

D $\in \overrightarrow{BC}$, m ($\angle$ ABC) = m ($\angle$ BAC)

and m ($\angle$ ACD) = 128°

Find : m ($\angle$ ABC)



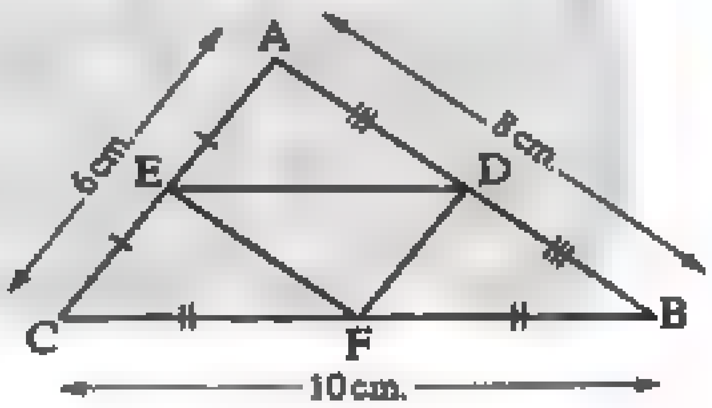
4 [a] Draw the image of Δ XYZ in which XY = 5 cm. , YZ = 4 cm. and ZX = 3 cm. by reflection in the straight line $\overleftrightarrow{YX}$

[b] **In the opposite figure :**

ABC is a triangle in which D , E and F are midpoints of $\overline{AB}$, $\overline{AC}$ and $\overline{CB}$ respectively.

If : AB = 8 cm. , BC = 10 cm. , AC = 6 cm.

Find the perimeter of : Δ DEF

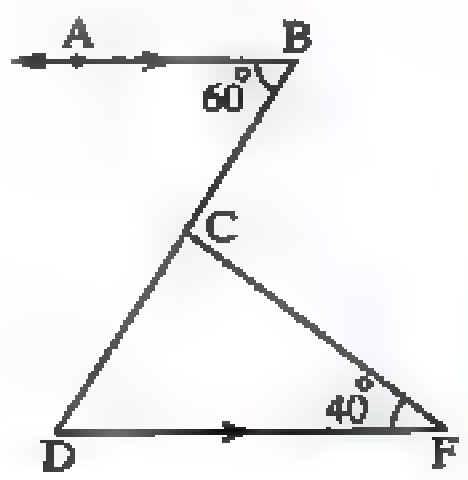


5 [a] **In the opposite figure :**

$\overrightarrow{BA}$ // $\overrightarrow{FD}$, m ($\angle$ B) = 60°

and m ($\angle$ F) = 40°

Find : m ($\angle$ DCF)



[b] Using the square lattice. Draw Δ ABC in which A (2 , 1) , B (1 , 4) and C (3 , 3) , **then find** its image by rotation about the origin point with an angle of measure 90°

5 Giza Governorate — Experimental Language Schools

Answer the following questions :

1 Choose the correct answer :

① ABCD is a parallelogram if m (∠ A) = 130° , then m (∠ B) = ...

(a) 130° (b) 50° (c) 40° (d) 65°

② The image of the point (3 , – 2) by reflection in X-axis is ....

(a) (3 , – 2) (b) (– 3 , – 2) (c) (3 , 2) (d) (– 3 , 2)

③ The sum of the measures of the interior angles of octagon is ....

(a) 720° (b) 900° (c) 1080° (d) 1260°

④ The sum of the measures of the exterior angles of pentagon is ....

(a) 540° (b) 360° (c) 180° (d) 720°

⑤ The measure of an interior angle of regular hexagon is ....

(a) 60° (b) 90° (c) 120° (d) 108°

2 Complete each of the following :

① The ray drawn from the midpoint of a side of a triangle parallel to another side ....

② The image of the point (3 , 2) by translation (– 1 , 3) is ....

③ ABC is a triangle , m (∠ B) = 90° , AB = 6 cm , AC = 10 cm , then BC = ... cm.

④ In the square , two diagonals are .... , ... and ....

⑤ A quadrilateral in which only two opposite sides are parallel is called a ....

3 [a] Using the square lattice draw Δ ABC : A = (5 , 5) , B = (5 , 1) , C = (1 , 1) , then find its image by rotation about the origin point with an angle of measure 180°

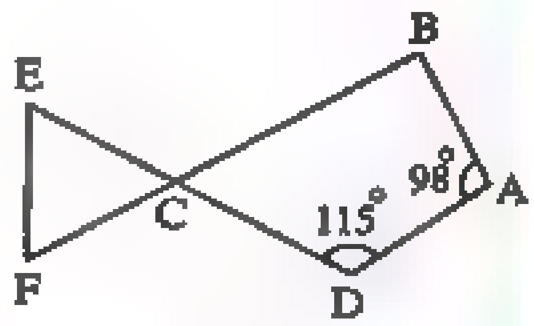
[b] In the opposite figure :

ABCD is a quadrilateral ,

m (∠ D) = 115° , m (∠ A) = 98°

, EC = CF = FE , $\overline{BF} \cap \overline{ED} = \{C\}$

Find with proof : m (∠ B)



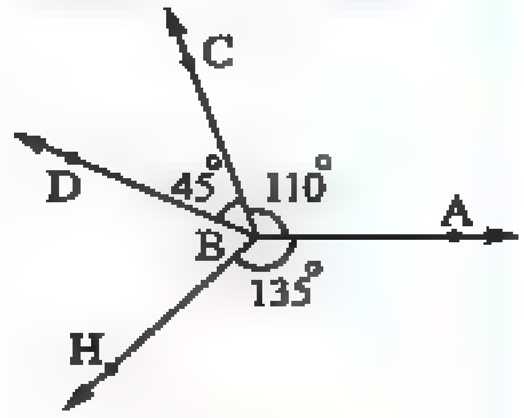
4 [a] In the opposite figure :

m (∠ ABC) = 110°

, m (∠ CBD) = 45°

, m (∠ ABH) = 135°

Find : m (∠ HBD)



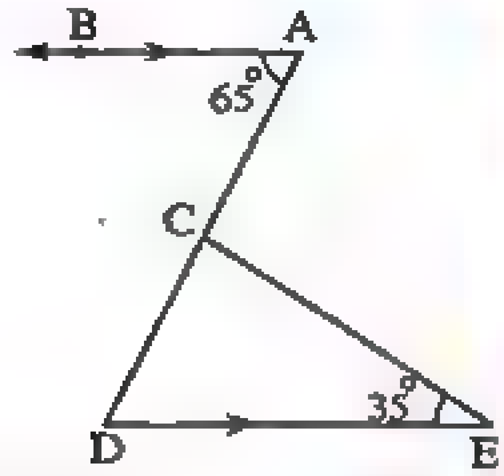
[b] In the opposite figure :

$\overrightarrow{AB} \mathbin{/\!/} \overrightarrow{ED}$,

m ($\angle$ A) = 65° ,

m ($\angle$ E) = 35°

Find with proof : m ($\angle$ ACE)



5 **[a] In the opposite figure :**

m ($\angle$ B) = m ($\angle$ ACD) = 90°

, AB = 3 cm. , BC = 4 cm. ,

CD = 12 cm.

Find : AD and perimeter of the figure ABCD

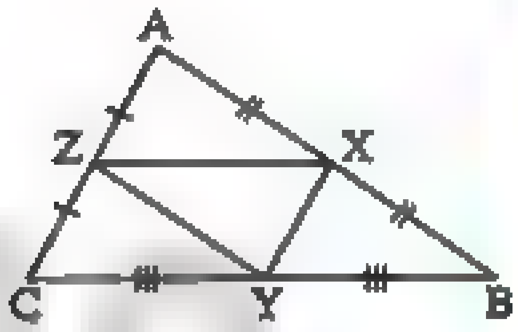
[b] In the opposite figure :

X , Y and Z are midpoints of $\overline{AB}$,

$\overline{BC}$ and $\overline{AC}$, AB = 10 cm ,

BC = 11 cm , AC = 9 cm

then find the perimeter of Δ XYZ with proof.



6 Alexandria Governorate — Al Montazah Zone

Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

① A parallelogram in which diagonals are equal in length is ....

(a) square. (b) rectangle. (c) rhombus. (d) trapezium.

② The image of the point (3 , 4) by reflection in y-axis is ....

(a) (– 3 , – 4) (b) (– 3 , 4) (c) (– 4 , 3) (d) (4 , – 3)

③ $(-2 , 1) \xrightarrow[\text{in } (0 , 0)]{\text{Reflection}}$ ....

(a) (2 , 1) (b) (2 , – 1) (c) (– 2 , – 1) (d) (1 , – 2)

④ The sum of measures of each two consecutive angles in a parallelogram is ....

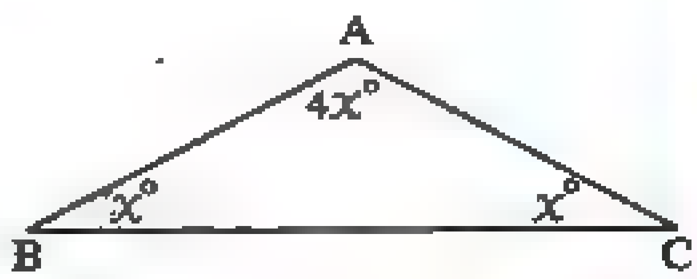
(a) 90° (b) 120° (c) 180° (d) 360°

⑤ **In the opposite figure :**

x = ...

(a) 20° (b) 30°

(c) 60° (d) 180°



2 **Complete each of the following :**

(1) The sum of the measures of the accumulative angles at a point is equal to ⋯⋯

(2) If ABCD is a rhombus , then ⋯⋯⋯ ⊥ ⋯⋯⋯

(3) The image of the point P (5 , 4) under translation $(x , y) \longrightarrow (x + 4 , y - 5)$ is ⋯⋯⋯

(4) If ABC is a right-angled triangle at B , then $(AB)^2$ = ⋯⋯⋯ – ⋯⋯⋯

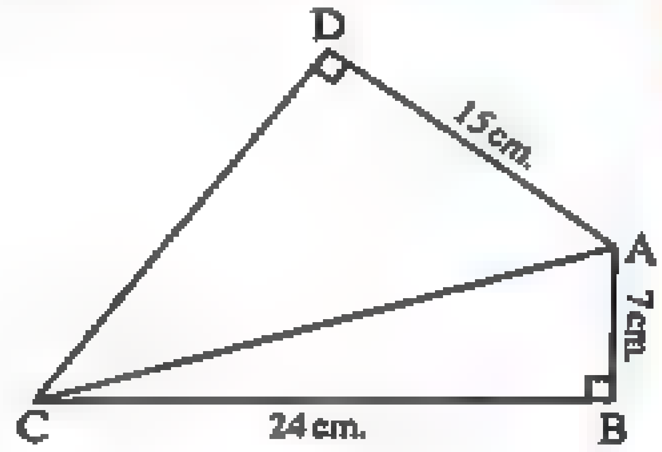
(5) The ray drawn from the midpoint of a side of a triangle parallel to another side ⋯⋯⋯

3 [a] **In the opposite figure :**

m (∠ D) = m (∠ B) = 90° , AB = 7 cm. ,

BC = 24 cm. and AD = 15 cm.

Find with proof : AC and DC.



[b] How many sides does a regular polygon have if the measure of each interior angle of it is 120° ?

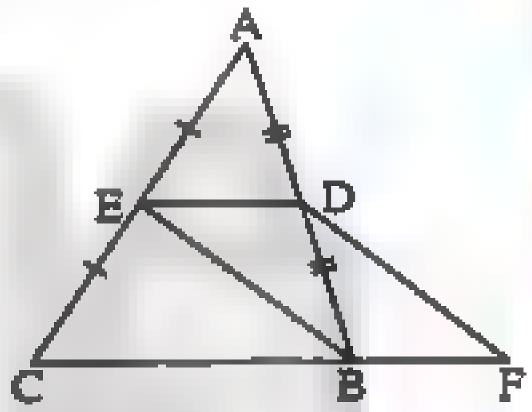
4 [a] **In the opposite figure :**

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively ,

$F \in \overrightarrow{CB}$ where $BF = \frac{1}{2}$ BC

Prove that :

BEDF is a parallelogram.



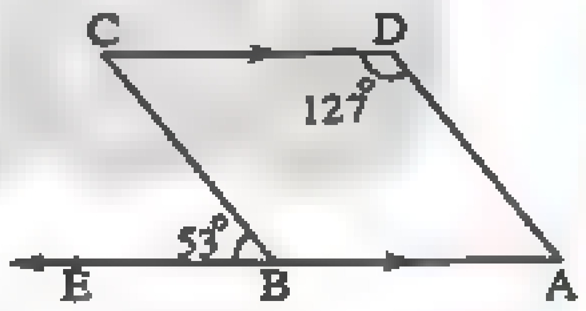
[b] On the lattice , find the image of the trianle ABC Where A (– 4 , – 1) , B (– 1 , – 3) and C (0 , – 1) by reflection on the x-axis.

5 [a] **In the opposite figure :**

$\overrightarrow{DC} \,//\, \overrightarrow{AB}$, $E \in \overleftrightarrow{AB}$,

m (∠ D) = 127° , m (∠ CBE) = 53°

Prove that : $\overline{AD} \,//\, \overline{BC}$.



[b] On graph paper draw the points A (0 , 0) , B (0 , 2) , C (4 , 2) and D (4 , 0). Draw the image formed by rotating the quadrilateral ABCD about the origin through an angle of measure 90°

7 Alexandria Governorate — Central Zone of Education E.G.C.

Answer the following questions :

1 **Choose the correct answer :**

(1) The sum of the measures of the interior angles of a triangle = the measure of the ⋯⋯⋯ angle.

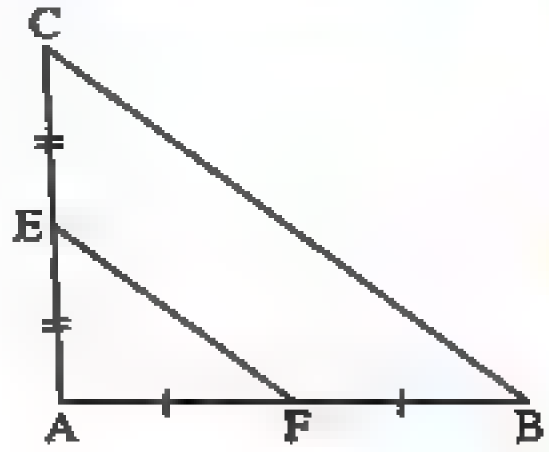
(a) acute (b) right (c) obtuse (d) straight

② The rhombus in which its two diagonals are equal in length is called a ....

(a) parallelogram. (b) square. (c) rectangle. (d) trapezium.

③ **In the opposite figure :**

ABC is a triangle in which F and E are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively. If BC = 16 cm. , AB = 12 cm. and AC = 10 cm. , then the perimeter of Δ AFE = ... cm.



(a) 24 (b) 27 (c) 19 (d) 25

④ The image of the point (– 3 , 7) by rotation about the origin point with an angle of measure – 90° is ....

(a) (– 7 , – 3) (b) (7 , – 3) (c) (7 , 3) (d) (3 , – 7)

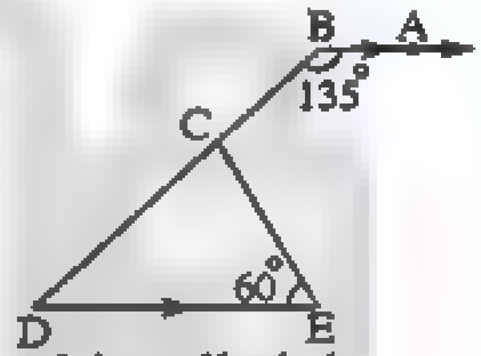
⑤ ABCD is a parallelogram in which m ($\angle$ A) = 55° , then m ($\angle$ B) = ...

(a) 35° (b) 125° (c) 55° (d) 180°

2 Complete each of the following :

① **In the opposite figure :**

If $\overrightarrow{BA}$ // $\overrightarrow{DE}$, m ($\angle$ DBA) = 135° and m ($\angle$ CED) = 60° , then m ($\angle$ DCE) = ...°

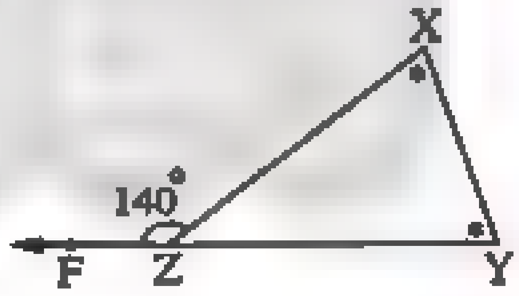


② The rotation about the origin point with an angle of measure ....° is called the neutral rotation.

③ The image of the point (4 , – 2) by reflection in the y-axis is ....

④ **In the opposite figure :**

If F $\in$ $\overrightarrow{YZ}$, m ($\angle$ X) = m ($\angle$ Y) and m ($\angle$ XZF) = 140° , then m ($\angle$ X) = ...°



⑤ In Δ ABC if m ($\angle$ C) > m ($\angle$ A) + m ($\angle$ B) , then $\angle$ C is .... angle.

3 [a] **In the opposite figure :**

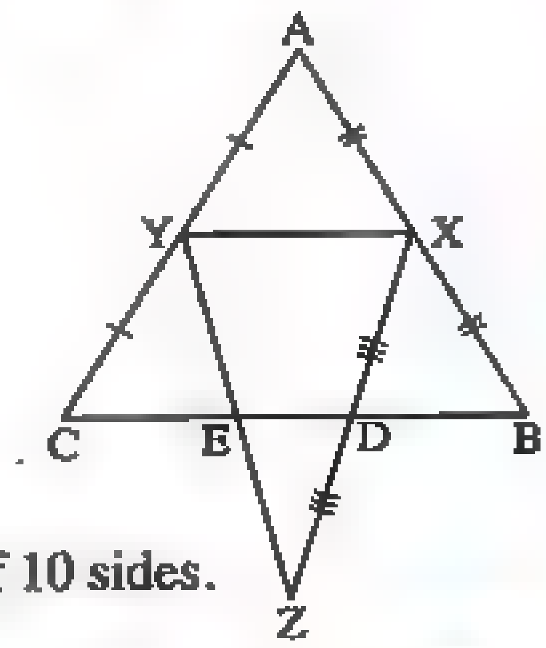
X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$

and XD = DZ

Prove that :

YE = EZ



[b] Calculate the measure of each interior angle of a regular polygon of 10 sides.

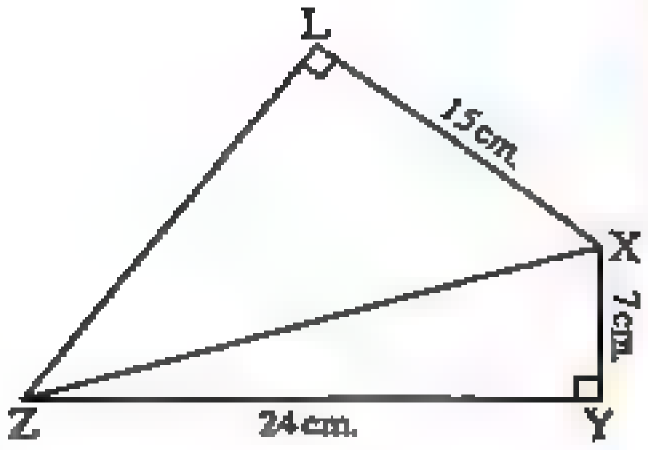
4 **[a] In the opposite figure :**

If m (∠ XYZ) = m (∠ XLZ) = 90° ,

XY = 7 cm. , YZ = 24 cm. and XL = 15 cm.

Find with proof :

The length of $\overline{XZ}$ and $\overline{LZ}$



[b] If the point $\grave{A}$ (– 6 , 4) is the image of point A by the translation of LM in the direction of $\overrightarrow{LM}$ where L (2 , 3) and M (1 , 5). **Find :** the point A

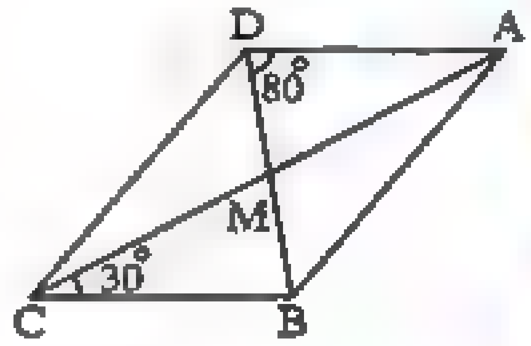
---

5 **[a] In the opposite figure :**

ABCD is a parallelogram ,

m (∠ BCA) = 30° , m (∠ ADB) = 80°

Find with proof : m (∠ CMB)



[b] Triangle ABC whose vertices A (1 , 5) , B (3 , 1) , C (5 , 3).

Draw the triangle and its image by the rotation about the origin point with an angle of measure 180°

8 El - Kalyoubia Governorate — Educational Directorate Zone

Answer the following questions :

1 **Choose the correct answer :**

① The image of the point (3 , – 2) by reflection in y-axis is ....

(a) (3 , 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (– 2 , 3)

② The measure of each interior angle of a regular pentagon is ....°

(a) 90 (b) 120 (c) 108 (d) 131

③ The measure of any of the exterior angle of an equilateral triangle = ...°

(a) 120 (b) 60 (c) 45 (d) 30

④ In Δ ABC : if m (∠ A) > m (∠ B) + m (∠ C) , then angle A is ....

(a) acute. (b) right. (c) straight. (d) obtuse.

⑤ The image of the point (– 1 , 3) by translation (4 , – 2) is ....

(a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)

2 **Complete each of the following :**

(1) The rectangle is a parallelogram in which one of its angles is ....

(2) The line segment joining the midpoints of two sides of a triangle is ....

(3) The image of the point (3 , 2) by rotation with an angle of measure 180° about the origin is ....

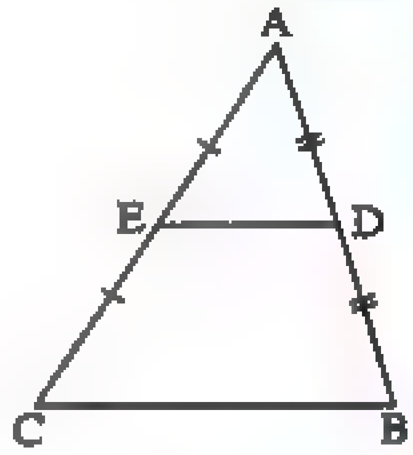
(4) If Δ ABC is a right-angled triangle at B , then $(AC)^2$ = ... + ...

(5) If ABCD is a square , then m ($\angle$ ABD) = ...°

3 **[a] In the opposite figure :** Δ ABC in which

AB = 12 cm. , BC = 10 cm. and AC = 8 cm.

Find with proof : the perimeter of Δ ADE



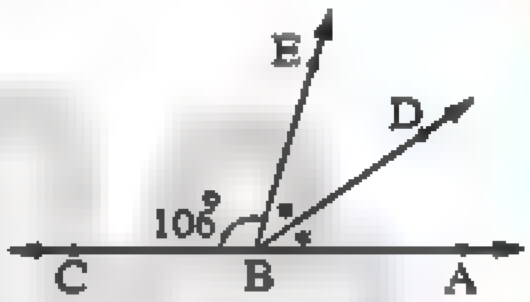
[b] In the opposite figure :

B $\in \overleftrightarrow{AC}$,

m ($\angle$ CBE) = 106° and

$\overrightarrow{BD}$ bisects $\angle$ ABE

Find with proof : m ($\angle$ ABD)



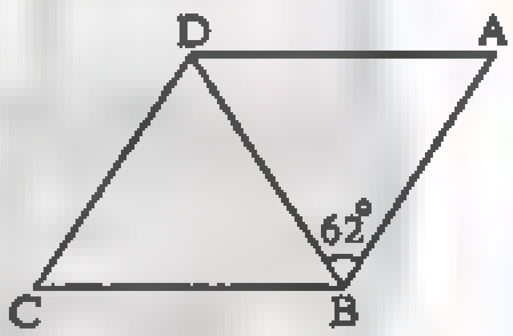
4 **[a]** Using the lattice , draw Δ ABC where A (1 , 0) , B (0 , 2) and C (– 3 , 1)

Then draw its image by reflection in x-axis

[b] In the opposite figure :

ABCD is a rhombus in which m ($\angle$ ABD) = 62°

Find with proof : m ($\angle$ A)

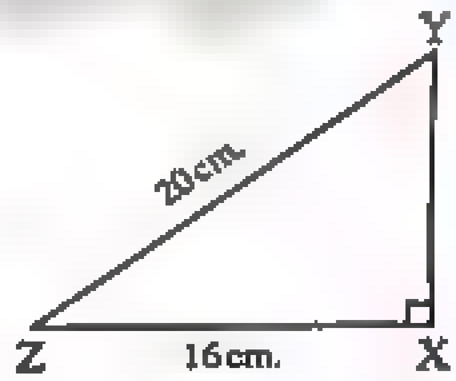


5 **[a] In the opposite figure :**

XYZ is a triangle in which m ($\angle$ X) = 90° ,

YZ = 20 cm. and XZ = 16 cm. ,

Find with proof : the length of $\overline{XY}$



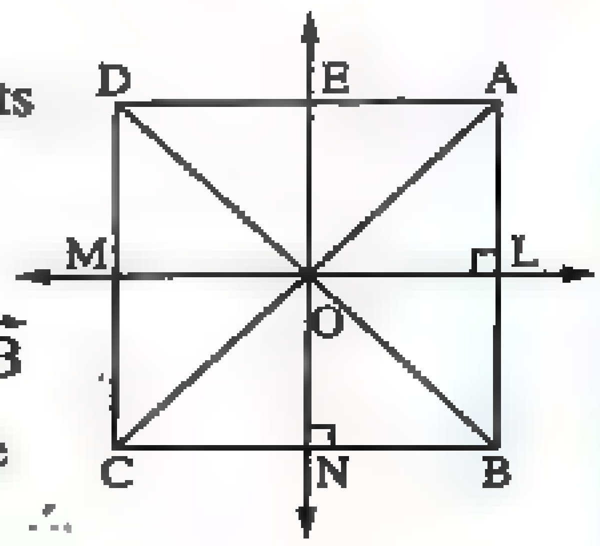
[b] In the opposite figure :

ABCD is a square of side length 6 cm and the origin point its center. **Find :**

(1) The image of Δ AOE by reflection around point O

(2) The image of Δ AOL by translation 3 cm in direction $\overrightarrow{AB}$

(3) The image Δ AOL by rotation about O and with an angle of measure (– 90°)



9 El-Sharkia Governorate — Directorate of Education

Mathematics Supervision for E.L.S

Answer the following questions :

1 Complete each of the following :

(1) The sum of measures of the interior angles of a triangle is .... °

(2) If Δ ABC is right-angled at B , then $(AC)^2$ = ... + ...

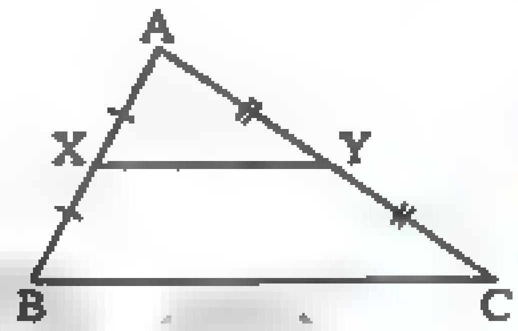
(3) The line segment joining the midpoints of two sides of a triangle is parallel to .. ..

(4) Sum of measures of the interior angles of a pentagon = ...°

(5) The image of the point (3 , 5) by the transformation $(x , y) \longrightarrow (x + 1 , y - 3)$ is ....

2 Choose the correct answer :

(1) **In the opposite figure :** XY = ... BC



(a) $\frac{1}{2}$ (b) 2 (c) 3 (d) $\frac{1}{3}$

(2) The polygon of 6 sides is called ....

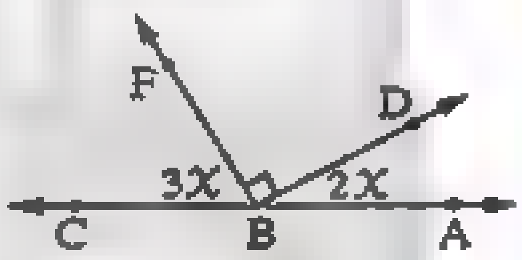
(a) pentagon. (b) hexagon. (c) octagon. (d) heptagon.

(3) The rhombus with right angle is called ....

(a) square. (b) triangle. (c) rectangle. (d) trapezium.

(4) **In the opposite figure :** B $\in \overleftrightarrow{AC}$ and

m ($\angle$ DBF) = 90° , then m ($\angle$ ABD) = ...



(a) 180° (b) 90° (c) 36° (d) 45°

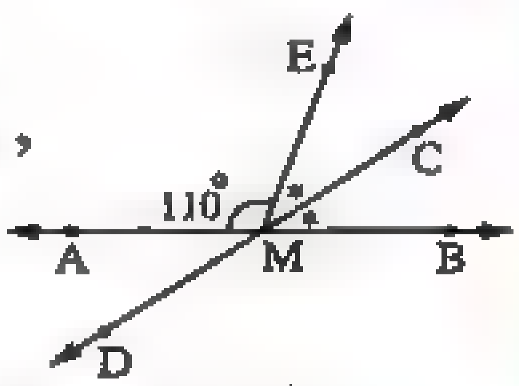
(5) The triangle has at least two .... angles.

(a) obtuse (b) right (c) acute (d) straight

3 [a] In the opposite figure :

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, m ($\angle$ AME) = 110° , $\overrightarrow{MC}$ bisects $\angle$ BME ,

Find with proof : m ($\angle$ AMD)



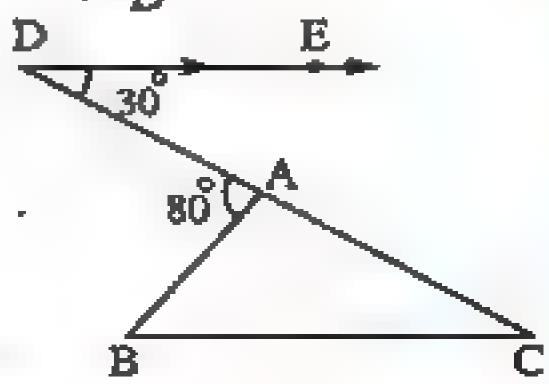
[b] In the opposite figure :

D $\in \overrightarrow{CA}$, $\overrightarrow{DE}$ // $\overrightarrow{BC}$,

m ($\angle$ DAB) = 80°

, m ($\angle$ D) = 30°

Find : m ($\angle$ ABC)



4 [a] Find the measure of each interior angle of regular hexagon.

[b] In a coordinates plane. Draw the points A (2 , 2) , B (5 , 2) , C (5 , 5) , then find $\Delta \grave{A}\grave{B}\grave{C}$ the image of Δ ABC by reflection in they y-axis.

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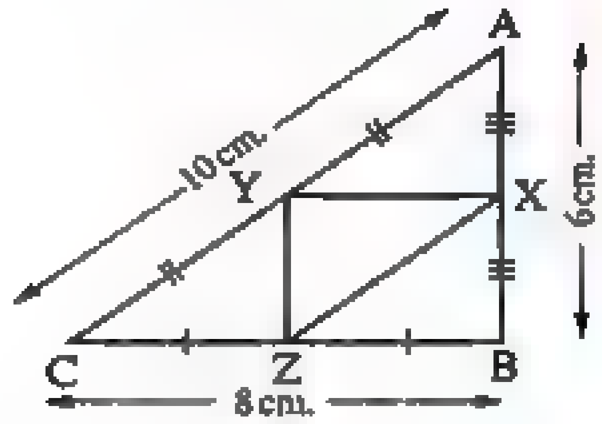
5 **In the opposite figure :**

AB = 6 cm. , BC = 8 cm. , AC = 10 cm.

X , Y and Z are midpoints of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ respectively ,

Find : the perimeter of Δ XYZ



10 Suez Governorate — Suez Educational Directorate

Mathematics inspectorate

Answer the following questions :

1 **Choose the correct answer :**

(1) In parallelogram if the two adjacent sides are equal in the length , then the shape is ....

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

(2) The measure of any of exterior angles of an equilateral triangle equals ....

(a) 60° (b) 180° (c) 120° (d) 90°

(3) Any triangle has at least two .... interior angles.

(a) acute (b) right (c) obtuse (d) straight

(4) The measures of each angle of regular hexagon equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

(5) The image of the point (2 , – 5) by reflection in y-axis is ....

(a) (2 , – 5) (b) (2 , 5) (c) (– 2 , – 5) (d) (5 , 2)

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2 **Complete each of the following :**

(1) The length of a line segment joining the midpoints of two sides of a triangle equals ....

(2) If the image of the point (– 1 , 3) by translation is (1 , 4) , then the image of the point (3 , – 2) by the same translation is ....

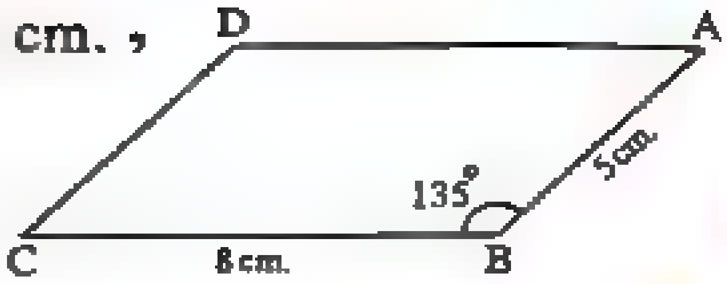
(3) The sum of the measure of interior angles of a triangle equals ....

(4) The parallelogram whose diagonals are equal in length and perpendicular is ....

(5) The image of the point (3 , 2) by rotation with an angle of measure 180° about the origin is ....

3 **[a] In the opposite figure :**

ABCD is parallelogram in which AB = 5 cm. , BC = 8 cm. , m (∠ B) = 135°



Find : ① m (∠ C)

② The perimeter of parallelogram ABCD

[b] Draw Δ OBC on a square lattice where O (0 , 0) , B (3 , 0) , C (0 , 4) , then find its image by rotation about the origin point with an angle of measure 180°

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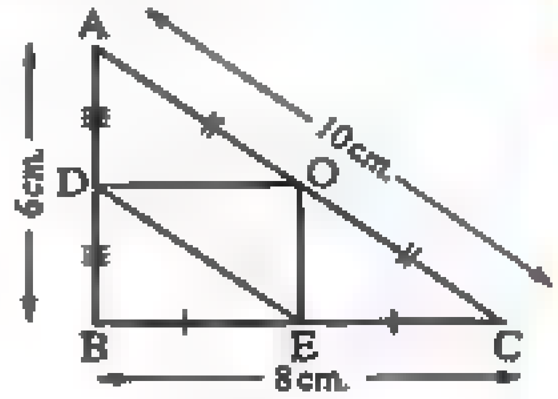
4 **[a] In the opposite figure :**

D , E and O , are midpoint of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively AB = 6 cm. , BC = 8 cm. , AC = 10 cm.

Find the perimeter of : Δ DEO



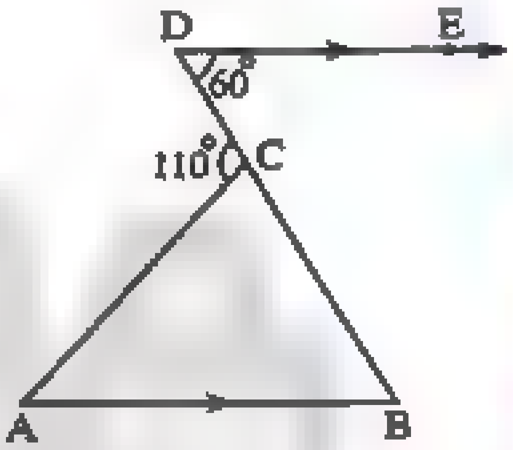
[b] In the opposite figure :

$\overrightarrow{AB}$ // $\overrightarrow{DE}$,

m (∠ D) = 60°

, m (∠ ACD) = 110°

Find with proof : m (∠ A)



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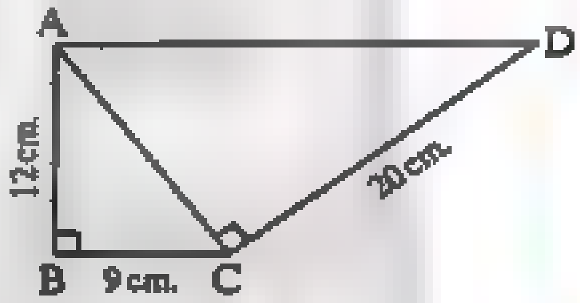
5 **[a] In the opposite figure :**

m (∠ B) = m (∠ ACD) = 90° , AB = 12 cm. , BC = 9 cm. , CD = 20 cm.

Find the length of : $\overline{AC}$, $\overline{AD}$



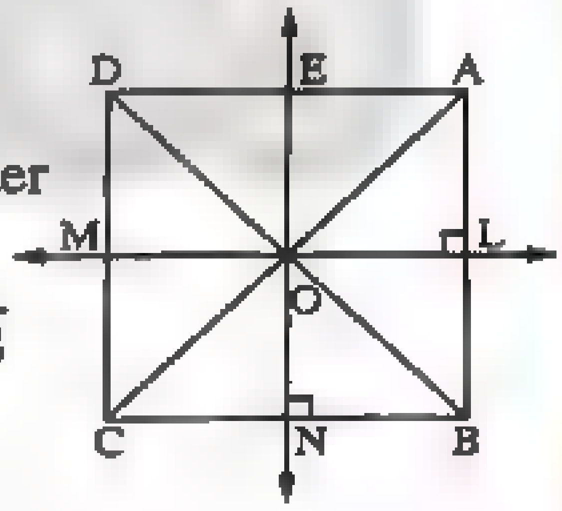
[b] In the opposite figure :

ABCD is square of side length 6 cm. and origin point its center

Find :

① The image of Δ AOL by translation 3 cm. in direction $\overrightarrow{AB}$

② The image of Δ AOL by reflection in $\overleftrightarrow{EN}$



11 El-Beheira Governorate — Bandar Kafr El-Dawar Educational

Mathematics inspectorate

Answer the following questions :

1 **Choose the correct answer :**

① ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = ……

(a) 70° (b) 110° (c) 140° (d) 210°

(2) The measure of the interior angle of regular pentagon = ...

(a) 108° (b) 120° (c) 135° (d) 540°

(3) If two adjacent sides are equal in length in a parallelogram , then the figure is a ....

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

(4) The sum of the measure of the exterior angles of a triangle = ...

(a) 90° (b) 180° (c) 360° (d) 720°

(5) The triangle contain two .... angles at least.

(a) acute (b) obtuse (c) right (d) reflex

2 **Complete each of the following :**

(1) The line segment joining the midpoints of two sides of a triangle is .... the third side.

(2) In Δ ABC : If $(AB)^2 = (BC)^2 + (AC)^2$, then m ($\angle$ ....) = 90°

(3) The image of the point (– 3 , 2) by reflection in y-axis is ....

(4) The image of the point (– 3 , – 4) by the translation (x – 4 , y + 1) is ....

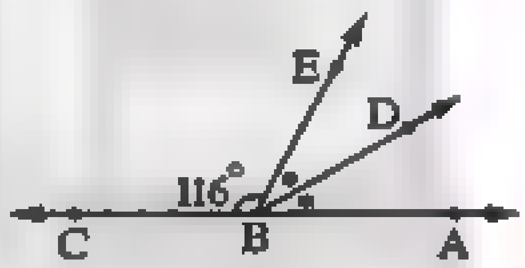
(5) The image of the point (– 4 , 2) by rotation around the origin point with an anlge of measure 90° is ....

3 **[a] In the opposite figure :**

B $\in \overleftrightarrow{AC}$, m ($\angle$ CBE) = 116°

and $\overrightarrow{BD}$ bisects $\angle$ ABE

Find : m ($\angle$ ABD)



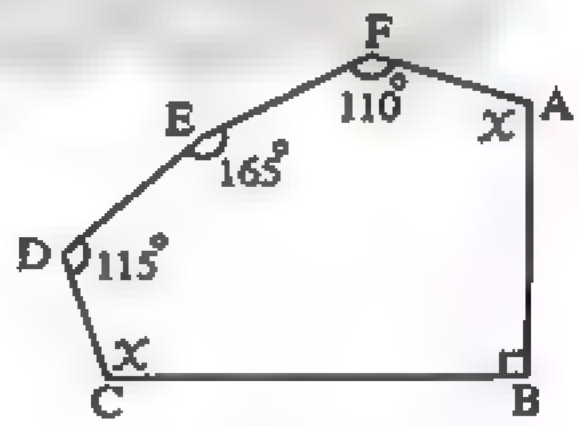
[b] In the opposite figure :

ABCDEF is a hexagon , m ($\angle$ B) = 90° ,

m ($\angle$ F) = 110° , m ($\angle$ E) = 165° , m ($\angle$ D) = 115° ,

m ($\angle$ FAB) = m ($\angle$ DCB) = x

Find : the value of x



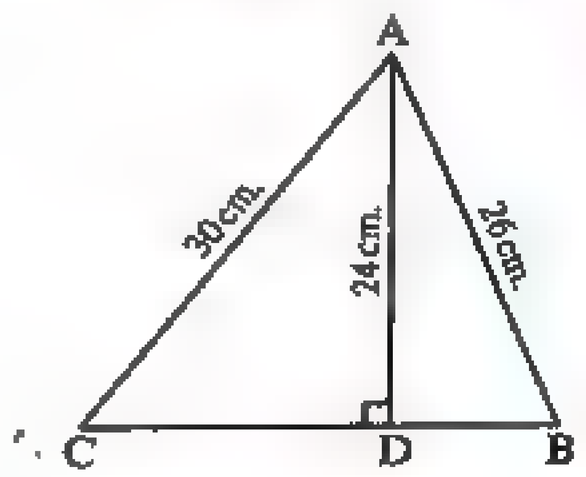
4 **[a] In the opposite figure :**

ABC is a triangle in which $\overline{AD} \perp \overline{BC}$,

If AD = 24 cm. , AB = 26 cm.

, AC = 30 cm.

Find the length of : $\overline{BC}$



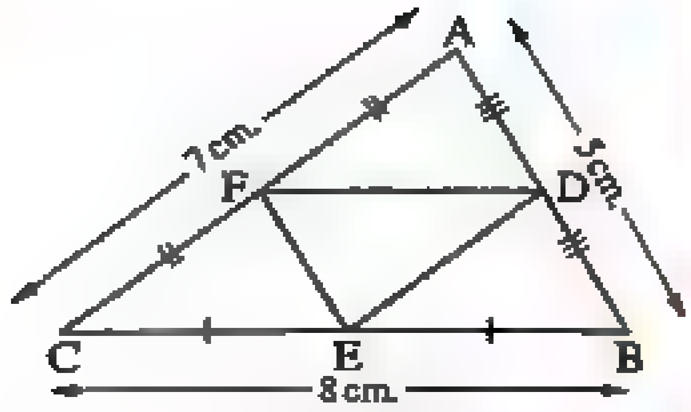
[b] In the opposite figure :

AB = 5 cm. , BC = 8 cm. ,

AC = 7 cm. , D , E and F are

the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

Calculate the perimeter of : Δ DEF



5 Draw the triangle ABC in which A (1 , – 1) , B (2 , 3) and C (0 , 4) then find :

(1) The image of Δ ABC by reflection in the origon point.

(2) The image of Δ ABC by rotation R (0 , – 90°)

12 Beni Suef Governorate Directorate of Experimental Language

Education Administration

Answer the following questions :

1 Choose the correct answer :

(1) A quadrilateral in which only two opposite sides are parallel is called a ....

(a) parallelogram. (b) rhombus. (c) trapezium. (d) square.

(2) The parallelogram whose diagonals are equal in length and not perpendicular is called ....

(a) square. (b) rectangle. (c) rhombus. (d) trapezium.

(3) The sum of the measures of the interior angles of a triangle is ....

(a) 90° (b) 180° (c) 270° (d) 360°

(4) The image of the point (– 4 , 5) by translation (2 , – 3) is ....

(a) (– 6 , – 8) (b) (– 2 , 2) (c) (– 2 , – 2) (d) (2 , – 2)

(5) The image of the point (5 , – 3) by rotation about the origin point with an angle of measure 90° is ....

(a) (– 5 , – 3) (b) (5 , 3) (c) (3 , 5) (d) (– 3 , 5)

2 Complete each of the following :

(1) A square is a .... with a right angle.

(2) The line segment joining the midpoints of two sides of a triangle is ....

(3) The ray drawn from the midpoint of a side of a triangle parallel to another side ....

(4) The image of the point (4 , 6) by geometric transformation $(x , y) \longrightarrow (-x , y - 7)$ is ....

(5) The measure of the exterior angle of the equilateral triangle at any one of its vertices equals ....°

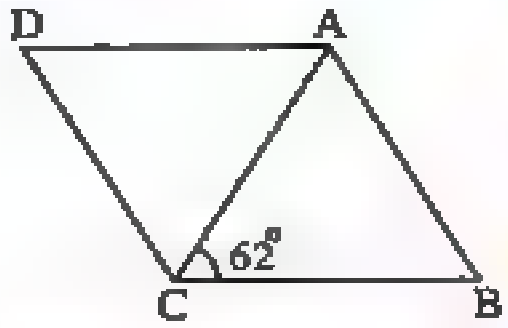
3 [a] Calculate the sum of the measures of the interior angles of a hexagon.

[b] In the opposite figure :

ABCD is a rhombus in which ,

m (∠ ACB) = 62° ,

Find with proof : m (∠ B)



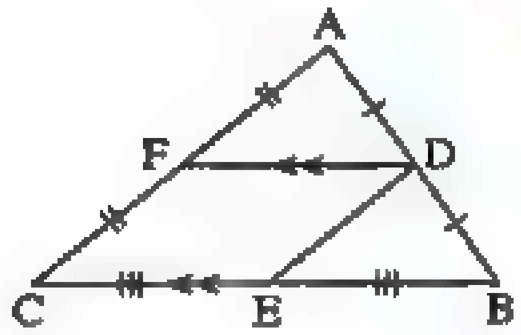
4 **[a] In the opposite figure :**

ABC is a triangle in which ,

D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

respectively. BC = 12 cm. , AC = 10 cm.

Find the perimeter of the quadrilateral DECF

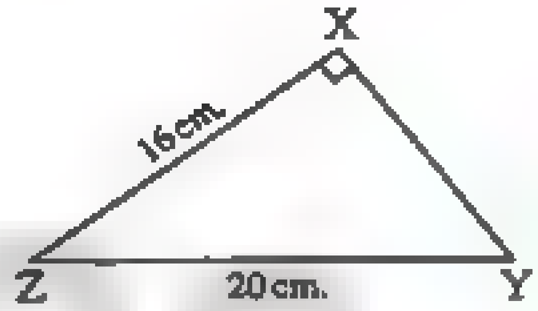


[b] In the opposite figure :

XYZ is a triangle in which ,

m (∠ X) = 90° , YZ = 20 cm. , XZ = 16 cm.

Find the length of $\overline{XY}$



5 On a square lattice , draw Δ XYZ where : X (2 , 4) , Y (4 , 2) and Z (1 , 1) , then draw :

① The image of the Δ XYZ by reflection in the X-axis.

② The image of the Δ XYZ by rotation about the origin point with an angle of measure 180°

13 El - Menia Governorate — El-Menia Educational Directorate

Answer the following questions :

1 **Choose the correct answer :**

① The measure of each interior angle of the regular hexagon = ...

(a) 108° (b) 120° (c) 136° (d) 114°

② ABCD is a parallelogram in which m (∠ A) = 60° , then angle m (∠ C) = ...

(a) 60° (b) 180° (c) 120° (d) 90°

③ The image of the point (3 , – 5) by reflection in the y-axis is ....

(a) (3 , 5) (b) (– 3 , – 5) (c) (– 3 , 5) (d) (– 5 , 3)

④ In Δ ABC , m (∠ A) = m (∠ B) + m (∠ C) , then m (∠ A) = ...

(a) 30° (b) 90° (c) 60° (d) 150°

(5) The image of the point (3 , 2) by rotation about the origin point with an angle of 180° is ....

(a) (– 3 , – 2) (b) (– 2 , 3) (c) (2 , – 3) (d) (3 , 2)

2 **Complete each of the following :**

(1) The parallelogram whose diagonals are perpendicular and not equal in length is a ....

(2) The image of the point (5 , 3) by translation $(x , y) \longrightarrow (x + 3 , y - 1)$ is ....

(3) The measure of the exterior angle of equilateral triangle is ....

(4) The sum of measures of the interior angles of a triangle = ...°

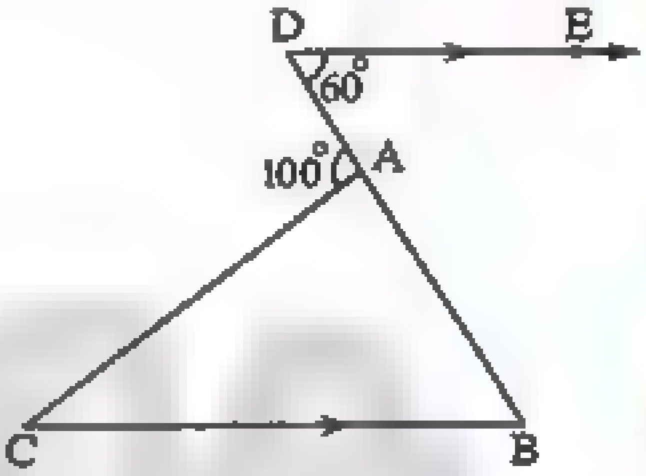
(5) The ray drawn from the midpoint of a side of a triangle parallel to another side ....

3 **[a] In the opposite figure :**

$\overrightarrow{DE} \,/\!/\, \overline{BC}$, m (∠ D) = 60° ,

m (∠ DAC) = 100° ,

Find : m (∠ C)

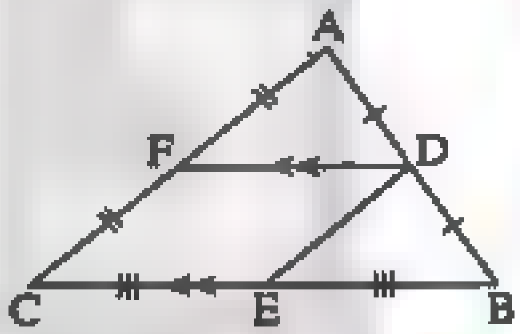


[b] In the opposite figure :

Δ ABC , D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively , BC = 12 cm.

, AC = 10 cm. ,

Find : the perimeter of the quadrilateral DECF

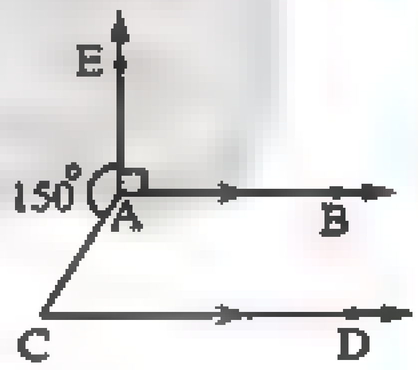


4 **[a] In the opposite figure :**

$\overrightarrow{AB} \,/\!/\, \overrightarrow{CD}$, m (∠ EAC) = 150°

, $\overrightarrow{AE} \perp \overrightarrow{AB}$

Find : m (∠ BAC) , m (∠ C)



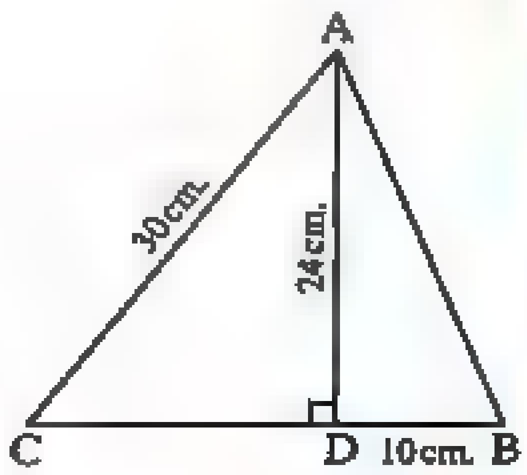
[b] On a square lattic. Draw Δ ABC where A (1 , 1) , B (4 , 1) , C (4 , 4) , then draw its image by reflection in x-axis.

5 **[a] In the opposite figure :**

Δ ABC , $\overline{AD} \perp \overline{BC}$, AD = 24 cm.

, BD = 10 cm. , AC = 30 cm.

Find : the length of $\overline{BC}$ and the area of Δ ABC



[b] On a square lattice. Draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1) , then draw its image by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

14 Aswan Governorate — Aswan Educational Directorate

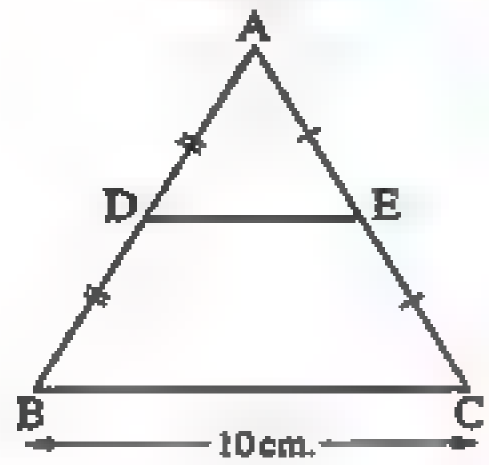
Answer the following questions :

1 Choose the correct answer :

① The measure of each interior angle of a regular pentagon = ...

(a) 90° (b) 120° (c) 108° (d) 135°

② **In the opposite figure :**



If BC = 10 cm. ,

then DE = ... cm.

(a) 5 (b) 10

(c) 20 (d) 40

③ The image of the point (3 , 0) by reflection in origin point is ....

(a) (– 3 , 0) (b) (0 , 3) (c) (0 , – 3) (d) (3 , 0)

④ ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 110° then m (∠ B) = ...°

(a) 77 (b) 55 (c) 180 (d) 125

⑤ The sum of the measures of interior angles of a triangle = ...°

(a) 360 (b) 180 (c) 120 (d) 60

2 Complete each of the following :

① The image of the point (– 1 , 2) by translation (1 , 3) is ....

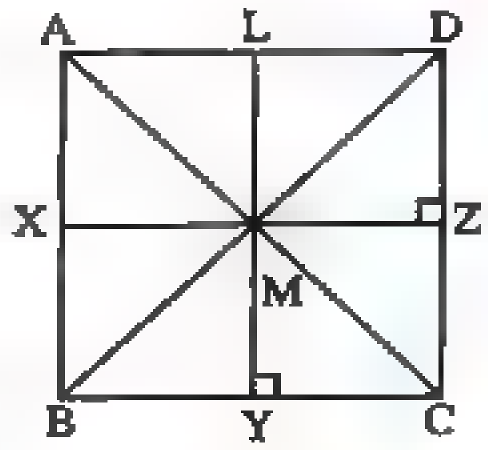
② If ABCD is a square , then m (∠ BAC) = ...°

③ The image of the point (4 , – 5) by rotation about the origin point with an angle of measure 90° is ....

④ The sum of the measure of the accumulative angles at a point = ...°

⑤ The measure of the exterior angle of the equilateral triangle = ...°

3 [a] In the opposite figure :



ABCD is a square , where M is the intersection of its diagonals X , Y , Z , L are midpoints of its sides.

Find : ① The image of the point A by reflecting in $\overleftrightarrow{LY}$

② The image of Δ ALM by reflection in $\overleftrightarrow{XZ}$

[b] In the opposite figure :

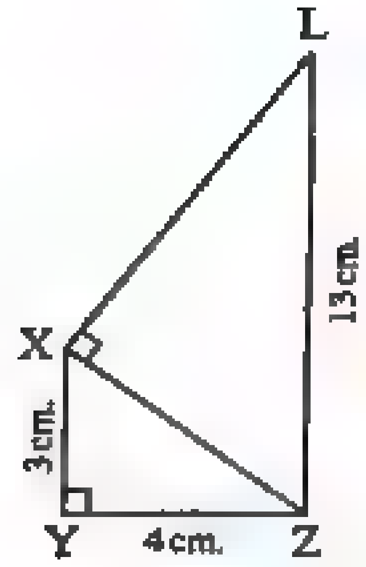
XYZL is quadrilateral

where m (∠ XYZ) = m (∠ LXZ) = 90°

XY = 3 cm. , YZ = 4 cm.

, LZ = 13 cm.

Find : the length of each $\overline{XZ}$, $\overline{XL}$



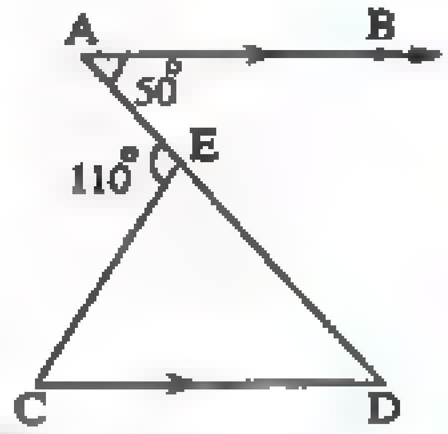
4 [a] If the ratio between the measure of angles of a quadrilateral is 2 : 3 : 3 : 4 , then find the measure of the smallest angle.

[b] In the opposite figure :

$\overrightarrow{AB}$ // $\overrightarrow{CD}$, m (∠ A) = 50° ,

m (∠ AEC) = 110°

Find with proof : the measures of interior angles in the Δ EDC



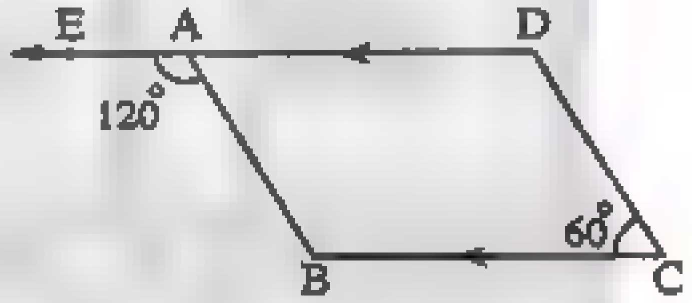
5 [a] Using the lattice. draw Δ ABC where : A (2 , 3) , B (4 , 2) , C (1 , 2) , then draw its image by rotation about the origin point with an angle of measure 180°

[b] In the opposite figure :

E ∈ $\overrightarrow{DA}$, m (∠ EAB) = 120°

, m (∠ C) = 60° , $\overrightarrow{DA}$ // $\overrightarrow{CB}$

Prove that : ABCD is a parallelogram.



15 Southern Sinai Governorate — Directorate of Education

Tur Sinai Educational Idara

Answer the following questions :

1 Choose the correct answer :

(1) The sum of the measures of the accumulative angles at a point is equal to ....°

(a) 360 (b) 270 (c) 180 (d) 90

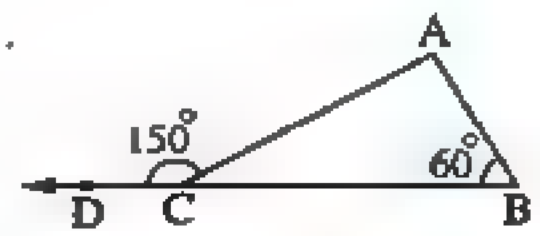
(2) The measure of each angle of a regular pentagon = ...°

(a) 90 (b) 108 (c) 120 (d) 180

(3) **In the opposite figure :** m (∠ A) = ...°

(a) 30 (b) 60

(c) 90 (d) 150



(4) Any triangle has two .... angles at least.

(a) straight (b) obtuse (c) right (d) acute

(5) The image of the point $(4 , -2)$ by translation $(-3 , 3)$ ....

(a) $(1 , -5)$ (b) $(7 , 1)$ (c) $(7 , -5)$ (d) $(1 , 1)$

2 **Complete each of the following :**

(1) The rectangle is .... but its angles are right angles.

(2) The line segment joining between the two midpoints of two sides of a triangle .... the third side.

(3) The image of the point $(3 , 2)$ by reflection in an origin is ....

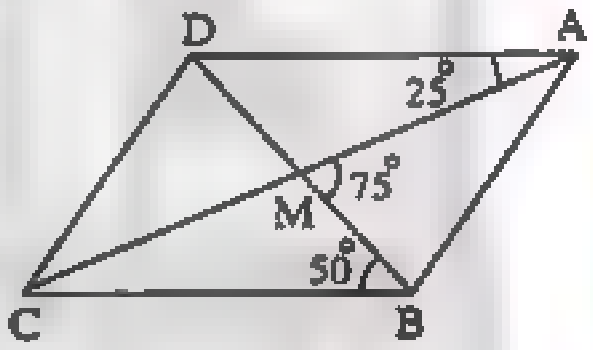
(4) In the right-angled triangle the area of the square on .... equals the sum of areas of the squares on the other two sides.

(5) If the image of the point $(-2 , 0)$ by rotation at origin is $(0 , -2)$, then the measure of the angle of rotation = ...°

3 [a] If the measure of the angle of a regular polygon is 135° ,

Find : the number of sides of this polygon.

[b] In the opposite figure :



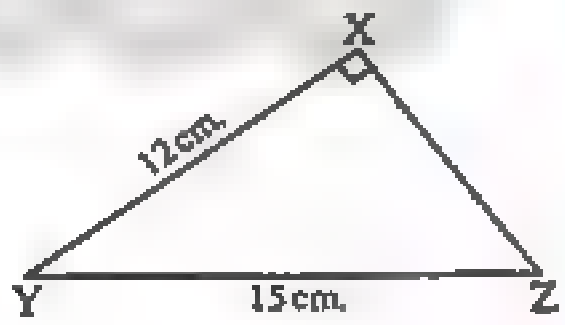
$\overline{AB} \parallel \overline{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$,

$m(\angle DBC) = 50°$, $m(\angle DAC) = 25°$,

$m(\angle AMB) = 75°$

Prove that : ABCD is a parallelogram.

4 **[a] In the opposite figure :**

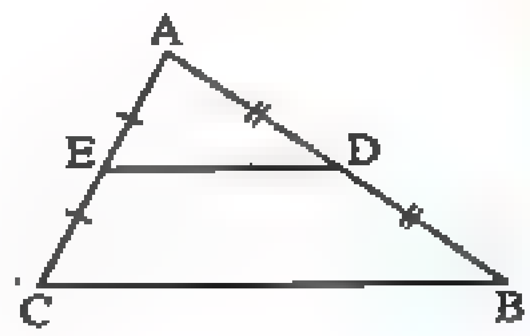


XYZ is a triangle such that

$m(\angle X) = 90°$, XY = 12 cm. , YZ = 15 cm.

Find : the length of $\overline{XZ}$

[b] In the opposite figure :



ABC is a triangle such that

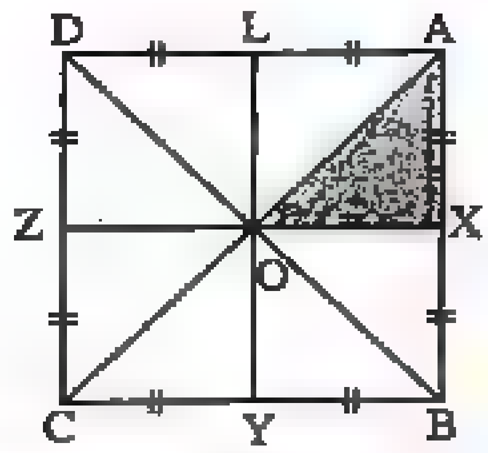
D is the midpoint of $\overline{AB}$,

E is the midpoint of $\overline{AC}$,

AB = 12 cm. , AC = 8 cm. , BC = 10 cm.

Find : the perimeter of Δ ADE

5 **In the opposite figure :**



ABCD is a square , $\overline{AC} \cap \overline{BD} = \{O\}$

X , Y , Z and L are midpoints of

$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ respectively.

Find : ① The image of Δ AXO by reflection in $\overleftrightarrow{AO}$

② The image of Δ AXO by translation of magnitude AO in direction $\overrightarrow{AO}$

③ The image of Δ AXO by rotation R (0 , 180°)

Answers of school book models in geometry

Model 1

1 (1) (b) (2) (a) (3) (d) (4) (b) (5) (b)

2

(1) (2 , – 1) (2) (– 2 , 1)

(3) its two diagonals are perpendicular

(4) 120° (5) (8 , 2)

3

[a] ∵ m (∠ A) = m (∠ B) = 25°

, ∠ ACD is an exterior angle of Δ ABC

∴ m (∠ ACD) = 25° + 25° = 50° (The req.)

[b] ∵ D is the midpoint $\overline{AB}$ ∴ AD = $\frac{1}{2}$ AB

∴ AD = $\frac{1}{2}$ × 12 = 6 cm. (1)

, ∵ E is the midpoint $\overline{AC}$ ∴ AE = $\frac{1}{2}$ AC

∴ AE = $\frac{1}{2}$ × 8 = 4 cm. (2)

, ∵ D is the midpoint $\overline{AB}$, E is the midpoint $\overline{AC}$

∴ ED = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 10 = 5 cm. (3)

∴ From (1) , (2) , (3) :

The perimeter of Δ ADE = 6 + 4 + 5 = 15 cm.

(The req.)

4

[a] ∵ $\overline{DE}$ // $\overline{YZ}$, $\overline{ZD}$ is a transversal

∴ m (∠ Z) = m (∠ D) = 50° (alternate angles)

In Δ XYZ :

∵ m (∠ Z) + m (∠ ZXY) + m (∠ Y) = 180°

∴ m (∠ Y) = 180° – (105° + 50°) = 25°

, ∵ X ∈ $\overline{DZ}$

∴ m (∠ YXD) = 180° – 105° = 75° (The req.)

[b] ∵ $\overline{AZ}$ // $\overline{YD}$ // $\overline{XE}$ // $\overline{CB}$

, AY = YX = XC

∴ AD = DE = EB ∴ EB = $\frac{18}{3}$ = 6 cm.

5

In Δ ABD : ∵ m (∠ ADB) = 90°

∴ $(BD)^2 = (AB)^2 - (AD)^2 = 676 - 576 = 100$

∴ BD = $\sqrt{100}$ = 10 cm.

In Δ ADC : ∵ m (∠ ADC) = 90°

∴ $(CD)^2 = (AC)^2 - (AD)^2 = 900 - 576 = 324$

∴ CD = $\sqrt{324}$ = 18 cm.

∴ BC = 10 + 18 = 28 cm.

∴ The area of Δ ABC = $\frac{1}{2}$ BC × AD

= $\frac{1}{2}$ × 28 × 24 = 336 cm^2

(The req.)

Model 2

1

(1) (b) (2) (c) (3) (b) (4) (c) (5) (c)

2

(1) 360° (2) (5 , 5) (3) 120°

(4) bisects the third side. (5) ZYC

3

[a] In Δ XYZ : m (∠ Y) = 90°

∴ $(XZ)^2 = (XY)^2 + (YZ)^2 = 49 + 576 = 625$

∴ XZ = $\sqrt{625}$ = 25 cm.

In Δ LXZ : m (∠ L) = 90°

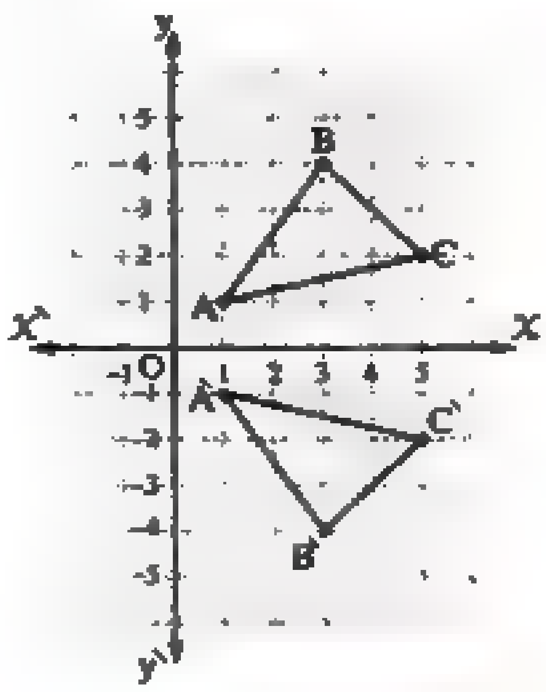
∴ $(LZ)^2 = (XZ)^2 - (LX)^2 = 625 - 225 = 400$

∴ LZ = $\sqrt{400}$ = 20 cm. (The req.)

[b]

4

[a]



[b] In ΔABC : $m(\angle ACB) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$

, $\because \overrightarrow{BD} \cap \overrightarrow{AO} = \{C\}$

$\therefore m(\angle ACB) = m(\angle OCD) = 60^\circ$ (V.O.A)

$\therefore m(\angle E) = 360^\circ - (60^\circ + 120^\circ + 90^\circ) = 90^\circ$

(The req.)

5

[a] $\because \overrightarrow{EO} \parallel \overrightarrow{CD}$, $\overleftrightarrow{EB}$ is a transversal

$\therefore m(\angle CBA) = m(\angle E) = 50^\circ$ (alternate angles)

From ΔABC :

$m(\angle BAC) = 180^\circ - (50^\circ + 30^\circ) = 100^\circ$

$\because \angle ABD$ is an exterior angle of ΔABC

$\therefore m(\angle ABD) = 30^\circ + 100^\circ = 130^\circ$ (The req.)

[b] $\because \overleftrightarrow{AD} \parallel \overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$, $\because AX = XB$

$\therefore DY = YC$

$\therefore Y$ is the midpoint of $\overline{CD}$

In ΔCDE : $\because \overleftrightarrow{ZY} \parallel \overline{DE}$

, Y is the midpoint of $\overline{CD}$

$\therefore Z$ is the midpoint of $\overline{CE}$

$\therefore CZ = ZE$ (Q.E.D.)

Model 3

1

(1) (b) (2) (c) (3) (a) (4) (c) (5) (b)

2

(1) two adjacent sides are equal in length

(2) equal in measure

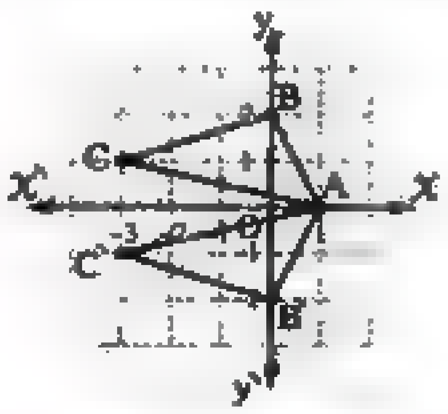
(3) y-axis

(4) parallel to the third side

(5) $(-4 , -1)$

3

[a]



[b] $\because ABCD$ is a parallelogram , M is the intersection point of its diagonals.

$\therefore M$ is the midpoint of $\overline{AC}$

In ΔABC : $\because M$ is the midpoint of $\overline{AC}$, $\overrightarrow{ME} \parallel \overrightarrow{BC}$

$\therefore E$ is the midpoint of $\overline{AB}$

$\therefore AE = EB$ (Q.E.D.)

4

[a] $\because \overrightarrow{OH} \parallel \overrightarrow{BC}$, $\overrightarrow{OB}$ is a transversal to them

$\therefore m(\angle B) + m(\angle O) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle B) = 180^\circ - 135^\circ = 45^\circ$

$\because \overrightarrow{DE} \parallel \overline{BC}$ and $\overrightarrow{DC}$ is a transversal to them

$\therefore m(\angle C) + m(\angle D) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ$

$\therefore$ In ΔABC :

$m(\angle BAC) = 180^\circ - (45^\circ + 60^\circ) = 75^\circ$ (The req.)

[b] In ΔABC : $m(\angle B) = 90^\circ$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 144 + 81 = 225$

$\therefore AC = \sqrt{225} = 15$ cm.

In ΔACD : $m(\angle ACD) = 90^\circ$

$\therefore (AD)^2 = (AC)^2 + (DC)^2 = 225 + 400 = 625$

$\therefore AD = \sqrt{625} = 25$ cm. (The req.)

5

(1) ΔLNB (2) ΔDOM (3) ΔBON

Model 4

1

(1) (a) (2) (b) (3) (a) (4) (b) (5) (a)

2

(1) parallel to the third side

(2) equal in measure

(3) each two opposite sides are parallel (there are another solution).

(4) $(2 , -3)$ (5) $(-3 , -2)$

3

In ΔCEO : $m(\angle COE) = 180^\circ - (35^\circ + 50^\circ) = 95^\circ$

, $\because \overline{DC} \cap \overline{BE} = \{O\}$

$\therefore m(\angle DOB) = m(\angle COE) = 95^\circ$ (V.O.A) (First req.)

From the quadrilateral ABOD

$m(\angle B) = 360^\circ - (95^\circ + 100^\circ + 85^\circ) = 80^\circ$

(Second req.)

4

[a] $\because$ The length $= \frac{48}{6} = 8$ cm.

$\therefore$ The length of the diagonal $= \sqrt{8^2 + 6^2} = 10$ cm.

[b] $\because \overline{AB} \parallel \overline{DE}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore m(\angle B) = m(\angle D) = 60°$ (alternate angles)

, $\because \angle AOD$ is an exterior angle of ΔABO

$\therefore m(\angle AOD) = m(\angle B) + m(\angle A)$

$\therefore m(\angle A) = 110° - 60° = 50°$ (The req.)

5

[a] The point $A = (1 , 2)$

$\therefore$ The image of the point $A (1 , 2)$ by translation $(-1 , 2)$ is $(0 , 4)$

[b] In ΔABC : $\because \overline{DE} \parallel \overline{BC}$, D is the midpoint of $\overline{AB}$

$\therefore$ E is the midpoint of $\overline{AC}$ $\therefore DE = \frac{1}{2} BC$

, $\because$ O is the midpoint of $\overline{BC}$

$\therefore BO = \frac{1}{2} BC$ $\therefore DE = BO$

, $\because \overline{DE} \parallel \overline{BC}$

$\therefore$ DBOE is a parallelogram (First req.)

, $\because ED = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6$ cm.

, $\because$ D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{BC}$

$\therefore DO = \frac{1}{2} AC = \frac{1}{2} \times 8 = 4$ cm.

, $\because$ E is the midpoint of $\overline{AC}$, O is the midpoint of $\overline{BC}$

$EO = \frac{1}{2} AB = \frac{1}{2} \times 10 = 5$ cm.

$\therefore$ The perimeter of $\Delta EDO = 6 + 4 + 5 = 15$ cm. (Second req.)

Model 5

1

(1) (b) (2) (d) (3) (a) (4) (d) (5) (c)

2

(1) The rhombus (2) right-angled

(3) acute (4) (2 , 4) (5) (2 , 4)

3

[a] Theoretical

[b] $\because$ D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{AC}$

$\therefore DO = \frac{1}{2} BC$ $\therefore DO = 4$ cm.

, $\because$ O is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

$\therefore OE = \frac{1}{2} AB$ $\therefore OE = 3$ cm.

, $\because$ E is the midpoint of $\overline{BC}$, D is the midpoint of $\overline{AB}$

$\therefore ED = \frac{1}{2} AC$ $\therefore ED = 5$ cm.

$\therefore$ The perimeter of $\Delta DEO = 4 + 3 + 5 = 12$ cm. (The req.)

4

[a] $\because$ ABCD is a parallelogram , M is the intersection point of its diagonals.

$\therefore$ M is the midpoint of $\overline{AC}$

In ΔABC : $\overrightarrow{MO} \parallel \overline{AB}$

, M is the midpoint of $\overline{AC}$

$\therefore$ O is the midpoint of $\overline{BC}$

$\therefore BO = OC$ (Q.E.D.)

[b] In ΔXYZ : $\because m(\angle X) = 90°$

$\therefore (XY)^2 = (YZ)^2 - (XZ)^2 = 400 - 256 = 144$

$\therefore XY = \sqrt{144} = 12$ cm. (The req.)

5

[a] $\because B \in \overline{AE}$

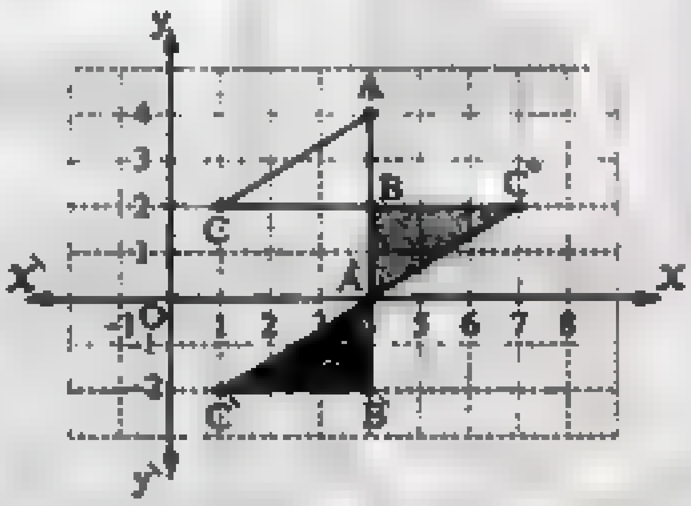
$\therefore m(\angle ABC) = 180° - 130° = 50°$

$\therefore$ From the quadrilateral ABCD :

$m(\angle C) = 360° - (50° + 80° + 120°)$

$= 360° - 250° = 110°$ (The req.)

[b]



(1) $\Delta A'B'C'$ is the image of ΔABC by translation $2\,AB$ in direction $\overrightarrow{AB}$

(2) $\Delta A''B''C''$ is the image of ΔABC by rotation about B with an angle of measure 180°

Model 6

1

(1) (b) (2) (c) (3) (a) (4) (a) (5) (b)

2

(1) equal to the sum of the measures of its non adjacent interior angles

(2) square (3) 130° (4) (−2 , 2)

3

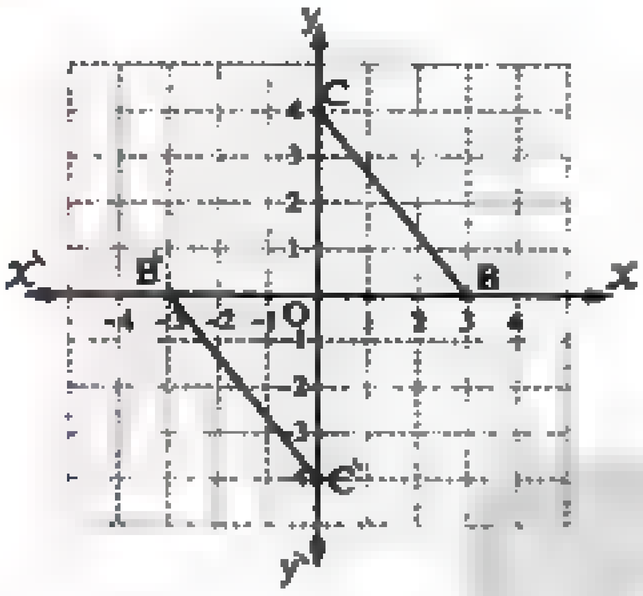
[a] ∵ $\overleftrightarrow{ED} // \overleftrightarrow{CB}$, $\overleftrightarrow{BE}$ is a transversal

∴ $m(\angle B) = m(\angle E) = 40°$ (alternate angles)

∴ In $\triangle ABC$: $m(\angle BAC) = 180° - (50° + 40°) = 90°$

∴ $\overline{AC} \perp \overline{BE}$ (Q.E.D.)

[b]



4

In $\triangle ABC$: $m(\angle ACB) = 180° - (90° + 40°) = 50°$

, ∵ $\overline{BD} \cap \overline{AO} = \{C\}$

∴ $m(\angle DCO) = m(\angle ACB) = 50°$ (V.O.A.)

∵ $\overline{AB} // \overline{DE}$, $\overleftrightarrow{BD}$ is a transversal

∴ $m(\angle B) = m(\angle D) = 90°$ (alternate angles)

From the quadrilateral CDEO

∴ $m(\angle E) = 360° - (90° + 50° + 130°) = 90°$ (The req.)

5

[a] In $\triangle XYZ$:

∵ D is the midpoint of $\overline{XY}$, E is the midpoint of $\overline{XZ}$

∴ $DE = \frac{1}{2} ZY = \frac{1}{2} \times 12 = 6$ cm.

, E is the midpoint of $\overline{XZ}$, O is the midpoint of $\overline{ZY}$

∴ $EO = \frac{1}{2} \times XY = \frac{1}{2} \times 6 = 3$ cm.

, O is the midpoint of $\overline{ZY}$, D is the midpoint of $\overline{XY}$

∴ $OD = \frac{1}{2} \times XZ = \frac{1}{2} \times 8 = 4$ cm.

∴ The perimeter of $\triangle DOE = 6 + 3 + 4 = 13$ cm.

(The req.)

[b] Construction : Draw $\overline{DE} \perp \overline{BC}$

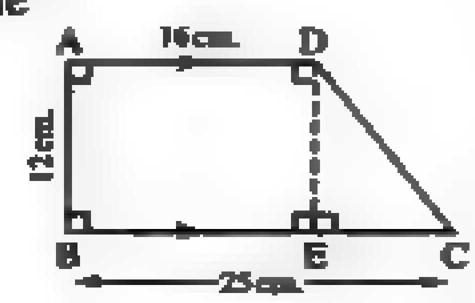
Proof : ABED is a rectangle

∴ $DE = AB = 12$ cm.

, $AD = BE = 16$ cm.

∴ $EC = 25 - 16 = 9$ cm.

, ∵ $m(\angle DEC) = 90°$



∴ $(DC)^2 = (DE)^2 + (EC)^2 = 144 + 81 = 225$

∴ $DC = \sqrt{225} = 15$ cm. (The req.)

Model 7

1

(1) (b) (2) (a) (3) (c) (4) (a) (5) (b)

2

(1) 540° (2) 120° (3) right (4) (− 2 , 2) (5) 90°

3

[a] ∵ $\overleftrightarrow{DE} // \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

∴ $m(\angle B) = m(\angle EAB) = 120°$ (alternate angles)

, ∵ $m(\angle B) + m(\angle C) = 120° + 60° = 180°$

and they are interior angles on the same side of the transversal

∴ $\overline{AB} // \overline{CD}$

, ∵ $\overleftrightarrow{DA} // \overline{CB}$

∴ ABCD is a parallelogram (Q.E.D.)

[b] In $\triangle ABC$:

∵ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

∴ $ED = \frac{1}{2} BC$ ∴ $BC = 2\,ED$

∴ $BC = 2 \times 5 = 10$ cm. (The req.)

4

[a] ∵ The sum of measures of the interior angles of the pentagon $= (5 - 2) \times 180° = 540°$

∴ $m(\angle D) = 540° - (110° + 70° + 120° + 150°)$

$= 90°$ (The req.)

[b] In $\triangle ABC$: ∵ $m(\angle B) = 90°$

∴ $(AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$

∴ $AB = 16$ cm.

∴ $AD = AB - DB = 16 - 9 = 7$ cm.

, ∵ $AE = 2\,BC$ ∴ $AE = 2 \times 12 = 24$

, ∵ $\overline{AE} // \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

∴ $m(\angle A) = m(\angle B) = 90°$ (alternate angles)

∴ $(ED)^2 = (AD)^2 + (AE)^2 = 49 + 576 = 625$

∴ $ED = \sqrt{625} = 25$ (The req.)

5

[a] ∵ ABCD is a parallelogram

∴ $m(\angle B) + m(\angle C) = 180°$

∴ $m(\angle C) = 180° - 135° = 45°$ (First req.)

, ∵ ABCD is a parallelogram

∴ $AB = CD = 5$ cm. , $AD = BC = 8$ cm.

∴ The perimeter of parallelogram ABCD

$= 5 + 8 + 5 + 8 = 26$ cm. (Second req.)

[b] (1) Δ ADC (2) Δ MBC

Model 8

1

(1) (a) (2) (c) (3) (c) (4) (b) (5) (d)

2

(1) half the length of the third side. (2) square

(3) 5 cm. (4) $(3, -4), (-3, 4)$ (5) $(5, -1)$

3

[a] ∵ $\overline{DA} \parallel \overline{BE}$, $\overline{BD}$ is a transversal

∴ $m(\angle CBD) = m(\angle ADB) = 50^\circ$ (alternate angles)

∴ $m(\angle DBE) = 180^\circ - 50^\circ = 130^\circ$

, ∵ $\overrightarrow{BA}$ bisects $\angle$ DBE

∴ $m(\angle ABE) = \frac{130^\circ}{2} = 65^\circ$

, ∵ $m(\angle C) = m(\angle ABE) = 65^\circ$

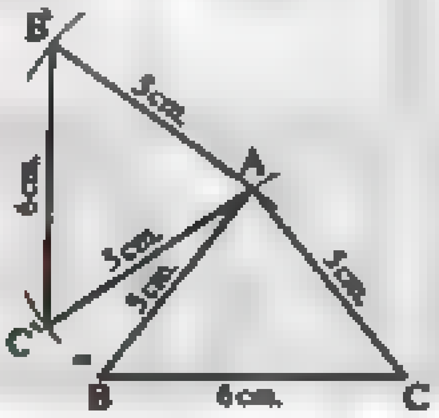
and they are corresponding angles

∴ $\overline{AB} \parallel \overline{CD}$

, ∵ $\overline{AD} \parallel \overline{BC}$

∴ ABCD is a parallelogram. (Q.E.D.)

[b]



4

[a] ∵ $\overline{DE} \parallel \overline{CB}$, $\overleftrightarrow{BD}$ is a transversal

∴ $m(\angle B) = m(\angle D) = 50^\circ$ (alternate angles)

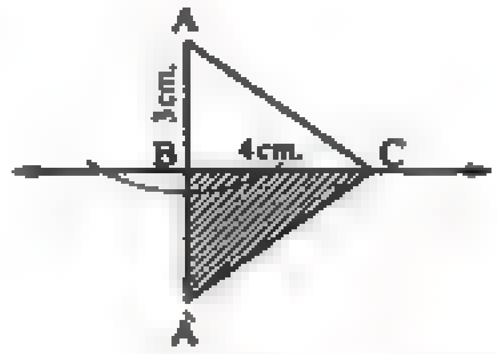
(First req.)

From Δ ABC :

$m(\angle BAC) = 180^\circ - (35^\circ + 50^\circ) = 95^\circ$

(Second req.)

[b]



5

[a] ∵ ABCD is a parallelogram

∴ $\overline{AB} \parallel \overline{DC}$ ∴ $\overline{XC} \parallel \overline{AB}$

In Δ ABE : ∵ C is the midpoint of $\overline{BE}$

, $\overline{XC} \parallel \overline{AB}$

∴ X is the midpoint of $\overline{AE}$

∴ AX = XE (Q.E.D.)

[b] In Δ BCD : ∵ $m(\angle B) = 90^\circ$

∴ $(BD)^2 = (DC)^2 - (BC)^2 = 169 - 144 = 25$

∴ $BD = \sqrt{25} = 5$ cm.

∴ $AB = 5 + 11 = 16$ cm.

In Δ ABC : ∵ $m(\angle B) = 90^\circ$

∴ $(AC)^2 = (AB)^2 + (BC)^2 = 256 + 144 = 400$

∴ $AC = \sqrt{400} = 20$ cm. (The req.)

Answers of school examinations in Geometry

1 Cairo

1

(1) (b) (2) (d) (3) (c) (4) (a) (5) (a)

2

(1) half the length of the third side

(2) equal in measure (3) The rectangle

(4) 180° (5) (0 , 5)

3

[a] ∵ ABCDE is a pentagon

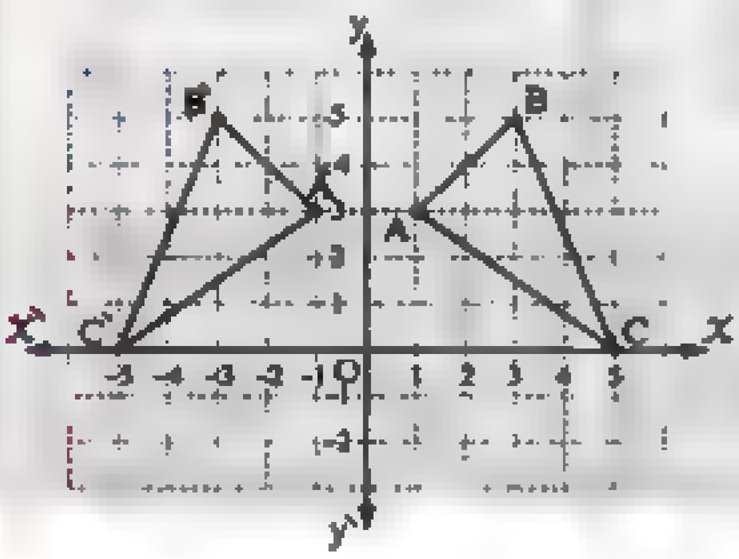
∴ The sum of measures of its interior angles

$= (5-2) \times 180° = 540°$

∴ m (∠ E) = 540° − (140° + 130° + 90° + 110°)

= 70° (The req.)

[b]



4

[a] ∵ ∠ DAB is an exterior angle of Δ ABC

∴ m (∠ C) = 120° − 50° = 70°

, ∵ $\overleftrightarrow{DE}$ // $\overleftrightarrow{CB}$, $\overleftrightarrow{CD}$ is a transversal

∴ m (∠ C) + m (∠ D) = 180°

(interior angles in the same side of the transversal)

∴ m (∠ D) = 180° − 70° = 110° (The req.)

[b] ∵ m (∠ ABC) + m (∠ CBD) + m (∠ DBE)

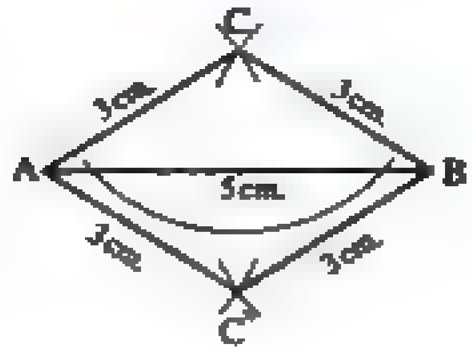
+ m (∠ EBA) = 360° (acumulative angles at B)

∴ m (∠ DBE) = 360° − (110° + 35° + 140°) = 75°

(The req.)

5

[a]



[b] In Δ ABC :

∵ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

∴ ED = $\frac{1}{2}$ BC ∴ BC = 12 cm.

, ∵ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$

∴ DF = $\frac{1}{2}$ AC ∴ AC = 8 cm.

, ∵ E is the midpoint of $\overline{AC}$, F is the midpoint of $\overline{BC}$

∴ EF = $\frac{1}{2}$ AB ∴ AB = 6 cm.

∴ The perimeter of Δ ABC

= 6 + 12 + 8 = 26 cm. (The req.)

Additional question

1 (1) (3 , −5)

(2) the lengths of line segments.

, the measures of angles.

, the parallelism , betweeness

(3) two

2 In Δ ABD : ∵ m (∠ ADB) = 90°

∴ $(DB)^2 = (AB)^2 - (AD)^2 = 676 - 576 = 100$

∴ DB = $\sqrt{100}$ = 10 cm.

In Δ ACD : ∵ m (∠ ADC) = 90°

∴ $(CD)^2 = (AC)^2 - (AD)^2 = 900 - 576 = 324$

∴ CD = $\sqrt{324}$ = 18 cm.

∴ BC = 10 + 18 = 28 (First req.)

∴ The area of Δ ABC = $\frac{1}{2} \times 28 \times 24$

= 336 cm. (Second req.)

2 Cairo

1 (1) (d) (2) (a) (3) (b) (4) (d) (5) (c)

2 (1) (5 , 3) (2) perpendicular , bisect each other

(3) bisects the third side (4) trapezium

(5) (0 , 2)

3

[a] $\overleftrightarrow{LM} \parallel \overleftrightarrow{YZ}$, $\overleftrightarrow{LY}$ is a transversal

$\therefore m(\angle L) + m(\angle Y) = 180^\circ$

(interior angles in the same side of the transversal)

$\therefore m(\angle Y) = 180^\circ - 100^\circ = 80^\circ$

, $\because \overleftrightarrow{XN} \parallel \overleftrightarrow{YZ}$, $\overleftrightarrow{XZ}$ is a transversal

$\therefore m(\angle Z) = m(\angle ZXN) = 40^\circ$ (alternate angles)

$\therefore$ In ΔXZY : $m(\angle YXZ) = 180^\circ - (80^\circ + 40^\circ) = 60^\circ$

(The req.)

[b] $\because \overleftrightarrow{LZ} \parallel \overleftrightarrow{YX}$, $\overleftrightarrow{LX}$ is a transversal

$\therefore m(\angle L) = m(\angle MXL) = 47^\circ$ (alternate angles)

$\therefore m(\angle L) + m(\angle Z) = 47^\circ + 133^\circ = 180^\circ$

and they are interior angles in the same side of the transversal

$\therefore \overleftrightarrow{LX} \parallel \overleftrightarrow{ZY}$, $\because \overleftrightarrow{LZ} \parallel \overleftrightarrow{XY}$

$\therefore$ XYZL is a parallelogram. (Q.E.D.)

4

[a] $\because \Delta ABC$ is an equilateral triangle

$\therefore m(\angle ACB) = \frac{180^\circ}{3} = 60^\circ$

, $\because \overrightarrow{AF} \cap \overrightarrow{BD} = \{C\}$

$\therefore m(\angle DCF) = m(\angle ACB) = 60^\circ$ (V.O.A.)

$\therefore$ From the quadrilateral CDEF :

$m(\angle F) = 360^\circ - (60^\circ + 120^\circ + 110^\circ) = 70^\circ$

(The req.)

[b] In ΔEDF :

$\because$ A is the midpoint of $\overline{ED}$, B is the midpoint of $\overline{EF}$

$\therefore AB = \frac{1}{2} DF = DC = 8$ cm.

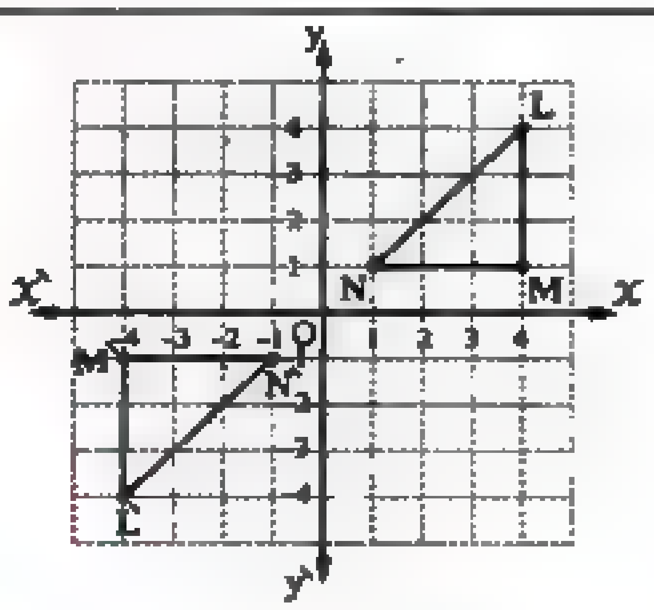
, $\because$ B is the midpoint of $\overline{EF}$, C is the midpoint of $\overline{DF}$

$\therefore BC = \frac{1}{2} DE = DA = 6$ cm.

$\therefore$ The perimeter of the quadrilateral ABCD

$= 8 + 6 + 8 + 6 = 28$ cm. (The req.)

5

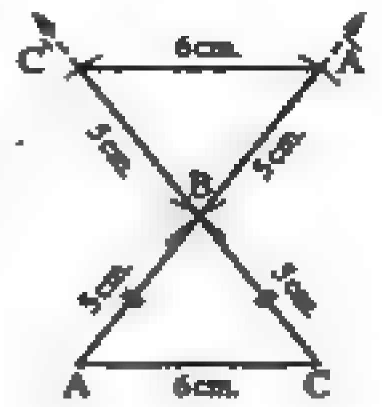


Additional question

1 (1) the sum of the squares of the lengths of the other two sides.

(2) 4 (3) (– 5 , 3)

2



3 Giza

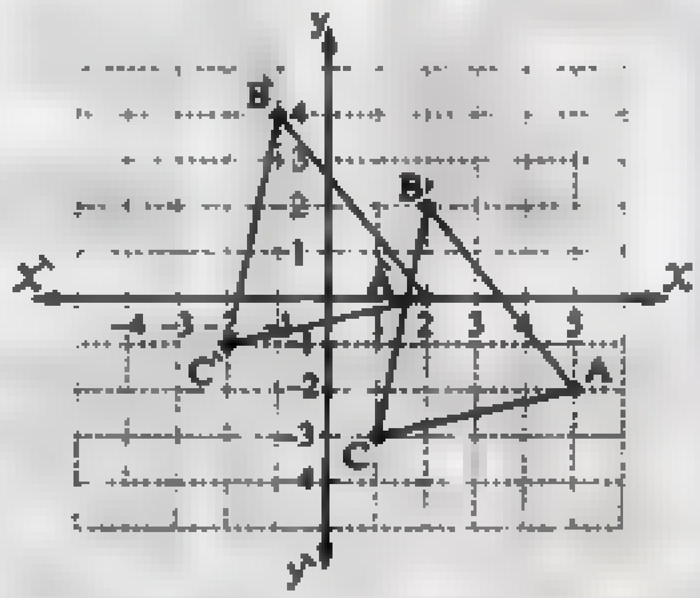
1 (1) (d) (2) (c) (3) (d) (4) (c) (5) (b)

2 (1) 116° (2) (3 , 5) (3) 120°

(4) parallel , equal in length (5) perpendicular

3

[a]



[b] $\because \overleftrightarrow{ED} \parallel \overleftrightarrow{ZY}$ and $\overleftrightarrow{DY}$ is a transversal.

$\therefore m(\angle XDE) = m(\angle Y) = 50^\circ$ (corresponding angles)

In ΔXDE :

$\therefore m(\angle XED) = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$

(The req.)

4

[a] $\because E \in \overleftrightarrow{AC}$

$\therefore m(\angle BEC) = 180^\circ - 65^\circ = 115^\circ$ (First req.)

$\therefore m(\angle BEC) + m(\angle CED) = 115^\circ + 85^\circ = 200^\circ$

$\therefore$ The points B , E and D are not on the same straight line. (Second req.)

Answers of final examinations

[b] ∵ ∠ AMB is an exterior angle of Δ MBC

∴ m (∠ BCM) = 80° − 50° = 30°

∴ m (∠ BCM) = m (∠ DAC)

, and they are alternate angles ∴ $\overline{AD}$ // $\overline{BC}$

, ∵ $\overline{AB}$ // $\overline{DC}$

∴ ABCD is a parallelogram (Q.E.D.)

5

[a] ∵ $\overleftrightarrow{YX}$ // $\overleftrightarrow{MZ}$, $\overleftrightarrow{YM}$ is a transversal

∴ a + 10° + 2 a + 50° = 180°

(interior angles in the same side of the transversal)

∴ 3 a + 60° = 180° ∴ 3 a = 120°

∴ a = 40° (The req.)

[b] In Δ ABC :

∵ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$

∴ FD = $\frac{1}{2}$ CB = EC = 8 cm.

∵ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$

∴ DE = $\frac{1}{2}$ AC = CF = 6 cm.

∴ The perimeter of the quadrilateral DECF

= 6 + 8 + 6 + 8 = 28 cm. (The req.)

Additional question

1 (1) 10 cm. (2) $\overline{BA}$

2 In Δ XYZ : ∵ m (∠ XYZ) = 90°

∴ $(XZ)^2 = (XY)^2 + (YZ)^2 = 49 + 576 = 625$

∴ XZ = $\sqrt{625}$ = 25 cm.

In Δ XLZ : ∵ m (∠ XLZ) = 90°

∴ $(LZ)^2 = (XZ)^2 - (XL)^2 = 625 - 225 = 400$

∴ LZ = $\sqrt{400}$ = 20 cm. (The req.)

4 Giza

1 (1) (b) (2) (c) (3) (a) (4) (c) (5) (d)

2 (1) (2 , −5) (2) right - angled

(3) is parallel to the third side

(4) rhombus (5) concave

3

[a] ∵ ABCD is a rhombus , $\overline{BD}$ is a diagonal in it.

∴ $\overrightarrow{BD}$ bisects ∠ ABC

, ∵ m (∠ ABD) = 62°

∴ m (∠ ABC) = 2 m (∠ ABD) = 2 × 62° = 124°

∴ m (∠ BAD) = 180° − 124° = 56° (The req.)

[b] The number of sides = $\frac{360°}{180° - 135°}$ = 8

4

[a] ∵ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$

∴ DF = $\frac{1}{2}$ BC ∴ DF = 4 cm.

, ∵ F is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

∴ FE = $\frac{1}{2}$ AB ∴ FE = 3 cm.

, E is the midpoint of $\overline{BC}$, D is the midpoint of $\overline{AB}$

∴ ED = $\frac{1}{2}$ AC ∴ ED = 5 cm.

∴ The perimeter of Δ DEF = 4 + 3 + 5 = 12 cm.

(The req.)

[b] In Δ ABC :

∵ E is the midpoint of $\overline{BA}$, F is the midpoint of $\overline{CA}$

∴ $\overline{EF}$ // $\overline{BC}$

, ∵ $\overline{AD}$ // $\overline{BC}$ ∴ $\overline{EF}$ // $\overline{AD}$ (1)

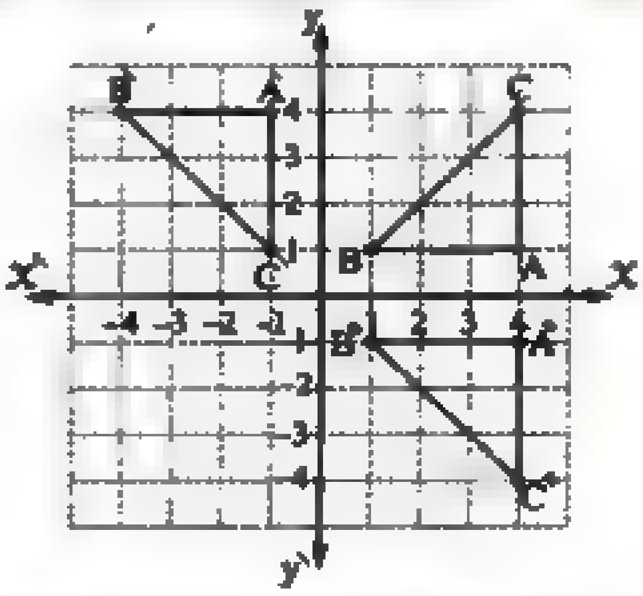
, EF = $\frac{1}{2}$ BC

, ∵ AD = $\frac{1}{2}$ BC ∴ EF = AD (2)

From (1) and (2) :

∴ AEFD is a parallelogram. (Q.E.D.)

5



(1) Δ A`B`C` is the image of Δ ABC by rotation about O with an angle of measure 90°

(2) Δ A``B``C`` is the image of Δ ABC by reflection in X-axis.

Additional question

1 (1) (c) (2) (d)

2 $\because$ The figure AECD is a rectangle

$\therefore$ EC = AD = 9 cm.

$\therefore$ BE = 17 – 9 = 8 cm.

$\therefore$ In Δ AEB : $\because$ m ($\angle$ AEB) = 90°

$\therefore (AE)^2 = (AB)^2 - (BE)^2 = 289 - 64 = 225$

$\therefore AE = \sqrt{225} = 15$ cm.

$\therefore$ DC = AE = 15 cm. (First req.)

The area of the trapezium ABCD

= The area of Δ ABE + The area of the rectangle AECD

$= \frac{1}{2} \times 8 \times 15 + 9 \times 15 = 195 \text{ cm}^2$. (Second req.)

5 Alexandria

1 (1) (b) (2) (d) (3) (b) (4) (b) (5) (b)

2 (1) parallelogram (2) (1 , –4) (3) parallel
(4) 360° (5) 115°

3

[a]

[b] $\because$ ABCD is a parallelogram , M is the intersection point of its diagonals

$\therefore$ M is the midpoint of $\overline{BD}$

In Δ ABD :

$\because$ M is the midpoint of $\overline{BD}$, $\overline{MX} \parallel \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AD}$ (Q.E.D.)

4

[a] $\because$ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2}$ BC $\therefore$ DF = 3.5 cm.

, $\because$ F is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

$\therefore FE = \frac{1}{2}$ AB $\therefore$ FE = 4 cm.

, $\because$ E is the midpoint of $\overline{BC}$, D is the midpoint of $\overline{AB}$

$\therefore ED = \frac{1}{2}$ AC $\therefore$ ED = 5 cm.

$\therefore$ The perimeter of Δ DEF = 4 + 3.5 + 5 = 12.5 cm.

(The req.)

[b] The number of sides $= \frac{360°}{180° - 135°} = 8$ sides

5

[a] 95°

[b] $\because \angle$ EDC is an exterior angle of Δ BCD

$\therefore$ m ($\angle$ EDC) = m ($\angle$ B) + m ($\angle$ C)

$\therefore$ m ($\angle$ C) = 105° – 55° = 50°

, $\because \overrightarrow{CA}$ bisects $\angle$ C $\therefore$ m ($\angle$ ACB) $= \frac{50°}{2} = 25°$

, $\because \angle$ CAD is an exterior angle of Δ ABC

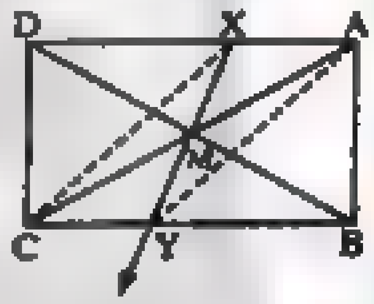
$\therefore$ m ($\angle$ CAD) = m ($\angle$ B) + m ($\angle$ ACB)

= 55° + 25° = 80° (The req.)

Additional question

1 (1) $(XY)^2 - (YZ)^2$ (2) (4 , –2) (3) one

2 $\because$ ABCD is a rectangle



$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore$ m ($\angle$ XAM)

= m ($\angle$ YCM)

(alternate angles)

$\Delta\Delta$ AMX , CMY in them :

m ($\angle$ XAM) = m ($\angle$ YCM)
m ($\angle$ AMX) = m ($\angle$ CMY) (V.O.A.)
AM = CM

$\therefore \Delta AMX \equiv \Delta CMY$, then we deduce that XM = MY

$\because Y \in \overrightarrow{XM}$

$\therefore$ Y is the image of X by reflection in the point M

(Q.E.D. 1)

$\therefore$ AM = CM , MX = MY

$\therefore$ The figure AXCY is a parallelogram. (Q.E.D.2)

6 Alexandria

1 (1) (c) (2) (b) (3) (c) (4) (d) (5) (b)

2 (1) trapezium (2) (5 , –3)
(3) parallel , equal in length
(4) 60° (5) 360°

Answers of final examinations

3

[a] 80°

[b] In the square ABCD :

$\because \overline{AD} \parallel \overline{BC}$, $E \in \overline{BC}$ $\quad \therefore \overline{AD} \parallel \overline{CE}$

, $\because \overline{AC} \parallel \overline{DE}$

$\therefore$ ACED is a parallelogram. (First req.)

$\because \overrightarrow{CA}$ bisects $\angle$ BCD $\quad \therefore m(\angle ACB) = 45°$

$\because m(\angle ACE) + m(\angle ACB) = 180°$

(Supplementary angles)

$\therefore m(\angle ACE) = 180° - 45° = 135°$ (Second req.)

4

[a] The sum of measures of the interior angles of hexagon $= (6-2) \times 180° = 720°$

[b] $\because$ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2} BC \quad \therefore DF = 4$ cm.

, $\because$ F is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

$\therefore FE = \frac{1}{2} AB \quad \therefore FE = 2.5$ cm.

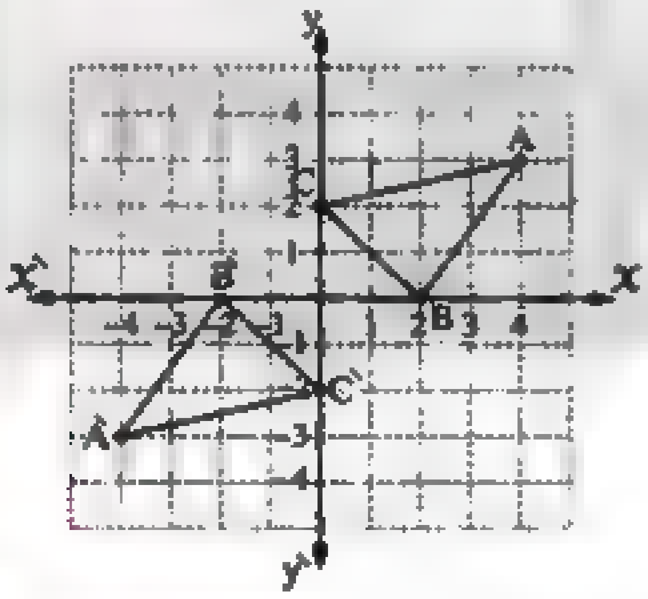
, $\because$ E is the midpoint of $\overline{BC}$, D is the midpoint of $\overline{AB}$

$\therefore ED = \frac{1}{2} AC \quad \therefore ED = 3.5$ cm.

$\therefore$ The perimeter of Δ DEF $= 4 + 2.5 + 3.5 = 10$ cm.

(The req.)

5



Additional question

1 (1) $(-2 , 7)$ (2) $(-2 , -1)$

2 In Δ ADC : $\because \overline{AD} \perp \overline{BC}$

$\therefore m(\angle ADC) = 90°$

$\therefore (AD)^2 = (AC)^2 - (DC)^2 = 400 - 256 = 144$

$\therefore AD = \sqrt{144} = 12$ cm.

In Δ ADB : $\because m(\angle ADB) = 90°$

$\therefore (AB)^2 = (AD)^2 + (DB)^2 = 144 + 81 = 225$

$\therefore AB = \sqrt{225} = 15$ cm. (The req.)

7 El-Kalyoubia

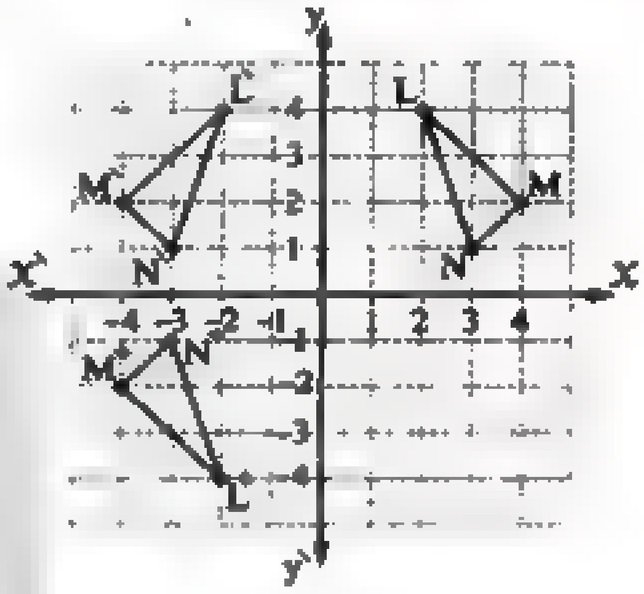
1 (1) $(n-2) \times 180°$ (2) trapezium

(3) bisects the third side

(4) $(5 , 1)$ (5) $(0 , 3)$

2 (1) (a) (2) (d) (3) (c) (4) (a) (5) (d)

3



(1) $\Delta \grave{L}\grave{M}\grave{N}$ is the image of Δ LMN by reflection in y-axis.

(2) $\Delta \grave{\grave{L}}\grave{\grave{M}}\grave{\grave{N}}$ is the image of Δ LMN by rotation about the origin with an angle of measure 180°

4

[a] In Δ ABC :

$\because$ D is the midpoint of $\overline{AB}$

, O is the midpoint of $\overline{AC}$

$\therefore OD = \frac{1}{2} CB = HC = 6$ cm.

$\because$ D is the midpoint of $\overline{AB}$

, H is the midpoint of $\overline{BC}$

$\therefore DH = \frac{1}{2} AC = CO = 5$ cm.

$\therefore$ The perimeter of the quadrilateral DHCO $= 6 + 5 + 6 + 5 = 22$ cm. (The req.)

[b] The number of sides $= \frac{360°}{180° - 108} = 5$ sides

5

[a] $\because \overrightarrow{OX} \parallel \overline{BC}$, $\overrightarrow{OB}$ is a transversal to them

$\therefore m(\angle B) + m(\angle O) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle B) = 180° - 135° = 45°$

$\because \overline{DH} \parallel \overline{BC}$ and $\overline{DC}$ is a transversal to them

$\therefore m(\angle C) + m(\angle D) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ$

$\therefore$ In ΔABC :

$m(\angle BAC) = 180^\circ - (45^\circ + 60^\circ) = 75^\circ$ (The req.)

[b] $\because m(\angle ABC) + m(\angle CBD) + m(\angle DBH)$

$+ m(\angle HBA) = 360^\circ$ (accumulative angles at B)

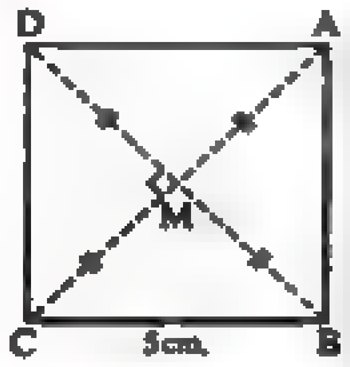
$\therefore m(\angle HBD) = 360^\circ - (110^\circ + 35^\circ + 140^\circ) = 75^\circ$

(The req.)

Additional question

1 (1) The origin point (2) 5 cm.

2



The image of the square ABCD by reflection in the point M is the square CDAB

We notice that we got the same square.

8 El-Kalyoubia

1 (1) 120° (2) square (3) $(-2, -4)$

(4) $(0, 2)$ (5) 180°

2 (1) (b) (2) (b) (3) (c) (4) (c) (5) (d)

3

[a] In ΔABC :

$\because$ X is the midpoint of $\overline{AB}$, Z is the midpoint of $\overline{AC}$

$\therefore XZ = \frac{1}{2} BC = BY = 5$ cm.

$\because$ Z is the midpoint of $\overline{AC}$, Y is the midpoint of $\overline{BC}$

$\therefore ZY = \frac{1}{2} AB = BX = 4$ cm.

$\therefore$ The perimeter of the figure XBYZ

$= 5 + 4 + 5 + 4 = 18$ cm. (The req.)

[b] In ΔABC :

$m(\angle BAC) = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$

$, \because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAD) = \frac{60^\circ}{2} = 30^\circ$

$\therefore$ From ΔABD :

$m(\angle ADB) = 180^\circ - (70^\circ + 30^\circ) = 80^\circ$

4

[a] $\because \overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{O\}$

$\therefore m(\angle BOD) = m(\angle AOC) = 50^\circ$ (V.O.A.)

$, \because \overrightarrow{OD}$ bisects $\angle BOH$

$\therefore m(\angle BOH) = 2 \times 50^\circ = 100^\circ$

$, \because O \in \overleftrightarrow{AB}$

$\therefore m(\angle AOH) = 180^\circ - 100^\circ = 80^\circ$ (The req.)

[b] $\because H \in \overline{BC}$

$\therefore m(\angle AHC) = 180^\circ - 70^\circ = 110^\circ$

$, \because$ AHCD is a quadrilateral.

$\therefore m(\angle HAD) = 360^\circ - (110^\circ + 115^\circ + 65^\circ) = 70^\circ$

$\therefore m(\angle BAD) = 70^\circ + 45^\circ = 115^\circ$

$\therefore m(\angle BAD) = m(\angle C)$ (1)

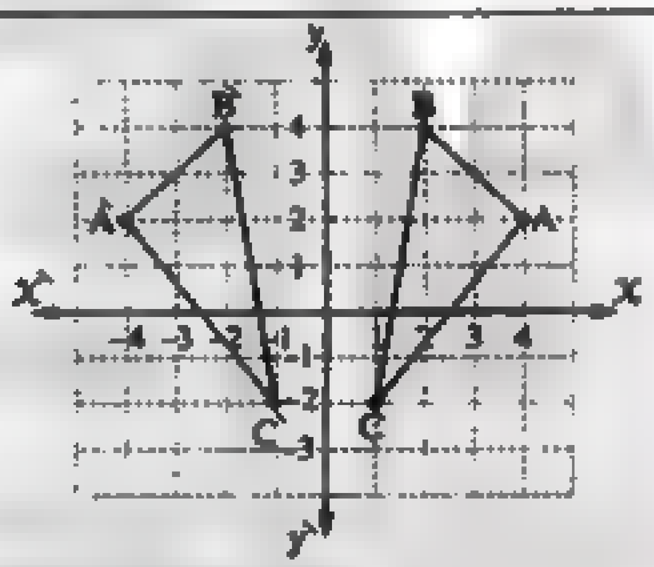
, In ΔABH : $\because m(\angle B) = 180^\circ - (45^\circ + 70^\circ) = 65^\circ$

$\therefore m(\angle B) = m(\angle D)$ (2)

From (1) and (2) :

$\therefore$ ABCD is a parallelogram. (Q.E.D.)

5

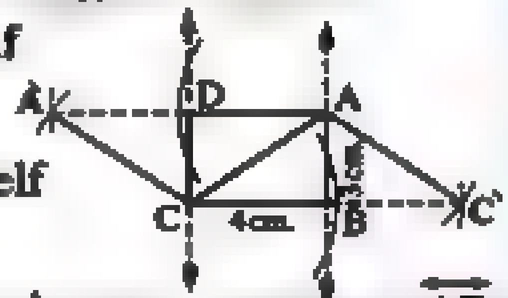


Additional question

1 (1) The sum of the squares of the lengths of the other two sides (2) 3

2 A is the image of itself by reflection in $\overleftrightarrow{AB}$

, B is the image of itself by reflection in $\overleftrightarrow{AB}$



$\therefore \Delta A\grave{B}C$ is the image of ΔABC by reflection in $\overleftrightarrow{AB}$

$\therefore m(\angle \grave{C}AB) = m(\angle CAB)$

$\therefore m(\angle \grave{C}AC) = 2\, m(\angle CAB)$ (Q.E.D. 1) (1)

Similarly we can prove that :

$m(\angle AC\grave{A}) = 2\, m(\angle ACD)$ (2)

$\because$ ABCD is a rectangle $\therefore \overline{AB} \,//\, \overline{CD}$

$\therefore m(\angle BAC) = m(\angle ACD)$ (alternate angles) (3)

From (1), (2), (3) : $\therefore m(\angle \grave{C}AC) = m(\angle AC\grave{A})$

but they are alternate angles

$\therefore \overline{A\grave{C}} \,//\, \overline{\grave{A}C}$ (Q.E.D. 2)

9 El-Dakahlia

1

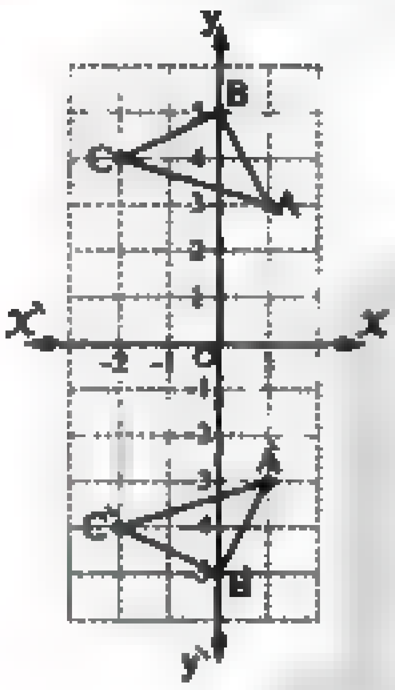
(1) (c)   (2) (b)   (3) (b)   (4) (d)   (5) (b)

2

(1) half the length of the third side.   (2) square
(3) rhombus   (4) 360°   (5) (2 , −3)

3

[a]



[b] In Δ ACD : m (∠ D) = 180° − (55° + 25°) = 100°

, ∵ ABCD is a parallelogram

∴ m (∠ B) = m (∠ D) = 100°   (The req.)

4

[a] From Δ BEF : m (∠ EBF) = 180° − (90° + 50°) = 40°

∵ $\overleftrightarrow{CF} \cap \overleftrightarrow{AE} = \{B\}$

∴ m (∠ ABC) = m (∠ EBF) = 40°   (V.O.A.)

, ∵ $\overline{AD}$ // $\overline{EF}$, $\overleftrightarrow{AE}$ is a transversal

∴ m (∠ A) = m (∠ E) = 90°   (alternate angles)

∴ from the quadrilateral ABCD

m (∠ C) = 360° − (40° + 90° + 110°) = 120° (The req.)

[b] ∵ ∠ AHB is an exterior angle of Δ ACH

∴ m (∠ HAC) = 70° − 40° = 30°

∵ $\overrightarrow{AH}$ bisects ∠ BAC

∴ m (∠ BAC) = 2 × 30° = 60°   (First req.)

, ∵ ∠ ABC is an exterior angle of Δ ABC

∴ m (∠ ABD) = 40° + 60° = 100°   (Second req.)

5

In Δ ABC :

∵ D is the midpoint of $\overline{AB}$

, H is the midpoint of $\overline{AC}$

∴ $\overline{DH}$ // $\overline{BC}$, DH = $\frac{1}{2}$ BC

, ∵ X is the midpoint of $\overline{DO}$, $\overline{XY}$ // $\overline{DH}$

∴ Y is the midpoint of $\overline{HO}$ (First req.)

, XY = $\frac{1}{2}$ HD   , ∵ DH = $\frac{1}{2}$ BC

∴ XY = $\frac{1}{2} \times \frac{1}{2}$ BC = $\frac{1}{4}$ BC

∴ XY = $\frac{1}{4}$ × 12 = 3 cm.   (Second req.)

Additional question

1 (1) (−4 , 1)   (2) Three

2 In Δ BCD : ∵ m (∠ B) = 90°

∴ $(BD)^2 = (CD)^2 - (CB)^2 = 169 - 144 = 25$

∴ BD = $\sqrt{25}$ = 5 cm.

In Δ ABC : ∵ m (∠ B) = 90°

, AB = 11 + 5 = 16 cm.

∴ $(AC)^2 = (AB)^2 + (BC)^2 = 256 + 144 = 400$

∴ AC = $\sqrt{400}$ = 20 cm.   (The req.)

10 Kafr El-Sheikh

1

(1) (d)   (2) (b)   (3) (c)   (4) (c)   (5) (a)

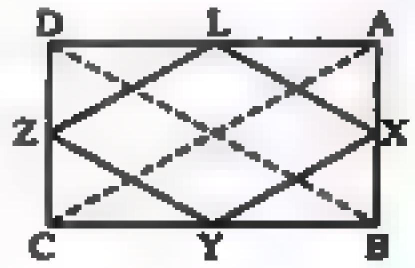
2

(1) are equal in measure
(2) a rhombus , a square   (3) convex
(4) (3 , −3)   (5) bisects the third side

3

[a] The measure of each interior angle of the regular octagon = $\frac{(8-2) \times 180°}{8}$ = 135°

[b] Construction :



Draw $\overline{BD}$ and $\overline{AC}$

Proof : In Δ ABD

∵ X is the midpoint of $\overline{AB}$

, L is the midpont of $\overline{AD}$

∴ $\overline{XL}$ // $\overline{BD}$, XL = $\frac{1}{2}$ BD   (1)

, In Δ DBC :

∵ Y is the midpoint of $\overline{BC}$, Z is the midpoint of $\overline{DC}$

∴ $\overline{YZ}$ // $\overline{BD}$, YZ = $\frac{1}{2}$ BD   (2)

Answers of final examinations

From (1) , (2) : $\overline{XL} // \overline{YZ}$, XL = YZ

$\therefore$ XYZL is a parallelogram , From Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC$ (3)

From (1) , (3) :

, $\because$ The two diagonals of the rectangle are equal in length

$\therefore$ LX = XY

$\therefore$ XYZL is a rhombus (Q.E.D. 1)

, the perimeter of the rhombus

$= 4\ XL = 4 \times \frac{1}{2} BD = 2\ BD$ (Q.E.D. 2)

4

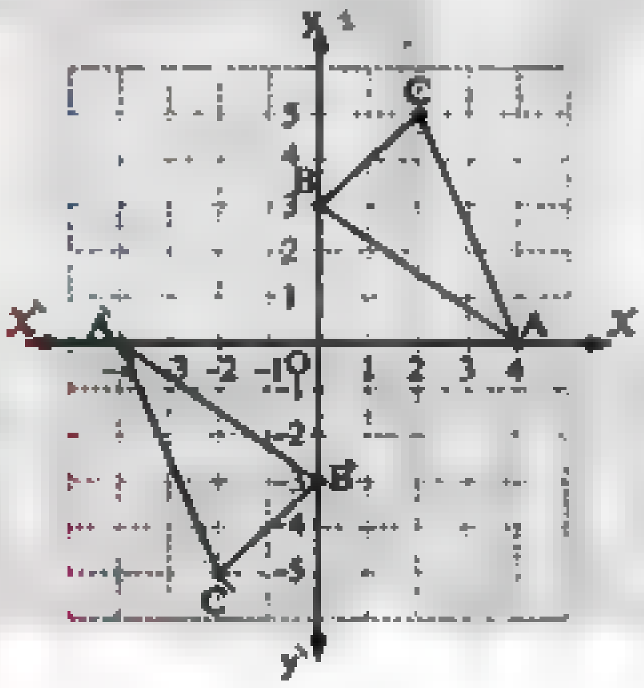
[a] $\because$ E is the midpoint of $\overline{AB}$

, D is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} BC = 6$ cm.

$\therefore$ The perimeter of Δ ADE = 6 + 6 + 6 = 18 cm. (The req.)

[b]



5

[a] $\Delta\Delta$ DXB , EYC in them :

$\begin{cases} DB = EC \\ DX = EY \\ m(\angle DXB) = m(\angle EYC) = 90^\circ \end{cases}$

$\therefore \Delta DXB \equiv \Delta EYC$, Then we deduce that :

$m(\angle B) = m(\angle C)$ (1)

$\because \overleftrightarrow{BC} // \overleftrightarrow{DE}$ and $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle ADE) = m(\angle B)$ (2)

(Corresponding angles)

Similarly : $m(\angle AED) = m(\angle C)$ (3)

(Corresponding angles)

From (1) , (2) and (3) :

$\therefore m(\angle ADE) = m(\angle AED)$ (Q.E.D.)

[b] $\because \angle$ ABE is an exterior angle of Δ ABC

$\therefore m(\angle ABE) = 60^\circ + 55^\circ = 115^\circ$

, $\because \overrightarrow{BD}$ bisects $\angle$ ABE

$\therefore m(\angle DBE) = \frac{115^\circ}{2} = 57.5^\circ$ (The req.)

Additional question

1 (1) (– a , b)

(2) The sum of the squares of the lengths of the other two sides.

2 In Δ ABC : $m(\angle B) = 90^\circ$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 81 + 144 = 225$

$\therefore AC = \sqrt{255} = 15$ cm.

In Δ ACD : $\because m(\angle ACD) = 90^\circ$

$\therefore (AD)^2 = (AC)^2 + (DC)^2 = 225 + 400 = 625$

$\therefore AD = \sqrt{625} = 25$ cm.

$\therefore$ The perimeter of the figure ABCD

= 9 + 12 + 20 + 25 = 66 cm.

11 El-Gharbia

1

(1) (b) (2) (a) (3) (b) (4) (a) (5) (a)

2

(1) parallel to the third side , its length is half the length of the third side.

(2) 50° (3) (– 3 , 1) (4) square (5) (3 , – 5)

3

[a] $\because$ ABCD is a parallelogram.

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore$ AC = 2 MA $\therefore$ AC = 7 cm.

, $\because$ AB = DC = 3 cm. , BC = AD = 6 cm.

$\therefore$ The perimeter of Δ ABC = 3 + 6 + 7 = 16 cm.

(The req.)

[b] $\because \overrightarrow{FE} // \overrightarrow{BC}$, $\overrightarrow{FB}$ is a transversal.

$\therefore m(\angle EFB) = m(\angle CBF) = 60^\circ$ (alternate angles)

In Δ ABC :

$\therefore m(\angle BAC) = 180^\circ - (60^\circ + 40^\circ) = 80^\circ$ (The req.)

Answers of final examinations

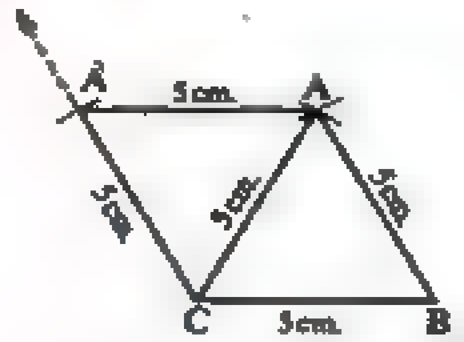
4

[a] In the pentagon ABCDE :

$\because$ m ($\angle$ A) + m ($\angle$ B) + m ($\angle$ C) + m ($\angle$ D) + m ($\angle$ E) = 540°

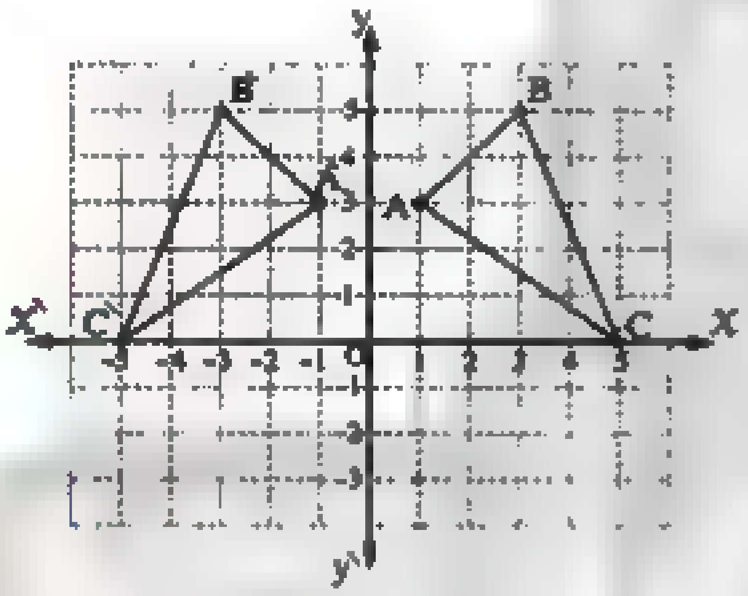
$\therefore$ m ($\angle$ E) = 540° – (140° + 130° + 90° + 110°) = 70° (The req.)

[b]



5

[a]



[b] $\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore$ XY = $\frac{1}{2}$ AC $\therefore$ XY = 3 cm.

$\because$ Y is the midpoint of $\overline{BC}$, Z is the midpoint of $\overline{AC}$

$\therefore$ YZ = $\frac{1}{2}$ AB $\therefore$ YZ = 3.5 cm.

$\because$ Z is the midpoint of $\overline{AC}$, X is the midpoint of $\overline{AB}$

$\therefore$ ZX = $\frac{1}{2}$ BC $\therefore$ ZX = 4.5 cm.

$\therefore$ The perimeter of Δ XYZ = 3 + 3.5 + 4.5 = 11 cm. (The req.)

Additional question

1 (1) 5 cm. (2) axis of symmetry

2 In Δ ADB : $\because$ m ($\angle$ ADB) = 90°

$\therefore$ $(BD)^2 = (AB)^2 - (AD)^2 = 676 - 576 = 100$

$\therefore$ BD = $\sqrt{100}$ = 10 cm.

In Δ ADC : $\because$ m ($\angle$ ADC) = 90°

$\therefore$ $(CD)^2 = (AC)^2 - (AD)^2 = 900 - 576 = 324$

$\therefore$ CD = $\sqrt{324}$ = 18 cm.

$\therefore$ BC = 18 + 10 = 28 cm. (The req.)

12 Damietta

1

(1) (a) (2) (a) (3) (b) (4) (d) (5) (c)

2

(1) equal in measure (2) 50° (3) bisects

(4) 120° (5) (3 , 2)

3

[a] $\because$ m ($\angle$ ABC) + m ($\angle$ CBD) + m ($\angle$ DBE) + m ($\angle$ EBA) = 360° (accumulative angles at B)

$\therefore$ m ($\angle$ EBD) = 360° – (110° + 35° + 140°) = 75° (The req.)

[b] In the square ABCD :

$\because$ AD = BC , BC = CE $\therefore$ AD = CE (1)

$\because$ $\overline{AD}$ // $\overline{BC}$, E $\in$ $\overrightarrow{BC}$ $\therefore$ $\overline{AD}$ // $\overline{CE}$ (2)

From (1) and (2) :

$\therefore$ The figure ACED is a parallelogram. (First req.)

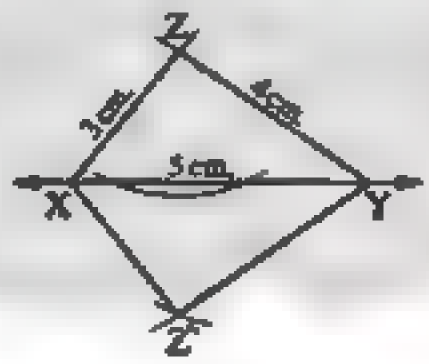
$\because$ $\overline{AC}$ bisects $\angle$ BAD

$\therefore$ m ($\angle$ CAD) = 45°

$\therefore$ m ($\angle$ E) = 45° (properties of parallelogram) (Second req.)

4

[a]



[b] $\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ BC $\therefore$ DE = 5 cm.

, $\because$ E is the midpoint of $\overline{AC}$, F is the midpoint of $\overline{BC}$

$\therefore$ FE = $\frac{1}{2}$ AB $\therefore$ FE = 4 cm.

, $\because$ F if the midpoint of $\overline{BC}$, D is the midpoint of $\overline{AB}$

$\therefore$ FD = $\frac{1}{2}$ AC $\therefore$ ED = 3 cm.

$\therefore$ The perimeter of Δ DEF = 4 + 3 + 5 = 12 cm. (The req.)

5

[a] $\because$ $\overrightarrow{BA}$ // $\overrightarrow{DF}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore$ m ($\angle$ D) = m ($\angle$ B) = 60° (alternate angles)

In Δ CDF :

m ($\angle$ DCF) = 180° − (60° + 40°) = 80° (The req.)

[b]

Additional question

1 (1) (c) (2) (b)

2 In Δ ABD : ∵ m ($\angle$ ADB) = 90°

∴ $(AD)^2 = (AB)^2 - (BD)^2 = 169 - 25 = 144$

∴ AD = $\sqrt{144}$ = 12 cm.

In Δ ADC : ∵ m ($\angle$ ADC) = 90°

∴ $(AC)^2 = (AD)^2 + (DC)^2 = 144 + 256 = 400$

∴ AC = $\sqrt{400}$ = 20 cm. (The req.)

13 El-Fayoum

1

(1) (a) (2) (d) (3) (c) (4) (d) (5) (a)

2

(1) parallel to the third side. (2) (1 , 0)

(3) a rectangle (4) 108° (5) 180°

3

[a] ∵ $\overleftrightarrow{DA} \cap \overleftrightarrow{EB}$ = {M}

∴ m ($\angle$ BMD) = m ($\angle$ AME) = 72° (V.O.A.)

, ∵ $\overrightarrow{MC}$ bisects $\angle$ BMD

∴ m ($\angle$ CMD) = $\frac{72°}{2}$ = 36° (The req.)

[b] ∵ m ($\angle$ BAC) = m ($\angle$ ABC)

, $\angle$ ACD is an exterior angle of Δ ABC

∴ m ($\angle$ ABC) = $\frac{128°}{2}$ = 64° (The req.)

4

[a] In Δ ABC :

∵ X is the midpoint of $\overline{AB}$, Z is the midpoint of $\overline{AC}$

∴ XZ = $\frac{1}{2}$ BC ∴ BC = 8 cm.

, ∵ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

∴ XY = $\frac{1}{2}$ AC ∴ AC = 5 cm.

, ∵ Y is the midpoint of $\overline{BC}$, Z is the midpoint of $\overline{AC}$

∴ YZ = $\frac{1}{2}$ AB ∴ AB = 6 cm.

∴ The perimeter of Δ ABC = 8 + 5 + 6 = 19 cm.

[b] ∵ ABCDEF is a regular hexagon

∴ The measure of each exterior angle of the regular hexagon = $\frac{360°}{6}$ = 60° (The req.)

5

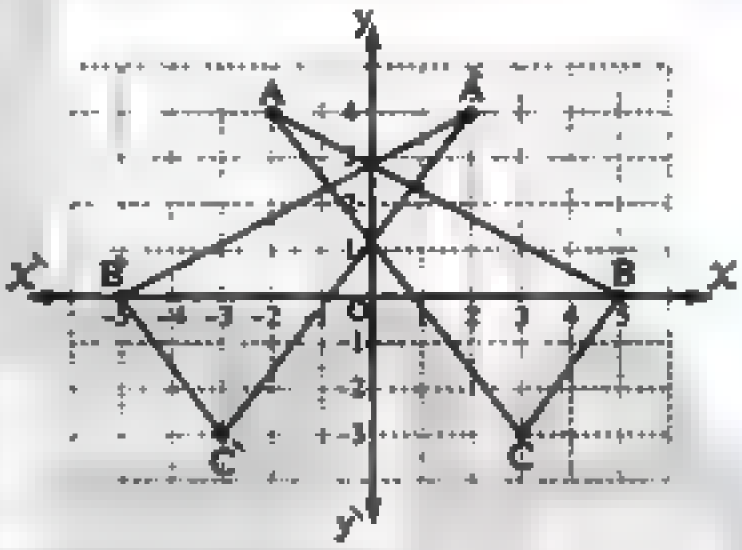
[a] ∵ $\overleftrightarrow{DE}$ // $\overline{BC}$, $\overleftrightarrow{DC}$ is a transversal

∴ m ($\angle$ C) = m ($\angle$ D) = 70° (alternate angles)

∵ $\angle$ DAB is an exterior angle of Δ ABC

∴ m ($\angle$ DAB) = 40° + 70° = 110° (The req.)

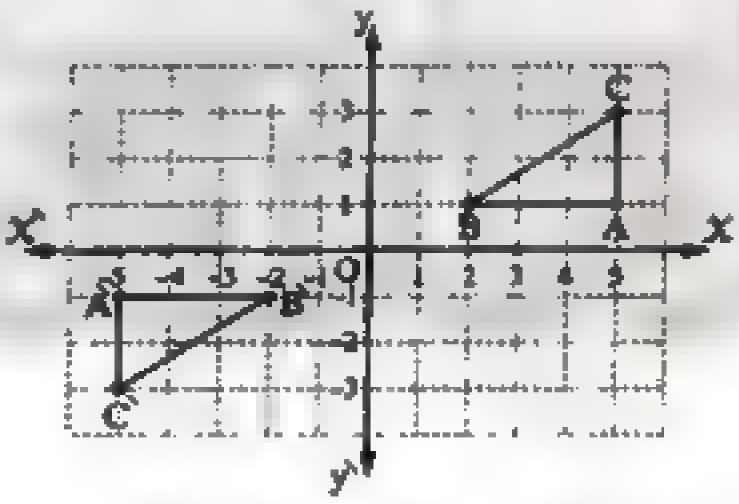
[b]



Additional question

1 (1) (b) (2) (a)

2



14 Beni Suef

1 (1) (c) (2) (a) (3) (b) (4) (a) (5) (b)

2 (1) equal in measure (2) (0 , 4)

(3) trapezium (4) a rhombus , a square

(5) 360°

3

[a] ∵ ABCD is a square and $\overline{BD}$ is a diagonal in it.

∴ m ($\angle$ C) = 90° and m ($\angle$ BDC) = 45°

In the quadrilateral CDME :

$\therefore$ m ($\angle$ MEC) = 360° – (90° + 45° + 102°) = 123° (The req.)

[b] The number of sides = $\frac{360°}{180° - 135°}$ = 8 sides

4

[a] $\because$ $\overline{AB}$ // $\overline{ED}$, $\overleftrightarrow{AE}$ is a transversal.

$\therefore$ m ($\angle$ BAE) + m ($\angle$ E) = 180°

(interior angles in the same side of the transversal)

$\therefore$ m ($\angle$ BAE) = 180° – 60° = 120°

$\therefore$ m ($\angle$ BAE) + m ($\angle$ B) = 180°

but they are interior angles in the same side of $\overline{AB}$

$\therefore$ $\overline{BC}$ // $\overline{AE}$ (Q.E.D.)

[b] $\because$ $\overleftrightarrow{AC} \cap \overleftrightarrow{DF}$ = {B}

$\therefore$ m ($\angle$ CBF) = m ($\angle$ ABD) = 30° (V.O.A.)

, $\because$ $\overrightarrow{BE}$ bisects $\angle$ CBF

$\therefore$ m ($\angle$ CBE) = m ($\angle$ EBF) = $\frac{30°}{2}$ = 15°

, $\because$ B $\in$ $\overleftrightarrow{AC}$

$\therefore$ m ($\angle$ ABF) = 180° – 30° = 150°

$\therefore$ m ($\angle$ ABE) = m ($\angle$ ABF) + m ($\angle$ EBF)

= 150° + 15° = 165° (The req.)

5

[a] In Δ ABC :

$\because$ E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$

$\therefore$ $\overline{EF}$ // $\overline{BC}$

, $\because$ $\overline{AD}$ // $\overline{BC}$ $\therefore$ $\overline{EF}$ // $\overline{AD}$ (1)

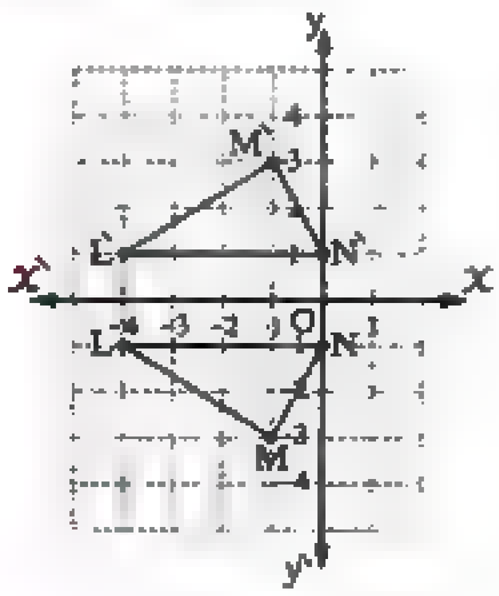
, EF = $\frac{1}{2}$ BC $\therefore$ BC = 2 EF

$\because$ BC = 2 AD $\therefore$ EF = AD (2)

From (1) , (2) :

$\therefore$ AEFD is a parallelogram. (Q.E.D.)

[b]



Additional question

1 (1) The point C (2) $\overline{CD}$ (3) Δ CYM

2 In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore$ $(AC)^2 = (AB)^2 + (BC)^2$ = 49 + 576 = 625

$\therefore$ AC = $\sqrt{625}$ = 25 cm.

In Δ ADC : $\because$ m ($\angle$ D) = 90°

$\therefore$ $(CD)^2 = (AC)^2 - (AD)^2$ = 625 – 225 = 400

$\therefore$ DC = $\sqrt{400}$ = 20 cm. (The req.)

15 Souhag

1 (1) 108° (2) 110°

(3) half the length of the third side.

(4) (6 , 1) (5) 50°

2 (1) (c) (2) (d) (3) (c) (4) (a) (5) (a)

3

[a] $\because$ $\overline{XY}$ // $\overline{EZ}$, $\overleftrightarrow{XE}$ is a transversal

$\therefore$ m ($\angle$ X) + m ($\angle$ E) = 180°

(interior angles in the same side of the transversal)

$\therefore$ m ($\angle$ E) = 180° – 115° = 65°

$\therefore$ m ($\angle$ E) = m ($\angle$ YZF) = 65°

but they are corresponding angles

$\therefore$ $\overline{XE}$ // $\overline{YZ}$

$\therefore$ XYZE is a parallelogram (Q.E.D.)

[b] $\because$ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ C) + m ($\angle$ A) = 180°

(interior angles and in the same side of the transversal)

$\therefore$ m ($\angle$ C) = 180° – 110° = 70°

$\because$ $\angle$ AED is an exterior angle of Δ CDE

$\therefore$ m ($\angle$ AED) = 50° + 70° = 120° (The req.)

4

[a] $\because$ m ($\angle$ BAE) + m ($\angle$ EAC) + m ($\angle$ CAB) = 360°

(accumulative angles at A)

$\therefore$ m ($\angle$ CAB) = 360° – (130° + 90°) = 140°

, $\because$ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{AC}$ is a transversal.

$\therefore$ m ($\angle$ CAB) + m ($\angle$ C) = 180°

(interior angles in the same side of the transversal)

$\therefore$ m ($\angle$ C) = 180° – 140° = 40° (The req.)

[b] $\because \overrightarrow{AE} \cap \overleftrightarrow{CD} = \{B\}$

$\therefore m(\angle DBE) = m(\angle ABC) = 50^\circ$ (V.O.A.)

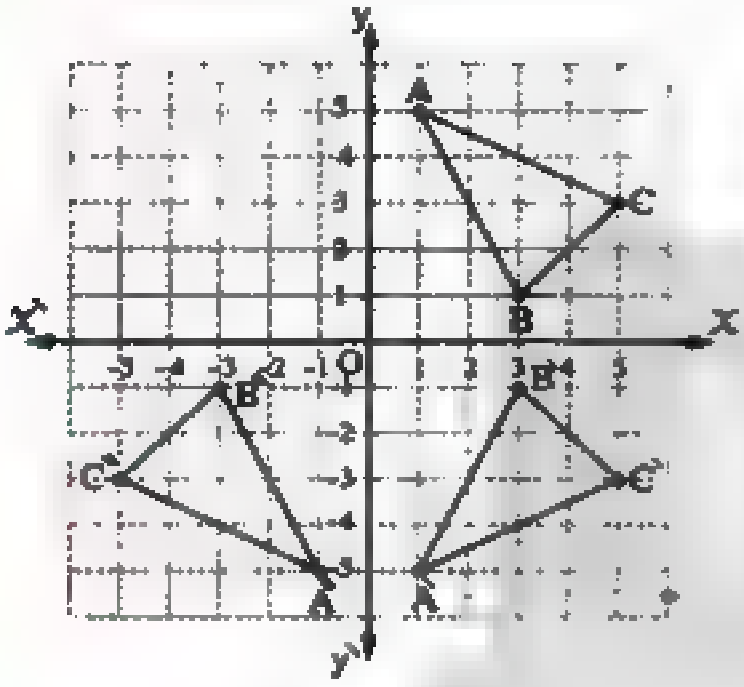
, $\because \overrightarrow{BE}$ bisects $\angle DBF$

$\therefore m(\angle EBF) = m(\angle DBE) = 50^\circ$

$\therefore m(\angle FBC) = 180^\circ - (50^\circ + 50^\circ) = 80^\circ$

(Q.E.D.)

5



(1) $\Delta \grave{A}\grave{B}\grave{C}$ is the image of ΔABC by reflection in X-axis.

(2) $\Delta \grave{A}\grave{B}\grave{C}$ is the image of ΔABC by rotation about origin point with an angle of measure 180°

Additional question

1 (1) two (2) -4 (3) axis of symmetry

2 In ΔABC : $\because m(\angle B) = 90^\circ$

$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$

$\therefore AB = \sqrt{256} = 16$ cm.

$\therefore AD = AB - BD = 16 - 9 = 7$ cm.

, $\because AE = 2\,BC$ $\therefore AE = 2 \times 12 = 24$ cm.

, $\because \overline{BC} \parallel \overline{AE}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle A) = m(\angle B) = 90^\circ$ (alternate angles)

In ΔADE : $\because m(\angle A) = 90^\circ$

$\therefore (DE)^2 = (AE)^2 + (AD)^2 = 576 + 49 = 625$

$\therefore DE = \sqrt{625} = 25$ cm. (The req.)

Answers of school book models in geometry and measurement

Model 1

1 (1) (b) (2) (a) (3) (d) (4) (b) (5) (b)

2

(1) (2 , – 1) (2) (– 2 , 1)

(3) its two diagonals are perpendicular

(4) 120° (5) (8 , 2)

3

[a] ∵ m (∠ A) = m (∠ B) = 25°

, ∠ ACD is an exterior angle of Δ ABC

∴ m (∠ ACD) = 25° + 25° = 50° (The req.)

[b] ∵ D is the midpoint $\overline{AB}$ ∴ AD = $\frac{1}{2}$ AB

∴ AD = $\frac{1}{2}$ × 12 = 6 cm. (1)

, ∵ E is the midpoint $\overline{AC}$ ∴ AE = $\frac{1}{2}$ AC

∴ AE = $\frac{1}{2}$ × 8 = 4 cm. (2)

, ∵ D is the midpoint $\overline{AB}$, E is the midpoint $\overline{AC}$

∴ ED = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 10 = 5 cm. (3)

∴ From (1) , (2) , (3) :

The perimeter of Δ ADE = 6 + 4 + 5 = 15 cm.

(The req.)

4

[a] ∵ $\overline{DE}$ // $\overline{YZ}$, $\overleftrightarrow{ZD}$ is a transversal

∴ m (∠ Z) = m (∠ D) = 50° (alternate angles)

In Δ XYZ :

∵ m (∠ Z) + m (∠ ZXY) + m (∠ Y) = 180°

∴ m (∠ Y) = 180° – (105° + 50°) = 25°

, ∵ X ∈ $\overline{DZ}$

∴ m (∠ YXD) = 180° – 105° = 75° (The req.)

[b] ∵ $\overleftrightarrow{AZ}$ // $\overleftrightarrow{YD}$ // $\overleftrightarrow{XE}$ // $\overleftrightarrow{CB}$

, AY = YX = XC

∴ AD = DE = EB ∴ EB = $\frac{18}{3}$ = 6 cm.

5

[a] In Δ ABD : ∵ m (∠ ADB) = 90°

∴ $(BD)^2 = (AB)^2 – (AD)^2$ = 676 – 576 = 100

∴ BD = $\sqrt{100}$ = 10 cm.

In Δ ADC : ∵ m (∠ ADC) = 90°

∴ $(CD)^2 = (AC)^2 – (AD)^2$ = 900 – 576 = 324

∴ CD = $\sqrt{324}$ = 18 cm.

∴ BC = 10 + 18 = 28 cm.

∴ The area of Δ ABC = $\frac{1}{2}$ BC × AD

= $\frac{1}{2}$ × 28 × 24 = 336 cm^2.

(The req.)

[b] ∵ ABCD is a squre

∴ $\overline{AD}$ // $\overline{BC}$, ∵ E ∈ $\overleftrightarrow{BC}$

∴ $\overline{AD}$ // $\overline{EC}$, ∵ $\overline{AC}$ // $\overline{DE}$

∴ ACED is a parallelogram (Q.E.D.)

Model 2

1 (1) (b) (2) (c) (3) (b) (4) (c) (5) (c)

2

(1) 360° (2) (5 , 5) (3) 120°

(4) bisects the third side. (5) ZYC

3

[a] In Δ XYZ : m (∠ Y) = 90°

∴ $(XZ)^2 = (XY)^2 + (YZ)^2$ = 49 + 576 = 625

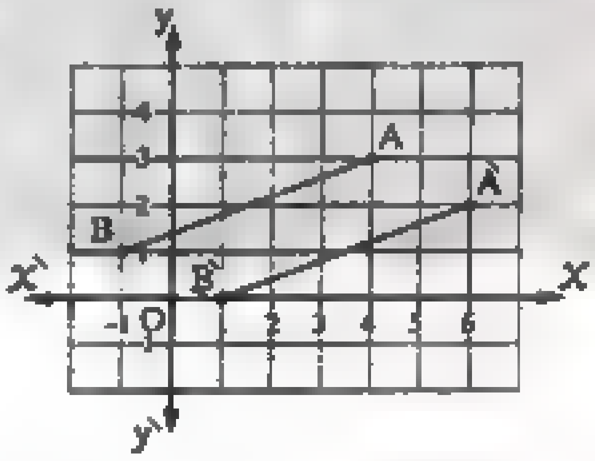
∴ XZ = $\sqrt{625}$ = 25 cm.

In Δ LXZ : m (∠ L) = 90°

∴ $(LZ)^2 = (XZ)^2 – (LX)^2$ = 625 – 225 = 400

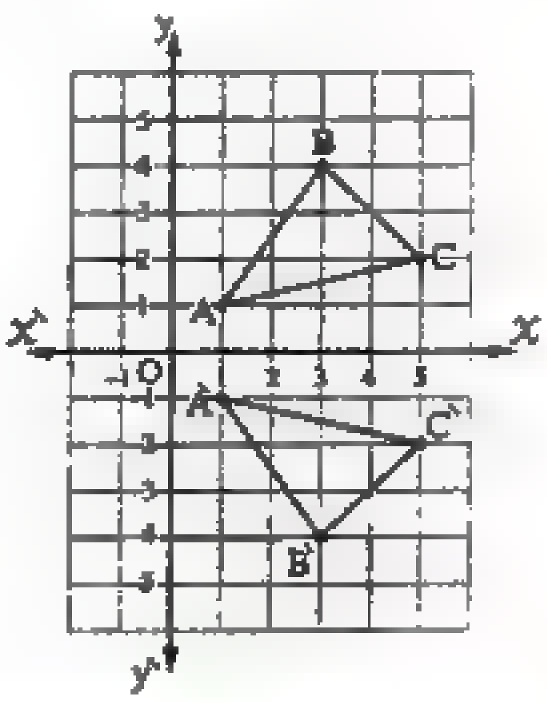
∴ LZ = $\sqrt{400}$ = 20 cm. (The req.)

[b]



4

[a]



[b] In Δ ABC : m ($\angle$ ACB) = 180° – (90° + 30°) = 60°

, $\because \overline{BD} \cap \overline{AO} = \{C\}$

$\therefore$ m ($\angle$ ACB) = m ($\angle$ OCD) = 60° (V.O.A)

$\therefore$ m ($\angle$ E) = 360° – (60° + 120° + 90°) = 90°

(The req.)

5

[a] $\because \overline{EO} \parallel \overline{CD}$, $\overleftrightarrow{EB}$ is a transversal

$\therefore$ m ($\angle$ CBA) = m ($\angle$ E) = 50° (alternate angles)

From Δ ABC :

m ($\angle$ BAC) = 180° – (50° + 30°) = 100°

$\because$ $\angle$ ABD is an exterior angle of Δ ABC

$\therefore$ m ($\angle$ ABD) = 30° + 100° = 130° (The req.)

[b] $\because \overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\because$ AX = XB

$\therefore$ DY = YC

$\therefore$ Y is the midpoint of $\overline{CD}$

In Δ CDE : $\because \overline{ZY} \parallel \overline{DE}$

, Y is the midpoint of $\overline{CD}$

$\therefore$ Z is the midpoint of $\overline{CE}$

$\therefore$ CZ = ZE (Q.E.D.)

Model 3

1

(1) (b) (2) (c) (3) (a) (4) (c) (5) (b)

2

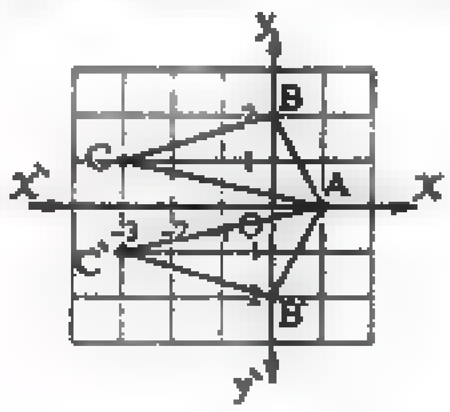
(1) two adjacent sides are equal in length

(2) equal in measure (3) y-axis

(4) parallel to the third side (5) (– 4 , – 1)

3

[a]



[b] $\because$ ABCD is a parallelogram , M is the intersection point of its diagonals.

$\therefore$ M is the midpoint of $\overline{AC}$

In Δ ABC : $\because$ M is the midpoint of $\overline{AC}$, $\overline{ME} \parallel \overline{BC}$

$\therefore$ E is the midpoint of $\overline{AB}$

$\therefore$ AE = EB (Q.E.D.)

4

[a] $\because \overrightarrow{OH} \parallel \overline{BC}$, $\overrightarrow{OB}$ is a transversal to them

$\therefore$ m ($\angle$ B) + m ($\angle$O) = 180°

(two interior angles in the same side of the transversal)

$\therefore$ m ($\angle$ B) = 180° – 135° = 45°

$\because \overrightarrow{DE} \parallel \overline{BC}$ and $\overleftrightarrow{DC}$ is a transversal to them

$\therefore$ m ($\angle$ C) + m ($\angle$ D) = 180°

(two interior angles in the same side of the transversal)

$\therefore$ m ($\angle$ C) = 180° – 120° = 60°

$\therefore$ In Δ ABC :

m ($\angle$ BAC) = 180° – (45° + 60°) = 75° (The req.)

[b] In Δ ABC : m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 144 + 81 = 225$

$\therefore AC = \sqrt{225} = 15$ cm.

In Δ ACD : m ($\angle$ ACD) = 90°

$\therefore (AD)^2 = (AC)^2 + (DC)^2 = 225 + 400 = 625$

$\therefore AD = \sqrt{625} = 25$ cm. (The req.)

5

(1) Δ LNB (2) Δ DOM (3) Δ BON

Model 4

1

(1) (a) (2) (b) (3) (a) (4) (b) (5) (a)

2

(1) parallel to the third side

(2) equal in measure

(3) each two opposite sides are parallel (there are another solution).

(4) (2 , – 3) (5) (– 3 , – 2)

3

In Δ CEO : m ($\angle$ COE) = 180° – (35° + 50°) = 95°

, $\because \overline{DC} \cap \overline{BE} = \{O\}$

$\therefore$ m ($\angle$ DOB) = m ($\angle$ COE) = 95° (V.O.A) (First req.)

From the quadrilateral ABOD

m ($\angle$ B) = 360° – (95° + 100° + 85°) = 80°

(Second req.)

4

[a] $\because$ The length = $\frac{48}{6}$ = 8 cm.

$\therefore$ The length of the diagonal = $\sqrt{8^2 + 6^2}$ = 10 cm.

[b] $\because \overline{AB} \parallel \overline{DE}$, $\overrightarrow{BD}$ is a transversal

$\therefore m(\angle B) = m(\angle D) = 60°$ (alternate angles)

, $\because \angle AOD$ is an exterior angle of ΔABO

$\therefore m(\angle AOD) = m(\angle B) + m(\angle A)$

$\therefore m(\angle A) = 110° - 60° = 50°$ (The req.)

5

[a] The point A = (1 , 2)

$\therefore$ The image of the point A (1 , 2) by translation (– 1 , 2) is (0 , 4)

[b] In ΔABC : $\because \overline{DE} \parallel \overline{BC}$, D is the midpoint of $\overline{AB}$

$\therefore$ E is the midpoint of $\overline{AC}$ $\qquad \therefore DE = \frac{1}{2} BC$

, $\because$ O is the midpoint of $\overline{BC}$

$\therefore BO = \frac{1}{2} BC$ $\qquad \therefore DE = BO$

, $\because \overline{DE} \parallel \overline{BC}$

$\therefore$ DBOE is a parallelogram (First req.)

, $\because ED = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6$ cm.

, $\because$ D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{BC}$

$\therefore DO = \frac{1}{2} AC = \frac{1}{2} \times 8 = 4$ cm.

, $\because$ E is the midpoint of $\overline{AC}$, O is the midpoint of $\overline{BC}$

$EO = \frac{1}{2} AB = \frac{1}{2} \times 10 = 5$ cm.

$\therefore$ The perimeter of $\Delta EDO = 6 + 4 + 5 = 15$ cm.

(Second req.)

Model 5

1

(1) (b) (2) (d) (3) (a) (4) (d) (5) (c)

2

(1) The rhombus (2) right-angled

(3) acute (4) (2 , 4) (5) (2 , 4)

3

[a] Theoretical

[b] $\because$ D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{AC}$

$\therefore DO = \frac{1}{2} BC$ $\qquad \therefore DO = 4$ cm.

, $\because$ O is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

$\therefore OE = \frac{1}{2} AB$ $\qquad \therefore OE = 3$ cm.

, $\because$ E is the midpoint of $\overline{BC}$, D is the midpoint of $\overline{AB}$

$\therefore ED = \frac{1}{2} AC$ $\qquad \therefore ED = 5$ cm.

$\therefore$ The perimeter of $\Delta DEO = 4 + 3 + 5 = 12$ cm.

(The req.)

4

[a] $\because$ ABCD is a parallelogram , M is the intersection point of its diagonals.

$\therefore$ M is the midpoint of $\overline{AC}$

In ΔABC : $\overline{MO} \parallel \overline{AB}$

, M is the midpoint of $\overline{AC}$

$\therefore$ O is the midpoint of $\overline{BC}$

$\therefore BO = OC$ (Q.E.D.)

[b] In ΔXYZ : $\because m(\angle X) = 90°$

$\therefore (XY)^2 = (YZ)^2 - (XZ)^2 = 400 - 256 = 144$

$\therefore XY = \sqrt{144} = 12$ cm. (The req.)

5

[a] $\because B \in \overrightarrow{AE}$

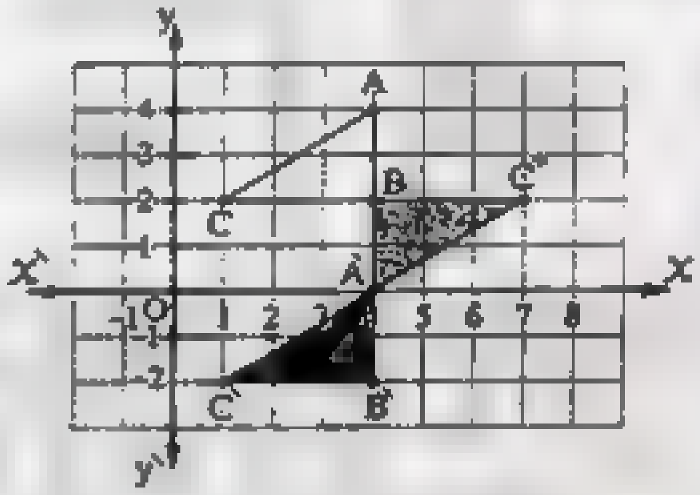
$\therefore m(\angle ABC) = 180° - 130° = 50°$

$\therefore$ From the quadrilateral ABCD :

$m(\angle C) = 360° - (50° + 80° + 120°)$

$= 360° - 250° = 110°$ (The req.)

[b]



(1) $\Delta A\grave{B}\grave{C}$ is the image of ΔABC by translation 2 AB in direction $\overrightarrow{AB}$

(2) $\Delta A\grave{B}\grave{C}$ is the image of ΔABC by rotation about B with an angle of measure 180°

Model 6

1

(1) (b) (2) (c) (3) (a) (4) (a) (5) (b)

2

(1) equal to the sum of the measures of its non adjacent interior angles

(2) square (3) 130°

(4) (– 2 , 2) (5) $(XY)^2 + (YZ)^2$

3

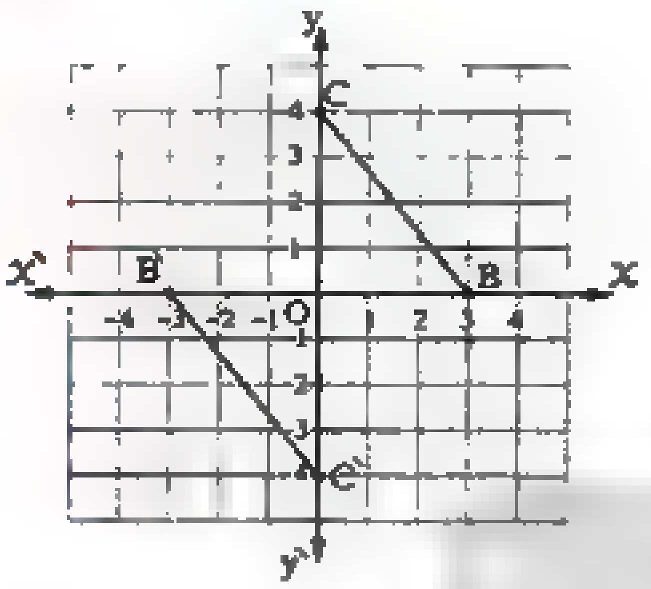
[a] $\because \overleftrightarrow{ED} \,//\, \overleftrightarrow{CB}$, $\overleftrightarrow{BE}$ is a transversal

$\therefore m(\angle B) = m(\angle E) = 40°$ (alternate angles)

$\therefore$ In ΔABC : $m(\angle BAC) = 180° - (50° + 40°) = 90°$

$\therefore \overline{AC} \perp \overline{BE}$ (Q.E.D.)

[b]



4

[a] In ΔABC : $m(\angle ACB) = 180° - (90° + 40°) = 50°$

, $\because \overline{BD} \cap \overline{AO} = \{C\}$

$\therefore m(\angle DCO) = m(\angle ACB) = 50°$ (V.O.A.)

$\because \overline{AB} \,//\, \overline{DE}$, $\overline{BD}$ is a transversal

$\therefore m(\angle B) = m(\angle D) = 90°$ (alternate angles)

From the quadrilateral CDEO

$\therefore m(\angle E) = 360° - (90° + 50° + 130°) = 90°$ (The req.)

[b] $\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore x + 3x + 2x = 180°$

$\therefore 6x = 180°$ $\therefore x = \frac{180°}{6} = 30°$

$\therefore m(\angle A) = x = 30°$, $m(\angle B) = 3x = 90°$

, $m(\angle C) = 2x = 60°$ (The req.)

5

[a] In ΔXYZ :

$\because$ D is the midpoint of $\overline{XY}$, E is the midpoint of $\overline{XZ}$

$\therefore DE = \frac{1}{2} ZY = \frac{1}{2} \times 12 = 6$ cm.

, E is the midpoint of $\overline{XZ}$, O is the midpoint of $\overline{ZY}$

$\therefore EO = \frac{1}{2} \times XY = \frac{1}{2} \times 6 = 3$ cm.

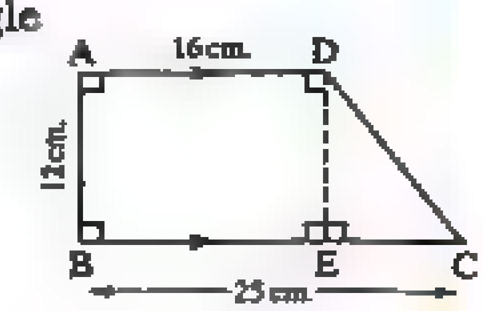
, O is the midpoint of $\overline{ZY}$, D is the midpoint of $\overline{XY}$

$\therefore OD = \frac{1}{2} \times XZ = \frac{1}{2} \times 8 = 4$ cm.

$\therefore$ The perimeter of $\Delta DOE = 6 + 3 + 4 = 13$ cm. (The req.)

[b] Construction : Draw $\overline{DE} \perp \overline{BC}$

Proof : ABED is a rectangle



$\therefore DE = AB = 12$ cm.

, $AD = BE = 16$ cm.

$\therefore EC = 25 - 16 = 9$ cm.

, $\because m(\angle DEC) = 90°$

$\therefore (DC)^2 = (DE)^2 + (EC)^2 = 144 + 81 = 225$

$\therefore DC = \sqrt{225} = 15$ cm. (The req.)

Model 7

1

(1) (b) (2) (a) (3) (c) (4) (a) (5) (b)

2

(1) 540° (2) 120° (3) right (4) $(-2, 2)$ (5) 90°

3

[a] $\because \overrightarrow{DE} \,//\, \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle B) = m(\angle EAB) = 120°$ (alternate angles)

, $\because m(\angle B) + m(\angle C) = 120° + 60° = 180°$

and they are interior angles on the same side of the transversal

$\therefore \overline{AB} \,//\, \overline{CD}$

, $\because \overline{DA} \,//\, \overline{CB}$

$\therefore$ ABCD is a parallelogram (Q.E.D.)

[b] In ΔABC :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore ED = \frac{1}{2} BC$ $\therefore BC = 2\, ED$

$\therefore BC = 2 \times 5 = 10$ cm. (The req.)

4

[a] $\because$ The sum of measures of the interior angles of the pentagon $= (5 - 2) \times 180° = 540°$

$\therefore m(\angle D) = 540° - (110° + 70° + 120° + 150°)$
$= 90°$ (The req.)

[b] In ΔABC : $\because m(\angle B) = 90°$

$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$

$\therefore AB = 16$ cm.

$\therefore AD = AB - DB = 16 - 9 = 7$ cm.

, $\because AE = 2\, BC$ $\therefore AE = 2 \times 12 = 24$

, $\because \overline{AE} \,//\, \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle A) = m(\angle B) = 90°$ (alternate angles)

$\therefore (ED)^2 = (AD)^2 + (AE)^2 = 49 + 576 = 625$

$\therefore ED = \sqrt{625} = 25$ (The req.)

5

[a] ∵ ABCD is a parallelogram

∴ m (∠ B) + m (∠ C) = 180°

∴ m (∠ C) = 180° − 135° = 45° (First req.)

, ∵ ABCD is a parallelogram

∴ AB = CD = 5 cm. , AD = BC = 8 cm.

∴ The perimeter of parallelogram ABCD

= 5 + 8 + 5 + 8 = 26 cm. (Second req.)

[b] (1) Δ ADC (2) Δ MBC

Model 8

1

(1) (a) (2) (c) (3) (c) (4) (b) (5) (d)

2

(1) half the length of the third side. (2) square

(3) 5 cm. (4) (3 , −4) , (−3 , 4) (5) (5 , −1)

3

[a] ∵ $\overrightarrow{DA}$ // $\overrightarrow{BE}$, $\overleftrightarrow{BD}$ is a transversal

∴ m (∠ CBD) = m (∠ ADB) = 50° (alternate angles)

∴ m (∠ DBE) = 180° − 50° = 130°

, ∵ $\overrightarrow{BA}$ bisects ∠ DBE

∴ m (∠ ABE) = $\frac{130°}{2}$ = 65°

, ∵ m (∠ C) = m (∠ ABE) = 65°

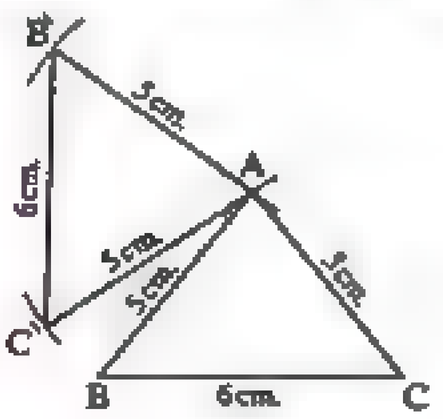
and they are corresponding angles

∴ $\overline{AB}$ // $\overline{CD}$

, ∵ $\overline{AD}$ // $\overline{BC}$

∴ ABCD is a parallelogram. (Q.E.D.)

[b]



4

[a] ∵ $\overline{DE}$ // $\overline{CB}$, $\overleftrightarrow{BD}$ is a transversal

∴ m (∠ B) = m (∠ D) = 50° (alternate angles)

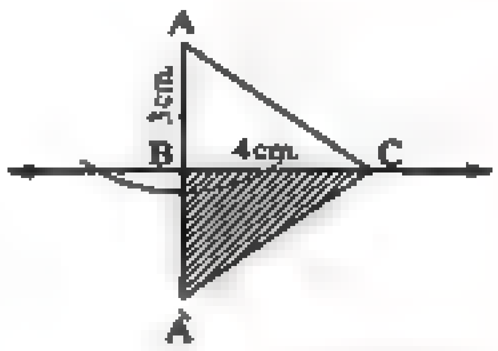
(First req.)

From Δ ABC :

m (∠ BAC) = 180° − (35° + 50°) = 95°

(Second req.)

[b]



5

[a] ∵ ABCD is a parallelogram

∴ $\overline{AB}$ // $\overline{DC}$ ∴ $\overline{XC}$ // $\overline{AB}$

In Δ ABE : ∵ C is the midpoint of $\overline{BE}$

, $\overline{XC}$ // $\overline{AB}$

∴ X is the midpoint of $\overline{AE}$

∴ AX = XE (Q.E.D.)

[b] In Δ BCD : ∵ m (∠ B) = 90°

∴ $(BD)^2 = (DC)^2 - (BC)^2 = 169 - 144 = 25$

∴ BD = $\sqrt{25}$ = 5 cm.

∴ AB = 5 + 11 = 16 cm.

In Δ ABC : ∵ m (∠ B) = 90°

∴ $(AC)^2 = (AB)^2 + (BC)^2 = 256 + 144 = 400$

∴ AC = $\sqrt{400}$ = 20 cm. (The req.)

Answers of school examinations in Geometry and measurement

1 Cairo

1

(1) (b) (2) (a) (3) (a) (4) (c) (5) (a)

2

(1) $(-5, 0)$ (2) $\overline{XZ}$, $\overline{YL}$ (3) acute

(4) bisects (5) $-90°$

3

[a] In Δ ADB : $\because$ m ($\angle$ ADB) = 90°

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$

$\therefore BD = \sqrt{100} = 10$ cm.

In Δ ADC : $\because$ m ($\angle$ ADC) = 90°

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$

$\therefore CD = \sqrt{324} = 18$ cm.

$\therefore BC = 10 + 18 = 28$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 28 \times 24 = 336$ cm^2.

(The req.)

[b] $\because \overline{AB} \parallel \overline{DE}$, $\overline{BD}$ is a transversal

$\therefore$ m ($\angle$ B) = m ($\angle$ D) = 60° (alternate angles)

, $\because \angle$ AOD is an exterior angle of Δ ABO

$\therefore$ m ($\angle$ AOD) = m ($\angle$ A) + m ($\angle$ B)

$\therefore$ m ($\angle$ A) = 100° − 60° = 40° (The req.)

4

[a] In Δ ABC : $\because$ X is the mid-point of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2}$ AC $\therefore$ AC = 28 cm.

, $\because$ X is the mid-point of $\overline{AB}$, Z is the mid-point of $\overline{AC}$

$\therefore XZ = \frac{1}{2}$ BC $\therefore$ BC = 20 cm.

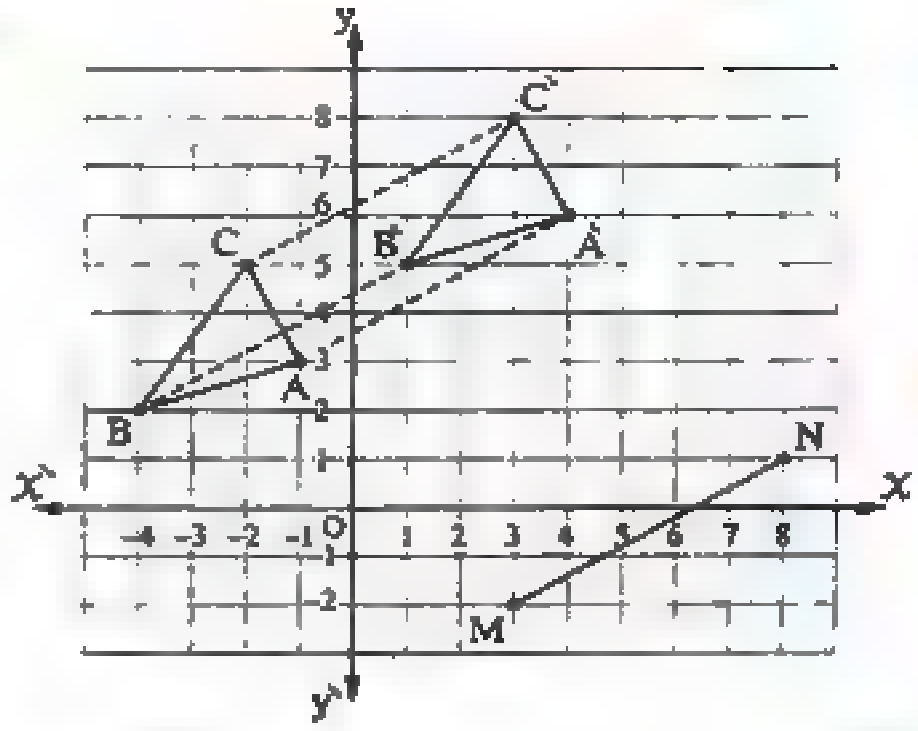
, $\because$ Y is the mid-point of $\overline{BC}$, Z is the mid-point of $\overline{AC}$

$\therefore YZ = \frac{1}{2}$ AB $\therefore$ AB = 24 cm.

$\therefore$ The perimeter of Δ ABC = 28 + 20 + 24 = 72 cm.

(The req.)

[b]



5

[a] $\because \overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\because$ AX = XB

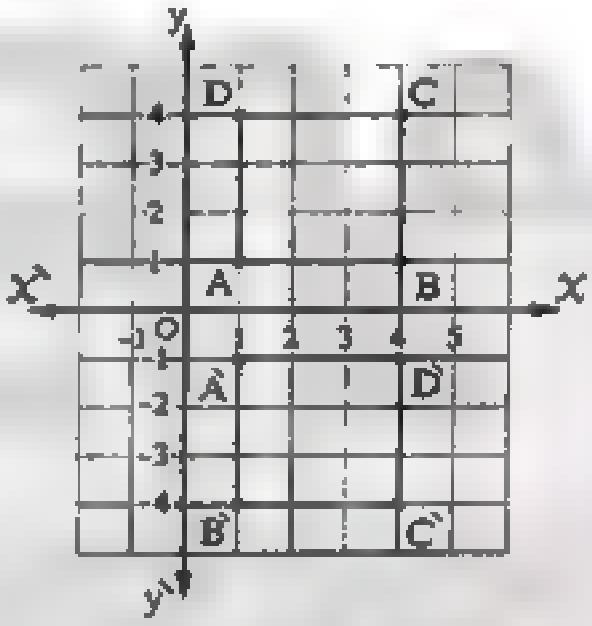
$\therefore$ DY = YC $\therefore$ Y is the midpoint of $\overline{CD}$

In Δ CDE : $\because \overline{ZY} \parallel \overline{DE}$, Y is the midpoint of $\overline{CD}$

$\therefore$ Z is the midpoint of $\overline{CE}$

$\therefore$ CZ = ZE (Q.E.D.)

[b]



2 Cairo

1

(1) (c) (2) (d) (3) (c) (4) (b) (5) (a)

2

(1) 12 (2) $(8, 2)$ (3) half the length of the third side

(4) y (5) rhombus

3

[a] $\because \overleftrightarrow{OH} \parallel \overline{BC}$, $\overrightarrow{OB}$ is a transversal

$\therefore$ m ($\angle$ B) + m ($\angle$ O) = 180°

(Two interior angles in the same side of the transversal)

$\therefore$ m ($\angle$ B) = 180° − 135° = 45°

$\because \overleftrightarrow{DE} \parallel \overline{BC}$, $\overleftrightarrow{CD}$ is a transversal

$\therefore m(\angle C) + m(\angle D) = 180°$

(Two interior angles in the same side of the transversal)

$\therefore m(\angle C) = 180° - 120° = 60°$

In ΔABC : $\therefore m(\angle BAC) = 180° - (45° + 60°) = 75°$

(The req.)

[b] $\because$ ABCD is a parallelogram , M is the intersection point of its diagonals.

$\therefore$ M is the midpoint of $\overline{AC}$

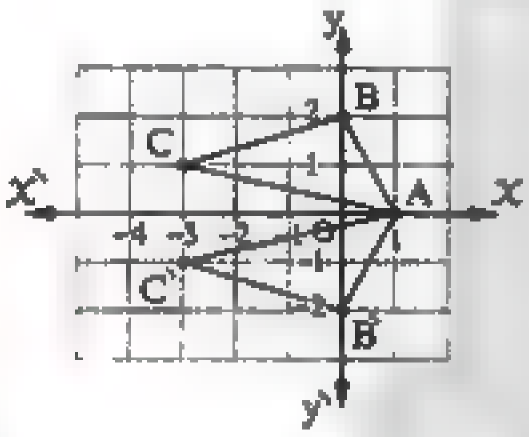
In ΔABC : $\because$ M is the midpoint of $\overline{AC}$, $\overline{MO} \parallel \overline{AB}$

$\therefore$ O is the midpoint of $\overline{BC}$ $\quad \therefore BO = OC$

(Q.E.D.)

4

[a]



[b] $\because C \in \overrightarrow{BF}$

$\therefore m(\angle BCD) = 180° - 30° = 150°$

From the pentagon ABCDE :

$\therefore m(\angle D) = 540° - (120° + 70° + 110° + 150°)$

$= 90°$ (The req.)

5

[a] In ΔADB : $\because m(\angle ADB) = 90°$

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$

$\therefore BD = \sqrt{100} = 10$ cm.

In ΔADC : $\because m(\angle ADC) = 90°$

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$

$\therefore CD = \sqrt{324} = 18$ cm.

$\therefore BC = 10 + 18 = 28$ cm. (First req.)

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 28 \times 24 = 336\ cm^2$.

(Second req.)

[b] From ΔABC : $m(\angle ACB) = 180° - (90° + 30°) = 60°$

$\because \overline{BD} \cap \overline{AO} = \{C\}$

$\therefore m(\angle ACB) = m(\angle DCO) = 60°$ (V.O.A)

$\therefore$ From the quadrilateral CDEO

$m(\angle E) = 360° - (120° + 60° + 90°) = 90°$

(The req.)

3 Cairo

1

(1) (c) (2) (b) (3) (d) (4) (c) (5) (c)

2

(1) 130° (2) (2 , 4) (3) 180°

(4) bisects the third side (5) 90

3

[a] From ΔBEF : $m(\angle EBF) = 180° - (90° + 50°) = 40°$

$\because \overline{AE} \cap \overline{CF} = \{B\}$

$\therefore m(\angle ABC) = m(\angle EBF) = 40°$ (V.O.A)

$\because \overline{AD} \parallel \overline{EF}$, $\overline{AE}$ is a transversal

$\therefore m(\angle A) = m(\angle E) = 90°$ (alternate angles)

$\therefore$ From the quadrilateral ABCD :

$m(\angle C) = 360° - (110° + 90° + 40°) = 120°$

(The req.)

[b] The number of sides $= \frac{360°}{180° - 135} = 8$ sides.

4

[a] In ΔABC : $\because$ D is the mid-point of $\overline{AB}$, E is the mid-point of $\overline{BC}$

$\therefore DE = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5$ cm.

, $\because$ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$ cm.

, $\because$ E is the midpoint of $\overline{BC}$, F is the midpoint of $\overline{AC}$

$\therefore EF = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3$ cm.

$\therefore$ The perimeter of $\Delta DEF = 5 + 4 + 3 = 12$ cm.

(The req.)

[b] In ΔCBD : $\because m(\angle B) = 90°$

$\therefore (BD)^2 = (CD)^2 - (BC)^2 = (13)^2 - (12)^2 = 25$

$\therefore BD = \sqrt{25} = 5$ cm.

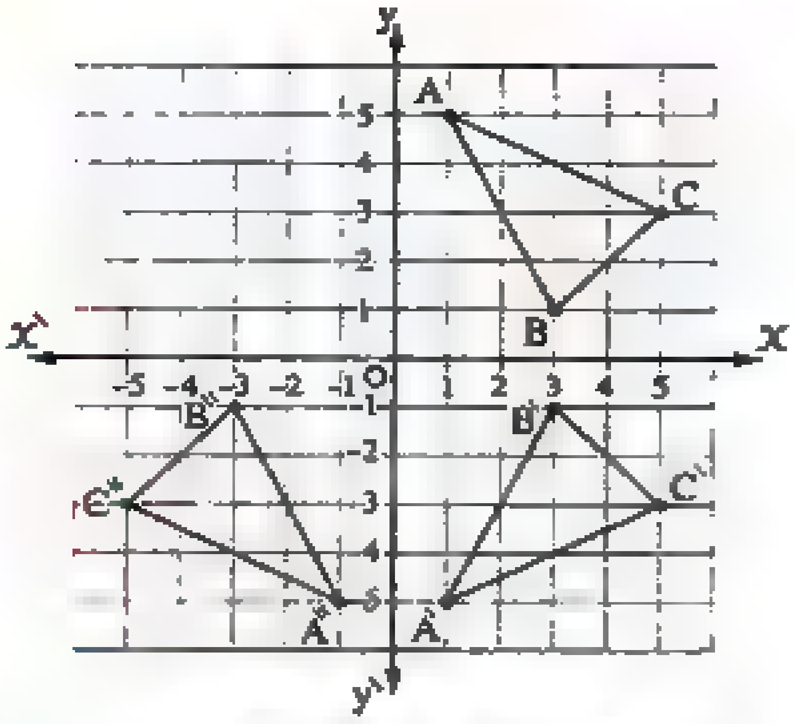
$\therefore AB = 11 + 5 = 16$ cm.

In ΔABC : $\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (16)^2 + (12)^2 = 400$

$\therefore AC = \sqrt{400} = 20$ cm. (The req.)

5



4 Giza

1

(1) (a) (2) (d) (3) (c) (4) (d) (5) (a)

2

(1) parallel to the third side (2) (1 , 0)
(3) rectangle (4) 108 (5) 180

3

[a] $\because \overleftrightarrow{DA} \cap \overleftrightarrow{EB} = \{M\}$

$\therefore m(\angle BMD) = m(\angle AME) = 72°$ (V.O.A)

$\because \overrightarrow{MC}$ bisects $\angle BMD$

$\therefore m(\angle CMD) = \frac{72°}{2} = 36°$ (The req.)

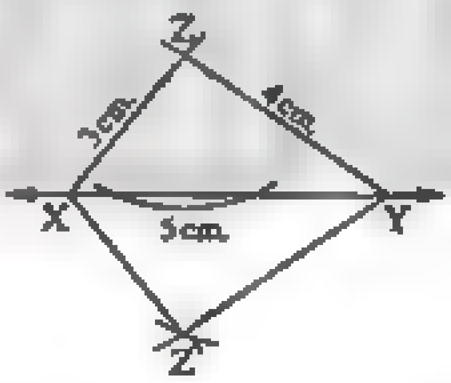
[b] $\because m(\angle ABC) = m(\angle BAC)$

, $\angle ACD$ is an exterior angle of ΔABC

$\therefore m(\angle ABC) = \frac{128°}{2} = 64°$ (The req.)

4

[a]



[b] In ΔABC :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore ED = \frac{1}{2} BC = \frac{1}{2} \times 10 = 5$ cm.

, $\because$ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$

$\therefore DF = \frac{1}{2} AC = \frac{1}{2} \times 6 = 3$ cm.

, $\because$ E is the midpoint of $\overline{AC}$, F is the midpoint of $\overline{BC}$

$\therefore EF = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4$ cm.

$\therefore$ The perimeter of $\Delta DEF = 5 + 3 + 4 = 12$ cm.

(The req.)

5

[a] $\because \overleftrightarrow{BA} \parallel \overleftrightarrow{DF}$, $\overleftrightarrow{BD}$ is a transversal

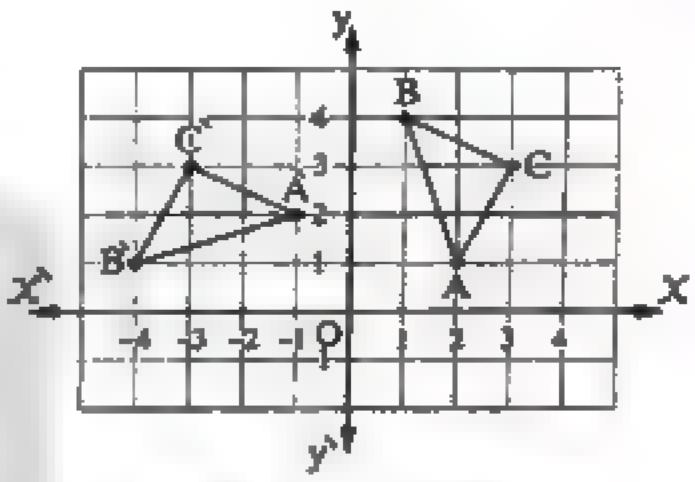
$\therefore m(\angle D) = m(\angle B) = 60°$ (alternate angles)

In ΔCDF :

$m(\angle DCF) = 180° - (40° + 60°) = 80°$

(The req.)

[b]



5 Giza

1

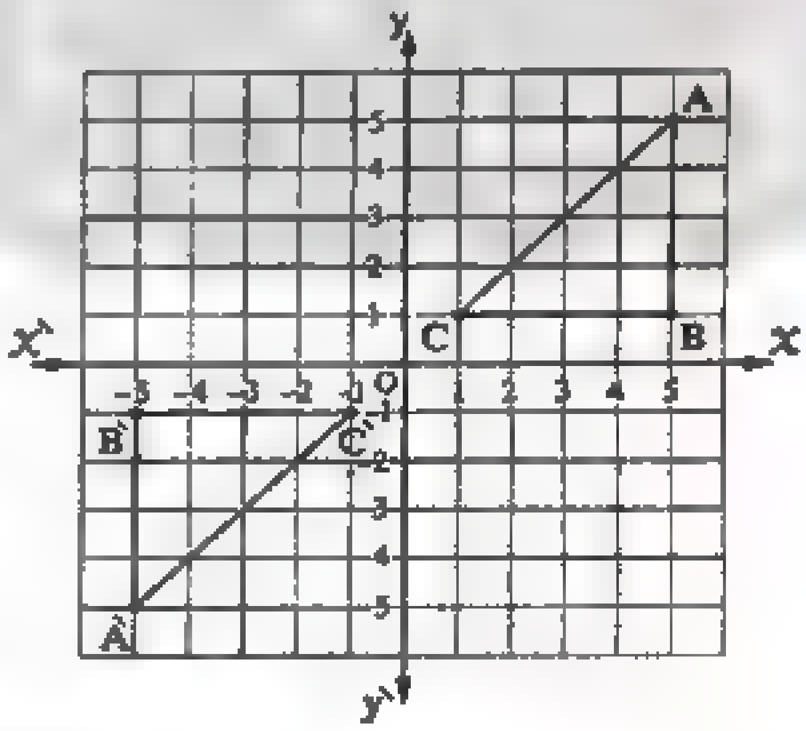
(1) (b) (2) (c) (3) (c) (4) (b) (5) (c)

2

(1) bisects the third side (2) (2 , 5) (3) 8
(4) equal in length , bisect each other , perpendicular
(5) trapezium

3

[a]



[b] $\because EC = CF = FE$

$\therefore \Delta CEF$ is an equilateral triangle

$\therefore m(\angle ECF) = \frac{180°}{3} = 60°$

$\because \overleftrightarrow{BF} \cap \overleftrightarrow{ED} = \{C\}$

$\therefore m(\angle BCD) = m(\angle ECF) = 60°$ (V.O.A)

$\therefore$ From quadrilateral ABCD :

$m(\angle B) = 360° - (115° + 98° + 60°) = 87°$

(The req.)

4

[a] $m(\angle HBD) = 360° - (135° + 110° + 45°) = 70°$

(The req.)

[b] $\because \overleftrightarrow{AB} \,/\!/\, \overrightarrow{ED}$, $\overrightarrow{AD}$ is a transversal

$\therefore m(\angle D) = m(\angle A) = 65°$ (alternate angles)

$\because \angle ACE$ is an exterior angle of Δ CDE

$\therefore m(\angle ACE) = 65° + 35° = 100°$ (The req.)

5

[a] In Δ ABC : $\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (3)^2 + (4)^2 = 25$

$\therefore AC = \sqrt{25} = 5$ cm.

In Δ ACD : $\because m(\angle ACD) = 90°$

$\therefore (AD)^2 = (AC)^2 + (CD)^2 = (5)^2 + (12)^2 = 169$

$\therefore AD = \sqrt{169} = 13$ cm.

$\therefore$ The perimeter of the figure ABCD

$= 3 + 4 + 12 + 13 = 32$ cm. (The req.)

[b] In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5$ cm.

, $\because$ X is the midpoint of $\overline{AB}$, Z is the midpoint of $\overline{AC}$

$\therefore XZ = \frac{1}{2} BC = \frac{1}{2} \times 11 = 5.5$ cm.

, $\because$ Y is the midpoint of $\overline{BC}$, Z is the midpoint of $\overline{AC}$

$\therefore YZ = \frac{1}{2} AB = \frac{1}{2} \times 10 = 5$ cm.

$\therefore$ The perimeter of Δ XYZ $= 4.5 + 5.5 + 5 = 15$ cm.

(The req.)

6 Alexandria

1

(1) (b) (2) (b) (3) (b) (4) (c) (5) (b)

2

(1) 360° (2) $\overline{AC}$, $\overline{BD}$ (3) (9 , − 1)

(4) $(AC)^2$, $(BC)^2$ (5) bisects the third side

3

[a] In Δ ABC : $\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$

$\therefore AC = \sqrt{625} = 25$ cm.

In Δ ADC : $\because m(\angle D) = 90°$

$\therefore (DC)^2 = (AC)^2 - (AD)^2 = (25)^2 - (15)^2 = 400$

$\therefore DC = \sqrt{400} = 20$ cm. (The req.)

[b] The number of sides $= \frac{360°}{180° - 120°} = 6$ sides

4

[a] In Δ ABC :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore \overline{DE} \,/\!/\, \overline{BC}$ $\therefore \overline{DE} \,/\!/\, \overline{BF}$ (1)

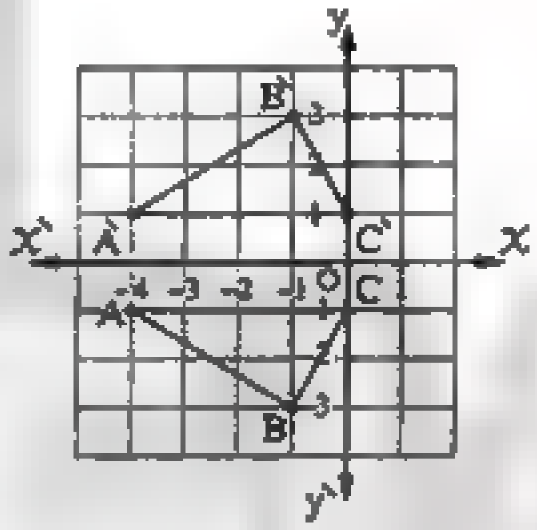
, $\because DE = \frac{1}{2} BC$, $\because BF = \frac{1}{2} BC$

$\therefore DE = BF$ (2)

From (1) and (2) : $\therefore$ BEDF is a parallelogram

(Q.E.D.)

[b]



5

[a] $\because \overline{DC} \,/\!/\, \overline{AB}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore m(\angle A) + m(\angle D) = 180°$

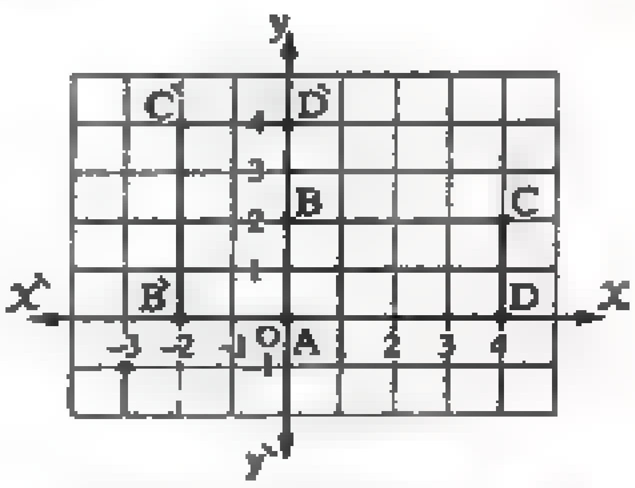
(Two interior angles in the same side of the transversal)

$\therefore m(\angle A) = 180° - 127° = 53°$

$\therefore m(\angle A) = m(\angle CBE) = 53°$ and they are corresponding angles

$\therefore \overleftrightarrow{AD} \,/\!/\, \overleftrightarrow{BC}$ (Q.E.D.)

[b]



7 Alexandria

1

(1) (d) (2) (b) (3) (c) (4) (c) (5) (b)

2

(1) 75 (2) ± 360 (3) $(-4 , -2)$
(4) 70 (5) obtuse

3

[a] In ΔABC :

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$

$\therefore \overline{XY} \parallel \overline{BC}$

In ΔXYZ : $\because$ D is the midpoint of $\overline{XZ}$

, $\overline{ED} \parallel \overline{XY}$

$\therefore$ E is the midpoint of $\overline{YZ}$

$\therefore YE = EZ$ (Q.E.D.)

[b] The measure of the interior angle $= \frac{(10-2) \times 180^\circ}{10}$

$= 144^\circ$

4

[a] In ΔXYZ : $\because m(\angle XYZ) = 90^\circ$

$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = (7)^2 + (24)^2 = 625$

$\therefore XZ = \sqrt{625} = 25$ cm.

In ΔXLZ : $\because m(\angle XLZ) = 90^\circ$

$\therefore (LZ)^2 = (XZ)^2 - (LX)^2 = (25)^2 - (15)^2 = 400$

$\therefore LZ = \sqrt{400} = 20$ cm. (The req.)

[b] The translation (LM) $= M - L = (1 , 5) - (2 , 3)$

$= (-1 , 2)$

$\therefore$ The point A = The image − The translation

$= (-6 , 4) - (-1 , 2) = (-5 , 2)$

5

[a] $\because$ ABCD is a parallelogram

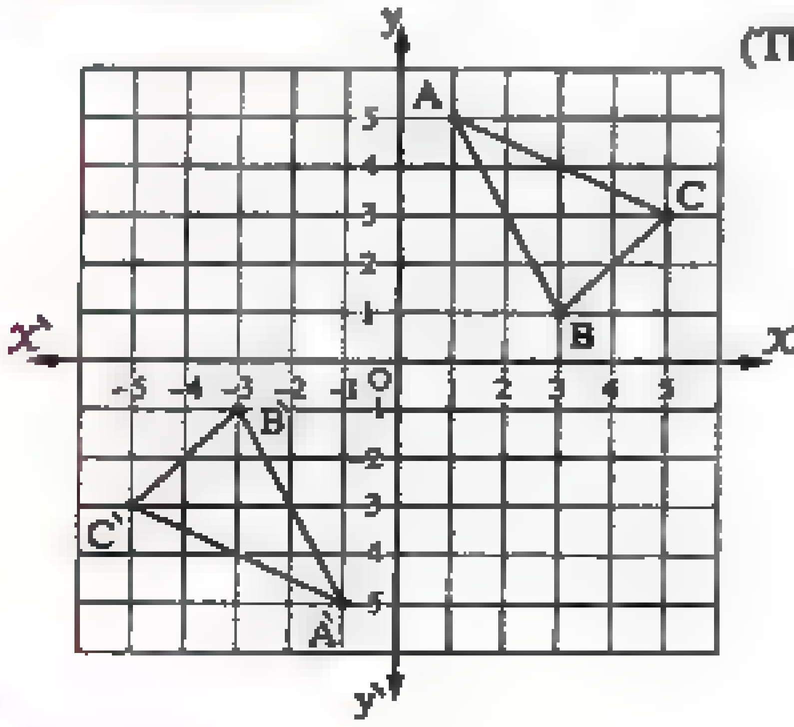
$\therefore \overline{AD} \parallel \overline{BC}$, $\overline{BD}$ is a transversal

$\therefore m(\angle DBC) = m(\angle ADB) = 80^\circ$ (alternate angles)

In ΔBMC : $\therefore m(\angle CMB) = 180^\circ - (30^\circ + 80^\circ) = 70^\circ$

(The req.)

[b]



8 El-Kalyoubia

1

(1) (b) (2) (c) (3) (a) (4) (d) (5) (a)

2

(1) right (2) parallel to the third side
(3) $(-3 , -2)$ (4) $(AB)^2 , (BC)^2$ (5) 45

3

[a] In ΔABC :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 10 = 5$ cm.

, $\because AD = \frac{1}{2} AB = \frac{1}{2} \times 12 = 6$ cm.

, $AE = \frac{1}{2} AC = \frac{1}{2} \times 8 = 4$ cm.

$\therefore$ The perimeter of $\Delta ADE = 5 + 6 + 4 = 15$ cm.

(The req.)

[b] $\because B \in \overline{AC}$

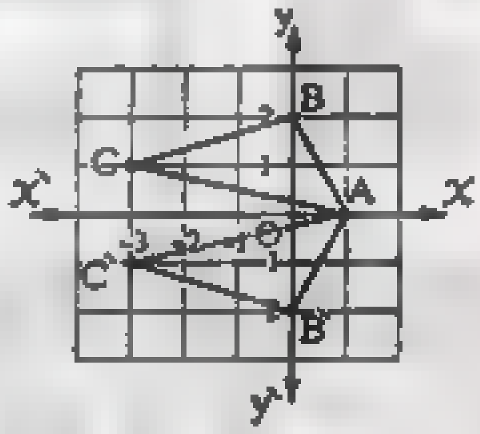
$\therefore m(\angle ABE) = 180^\circ - 106^\circ = 74^\circ$

$\because \overline{BD}$ bisects $\angle ABE$

$\therefore m(\angle ABD) = 74^\circ \div 2 = 37^\circ$ (The req.)

4

[a]



[b] $\because$ ABCD is a rhombus and $\overline{BD}$ is a diagonal in it.

$\therefore m(\angle ABC) = 2\, m(\angle ABD) = 2 \times 62^\circ = 124^\circ$

$\therefore m(\angle A) = 180^\circ - 124^\circ = 56^\circ$ (The req.)

5

[a] $\because m(\angle X) = 90^\circ$

$\therefore (XY)^2 = (YZ)^2 - (XZ)^2 = (20)^2 - (16)^2 = 144$

$\therefore XY = \sqrt{144} = 12$ cm. (The req.)

[b] (1) ΔCON (2) ΔLNB (3) ΔBON

9 El-Sharkia

1

(1) 180 (2) $(AB)^2 , (BC)^2$ (3) the third side
(4) 540 (5) $(4 , 2)$

2

(1) (a) (2) (b) (3) (a) (4) (c) (5) (c)

3

[a] $\because M \in \overleftrightarrow{AB}$

$\therefore m(\angle BME) = 180^\circ - 110^\circ = 70^\circ$

$\because \overrightarrow{MC}$ bisects $\angle BME$

$\therefore m(\angle BMC) = 70^\circ \div 2 = 35^\circ$

$\because \overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

$\therefore m(\angle AMD) = m(\angle BMC) = 35^\circ$ (V.O.A)

(The req.)

[b] $\because \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{DC}$ is a transversal

$\therefore m(\angle C) = m(\angle D) = 30^\circ$ (alternate angles)

$\because \angle DAB$ is an exterior angle of ΔABC

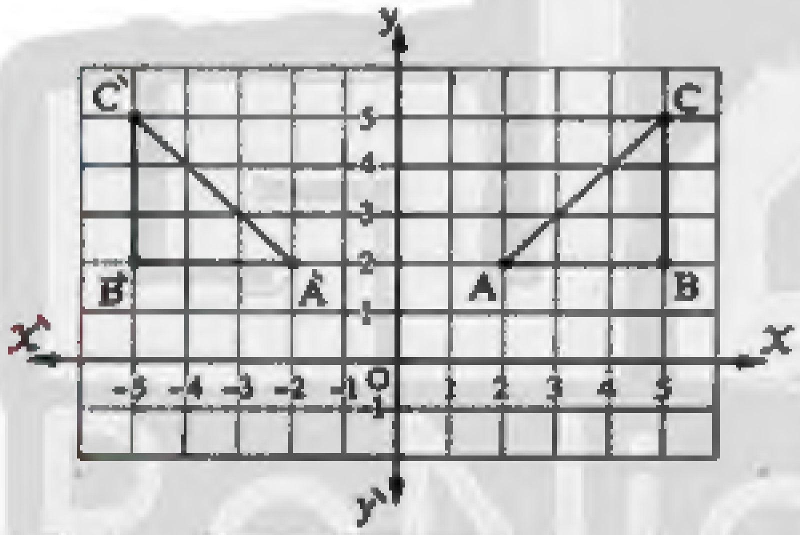
$\therefore m(\angle ABC) = 80^\circ - 30^\circ = 50^\circ$ (The req.)

4

[a] The measure of each interior angle $= \frac{(6-2) \times 180^\circ}{6}$

$= 120^\circ$

[b]



5

In ΔABC :

$\because X$ is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$

$\therefore XY = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$ cm.

$\because X$ is the midpoint of $\overline{AB}$, Z is the midpoint of $\overline{BC}$

$\therefore XZ = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5$ cm.

$\because Y$ is the midpoint of $\overline{AC}$, Z is the midpoint of $\overline{BC}$

$\therefore YZ = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3$ cm.

$\therefore$ The perimeter of $\Delta XYZ = 4 + 5 + 3 = 12$ cm.

(The req.)

10 Suez

1

(1) (b) (2) (c) (3) (a) (4) (c) (5) (c)

2

(1) half the length of the third side (2) $(5 , -1)$

(3) 180° (4) square (5) $(-3 , -2)$

3

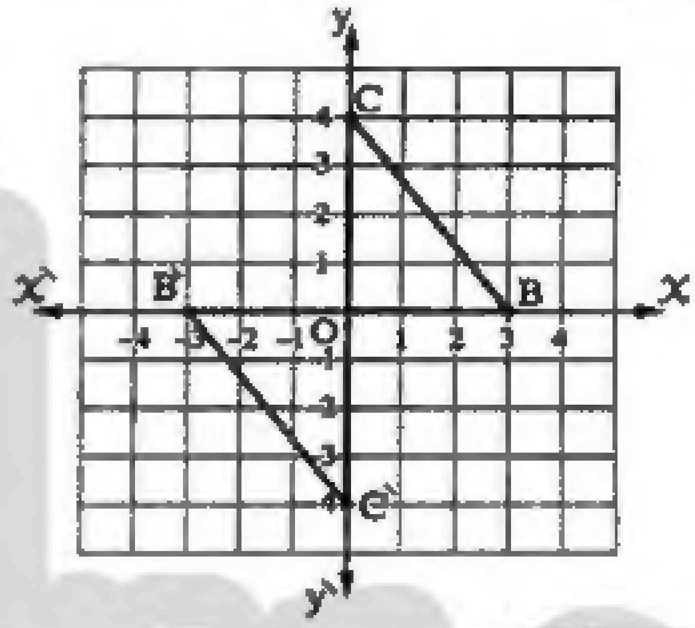
[a] $\because ABCD$ is a parallelogram

$\therefore m(\angle C) = 180^\circ - 135^\circ = 45^\circ$ (First req.)

, The perimeter of parallelogram ABCD

$= (8 + 5) \times 2 = 26$ cm. (Second req.)

[b]



4

[a] In ΔABC :

$\because D$ is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{AC}$

$\therefore DO = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$ cm.

$\because D$ is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$

$\therefore DE = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5$ cm.

$\because O$ is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

$\therefore OE = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3$ cm.

$\therefore$ The perimeter of $\Delta DEO = 4 + 5 + 3 = 12$ cm.

(The req.)

[b] $\because \overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore m(\angle B) = m(\angle D) = 60^\circ$ (alternate angles)

$\because \angle ACD$ is an exterior angle of ΔABC

$\therefore m(\angle A) = 110^\circ - 60^\circ = 50^\circ$ (The req.)

5

[a] In ΔABC : $\because m(\angle B) = 90^\circ$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (12)^2 + (9)^2 = 225$

$\therefore AC = \sqrt{225} = 15$ cm.

In ΔACD : $\because m(\angle ACD) = 90^\circ$

$\therefore (AD)^2 = (AC)^2 + (CD)^2 = (15)^2 + (20)^2 = 625$

$\therefore AD = \sqrt{625} = 25$ cm. (The req.)

[b] (1) ΔLNB (2) ΔDOM

11 El-Beheira

1

(1) (b) (2) (a) (3) (b) (4) (c) (5) (a)

2

(1) parallel to (2) C (3) (3 , 2)
(4) (– 7 , – 3) (5) (– 2 , – 4)

3

[a] $\because B \in \overleftrightarrow{AC}$ $\therefore m(\angle ABE) = 180° - 116° = 64°$
$\because \overrightarrow{BD}$ bisects $\angle ABE$
$\therefore m(\angle ABD) = 64° \div 2 = 32°$ (The req.)

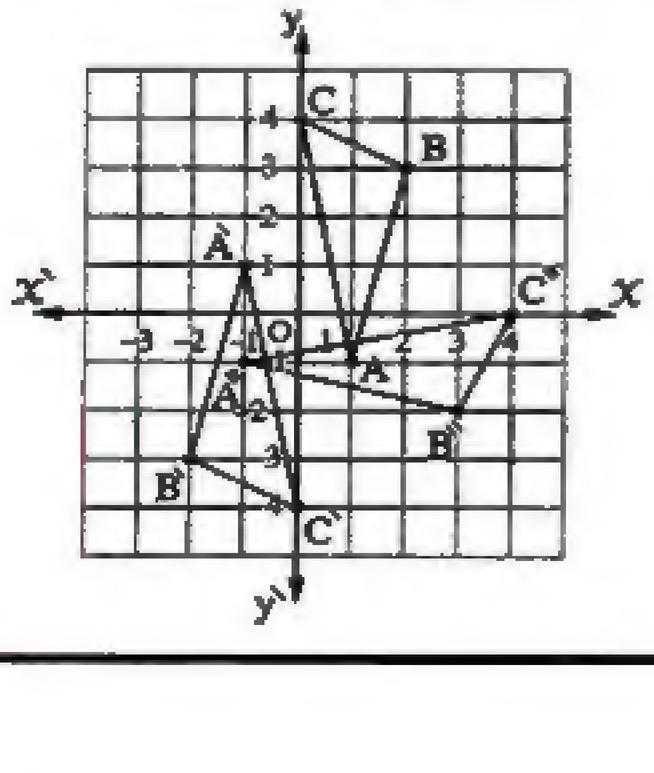
[b] $\because 2X = 720° - (110° + 90° + 165° + 115°) = 240°$
$\therefore X = 240° \div 2 = 120°$ (The req.)

4

[a] In ΔADB : $\because m(\angle ADB) = 90°$
$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$
$\therefore BD = \sqrt{100} = 10$ cm.
In ΔADC : $\because m(\angle ADC) = 90°$
$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$
$\therefore CD = \sqrt{324} = 18$ cm.
$\therefore BC = 10 + 18 = 28$ cm. (The req.)

[b] In ΔABC :
$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$
$\therefore DE = \frac{1}{2} AC = \frac{1}{2} \times 7 = 3.5$ cm.
$\because$ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$
$\therefore DF = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$ cm.
$\because$ E is the midpoint of $\overline{BC}$, F is the midpoint of $\overline{AC}$
$\therefore EF = \frac{1}{2} AB = \frac{1}{2} \times 5 = 2.5$ cm.
$\therefore$ The perimeter of $\Delta DEF = 3.5 + 4 + 2.5 = 10$ cm.
(The req.)

5



12 Beni Suef

1

(1) (c) (2) (b) (3) (b) (4) (b) (5) (c)

2

(1) rhombus (2) parallel to the third side
(3) bisects the third side (4) (– 4 , – 1) (5) 120°

3

[a] The sum $= (6 - 2) \times 180° = 720°$

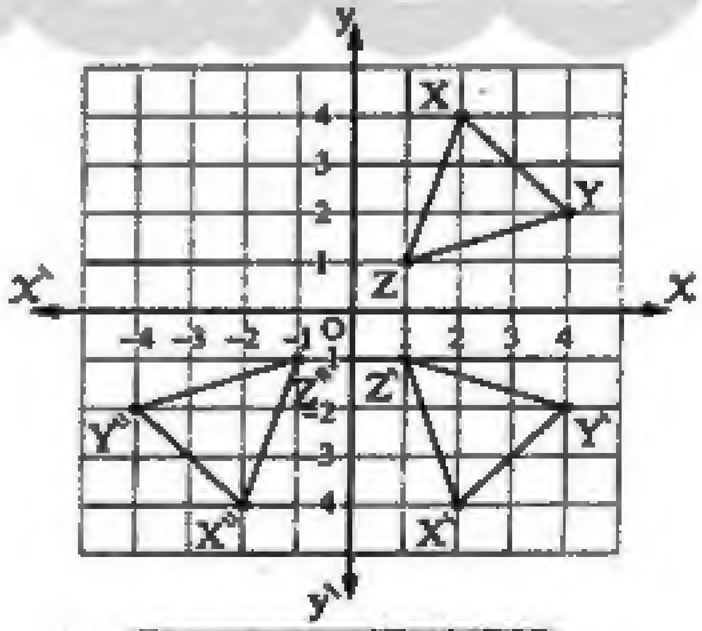
[b] $\because$ ABCD is a rhombus , $\overline{AC}$ is a diagonal in it
$\therefore m(\angle BCD) = 2\, m(\angle ACB) = 2 \times 62° = 124°$
$\therefore m(\angle B) = 180° - 124° = 56°$ (The req.)

4

[a] In ΔABC :
$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$
$\therefore DE = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5$ cm.
$\because$ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$
$\therefore DF = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6$ cm.
, $\because$ F is the midpoint of $\overline{AC}$
$\therefore FC = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5$ cm.
, $\because$ E is the midpoint of $\overline{BC}$
$\therefore EC = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6$ cm.
$\therefore$ The perimeter of DECF $= 5 + 6 + 5 + 6 = 22$ cm.
(The req.)

[b] $\because m(\angle X) = 90°$
$\therefore (XY)^2 = (YZ)^2 - (XZ)^2 = (20)^2 - (16)^2 = 144$
$\therefore XY = \sqrt{144} = 12$ cm. (The req.)

5



13 El-Menia

1

(1) (b) (2) (a) (3) (b) (4) (b) (5) (a)

2

(1) rhombus (2) (8 , 2) (3) 120° (4) 180°
(5) bisects the third side

3

[a] $\because \overrightarrow{DE} \parallel \overrightarrow{BC}$, $\overrightarrow{BD}$ is a transversal

$\therefore m(\angle B) = m(\angle D) = 60°$ (alternate angles)

$\because \angle CAD$ is an exterior angle of ΔABC

$\therefore m(\angle C) = 100° - 60° = 40°$ (The req.)

[b] In ΔABC :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$

$\therefore DE = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5$ cm.

$\because$ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6$ cm.

, $\because$ F is the midpoint of $\overline{AC}$

$\therefore FC = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5$ cm.

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore EC = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6$ cm.

$\therefore$ The perimeter of DECF $= 5 + 6 + 5 + 6 = 22$ cm.

(The req.)

4

[a] $\because m(\angle BAE) + m(\angle EAC) + m(\angle CAB) = 360°$

(accumulative at A)

$\therefore m(\angle CAB) = 360° - (150° + 90°) = 120°$

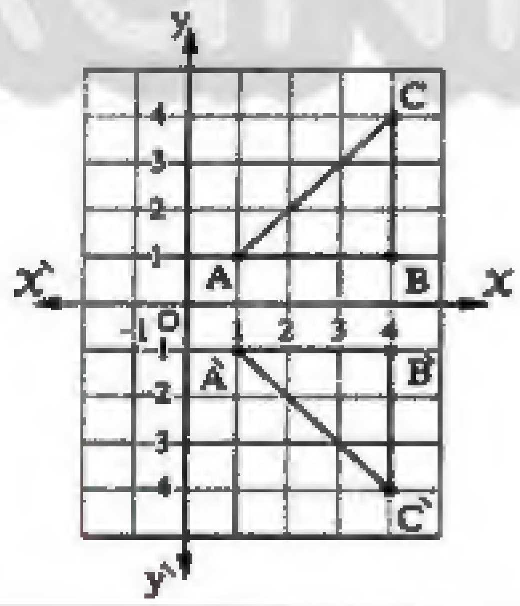
$\because \overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AC}$ is a transversal

$\therefore m(\angle CAB) + m(\angle C) = 180°$

(interior angles in the same side of the transversal)

$\therefore m(\angle C) = 180° - 120° = 60°$ (The req.)

[b]



5

[a] In ΔADC : $\because m(\angle ADC) = 90°$

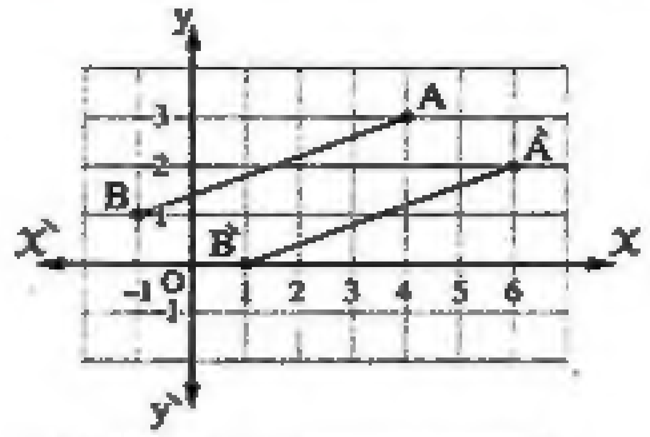
$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$

$\therefore CD = \sqrt{324} = 18$ cm. $\therefore BC = 10 + 18 = 28$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 28 \times 24 = 336$ cm^2.

(The req.)

[b]



14 Aswan

1

(1) (c) (2) (a) (3) (a) (4) (d) (5) (b)

2

(1) (0 , 5) (2) 45° (3) (5 , 4) (4) 360 (5) 120

3

[a] (1) D (2) ΔBYM

[b] In ΔXYZ : $\because m(\angle Y) = 90°$

$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = (3)^2 + (4)^2 = 25$

$\therefore XZ = \sqrt{25} = 5$ cm.

In ΔLXZ : $\because m(\angle LXZ) = 90°$

$\therefore (XL)^2 = (LZ)^2 - (XZ)^2 = (13)^2 - (5)^2 = 144$

$\therefore XL = \sqrt{144} = 12$ cm. (The req.)

4

[a] Let the measures of the angles of the quadrilateral be : $2x$, $3x$, $3x$ and $4x$

$\therefore 2x + 3x + 3x + 4x = 360°$

$\therefore 12x = 360°$ $\therefore x = 30°$

$\therefore$ The measure of the smallest angle is : 60°

[b] $\because \overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AD}$ is a transversal

$\therefore m(\angle D) = m(\angle A) = 50°$ (alternate angles)

$\because \angle AEC$ is an exterior angle of ΔCDE

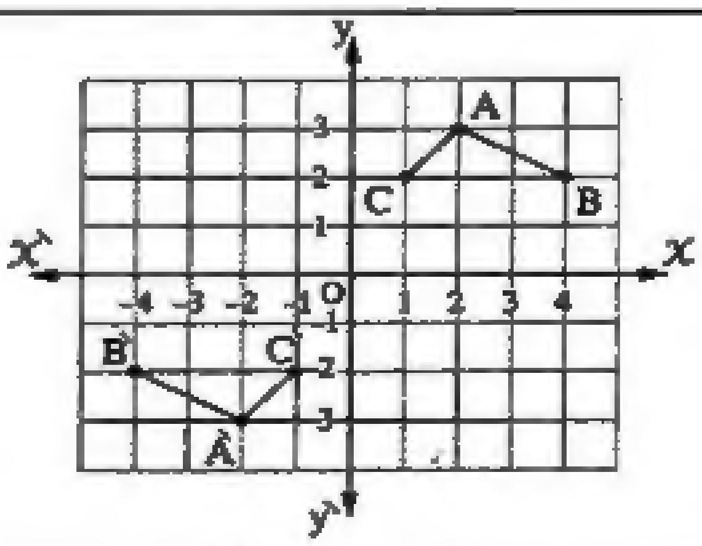
$\therefore m(\angle C) = 110° - 50° = 60°$

In ΔCDE : $\therefore m(\angle CED) = 180° - (60° + 50°) = 70°$

(The req.)

5

[a]



[b] $\because \overrightarrow{DA} // \overrightarrow{CB}$, $\overleftrightarrow{DC}$ is a transversal

$\therefore m(\angle D) + m(\angle C) = 180°$

(interior angles in the same side of the transversal)

$\therefore m(\angle D) = 180° - 60° = 120°$

$\therefore m(\angle D) = m(\angle BAE) = 120°$

but they are corresponding angles

$\therefore \overline{AB} // \overline{CD}$

, $\because \overline{DA} // \overline{CB}$

$\therefore$ ABCD a parallelogram (Q.E.D.)

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1

(1) (a) (2) (b) (3) (c) (4) (d) (5) (d)

2

(1) parallelogram (2) parallel to (3) (– 3 , – 2)

(4) The hypotenuse (5) 90

3

[a] The number of sides $= \frac{360°}{180° - 135°} = 8$ sides

[b] $\because M \in \overline{AC}$

$\therefore m(\angle BMC) = 180° - 75° = 105°$

In Δ BMC :

$\because m(\angle BCM) = 180° - (50° + 105°) = 25°$

$\therefore m(\angle BCA) = m(\angle CAD) = 25°$

but they are alternate angles

$\therefore \overline{AD} // \overline{BC}$

, $\because \overline{AB} // \overline{DC}$

$\therefore$ ABCD is a parallelogram (Q.E.D.)

4

[a] $\because m(\angle X) = 90°$

$\therefore (XZ)^2 = (YZ)^2 - (XY)^2 = (15)^2 - (12)^2 = 81$

$\therefore XZ = \sqrt{81} = 9$ cm. (The req.)

[b] In Δ ABC :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 10 = 5$ cm.

, $\because$ D is the midpoint of $\overline{AB}$

$\therefore AD = \frac{1}{2} AB = \frac{1}{2} \times 12 = 6$ cm.

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore AE = \frac{1}{2} AC = \frac{1}{2} \times 8 = 4$ cm.

$\therefore$ The perimeter of Δ ADE $= 6 + 4 + 5 = 15$ cm.

(The req.)

5

(1) Δ ALO (2) Δ OYC (3) Δ CZO